Resource Economics

Book · October 1999

DOI: 10.2277/0521640121 · Source: RePEc

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CAMBRIDGE
UNIVERSITY PRESS

published by the press syndicate of the university of cambridge
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

cambridge university press
The Edinburgh Building, Cambridge CB2 2RU, UK www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

First published 1999

Printed in the United States of America

Typeface Times Roman 10/12 pt. *System* QuarkXPress [BTS]

A catalog record for this book is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Conrad, Jon M.

Resource economics / Jon M. Conrad.

p. cm.

Includes index.

ISBN 0-521-64012-1. – ISBN 0-521-64974-9 (pbk.)

1. Natural resources – Management – Mathematical models.
2. Resource allocation – Mathematical models. 3. Microsoft Excel
(Computer file) I. Title.
HC59. 15. C656 1999 98–49529
333.7 – dc21 CIP

ISBN 0 521 64012 1 hardback
ISBN 0 521 64974 9 paperback

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CHAPTER 1

Basic Concepts

1.0 Renewable, Nonrenewable, and Environmental Resources

Economics might be defined as the study of how society allocates scarce resources. The field of resource economics would then be the study of how society allocates scarce natural resources such as stocks of fish, stands of trees, fresh water, oil, and other naturally occurring resources. A distinction is sometimes made between resource and environmental economics, where the latter field is concerned with the way wastes are disposed of and the resulting quality of air, water, and soil serving as waste receptors. In addition, environmental economics is concerned with the conservation of natural environments and biodiversity.

Natural resources are often categorized as being renewable or nonrenewable. A renewable resource must display a significant rate of growth or renewal on a relevant economic time scale. An economic time scale is a time interval for which planning and management are meaningful. The notion of an economic time scale can make the classification of natural resources a bit tricky. For example, how should we classify a stand of old-growth coast redwood or an aquifer with an insignificant rate of recharge? Whereas the redwood tree is a plant, and can be grown commercially, old-growth redwoods may be 800 to 1,000 years old, and their remaining stands might be more appropriately viewed as a nonrenewable resource. Whereas the water cycle provides precipitation that will replenish lakes and streams, the water contained in an aquifer with little or no recharge might be more economically similar to a pool of oil (a nonrenewable resource) than to a lake or reservoir that receives significant recharge from rain or melting snow.

A critical question in the allocation of natural resources is "How much of the resource should be harvested (extracted) today?" Finding the "best" allocation of natural resources over time can be regarded as a dynamic optimization problem. In such problems it is common to try to maximize some measure of net economic value, over some future horizon, subject to the dynamics of the harvested resource and any other relevant constraints. The solution to the dynamic optimization of a natural resource would be a schedule or "time path" indicating the optimal amount to be harvested (extracted) in each period. The optimal rate of harvest or extraction in a particular period may be zero. For example, if a fish stock has been historically mismanaged, and the current stock is below what is deemed optimal, then zero harvest (a moratorium on fishing) may be best until the stock recovers to a size at which a positive level of harvest is optimal.

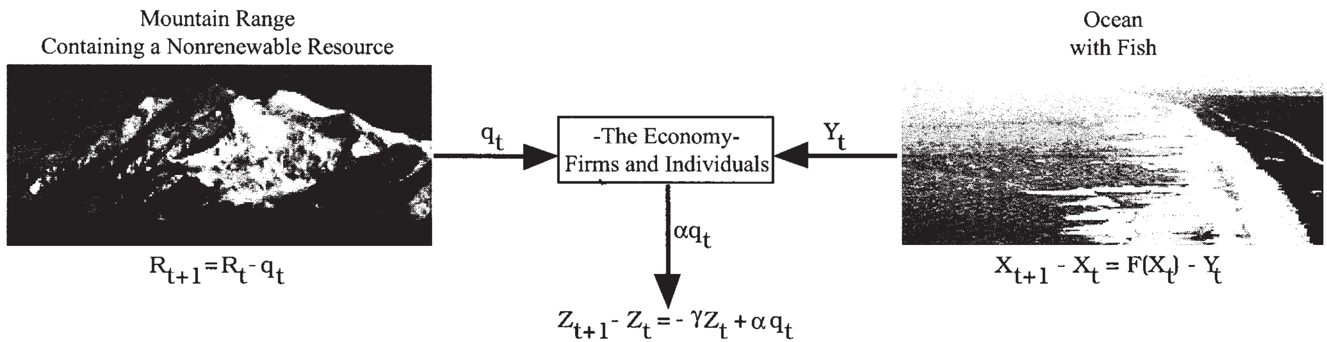
Aspects of natural resource allocation are depicted in Figure 1.1. On the right-hand side (RHS) of this figure we depict an ocean containing a stock of fish. The fish stock at the beginning of period t is denoted by the variable X_t, measured in metric tons. In each period the level of net growth depends on the size of the fish stock and is given by the function $F(X_t)$. We will postpone a detailed discussion of the properties of $F(X_t)$ until Chapter 3. For now, simply assume that if the fish stock is bounded by some "environmental carrying capacity," denoted K, so that $K \geq X_t \geq 0$, then $F(X_t)$ might be increasing as X_t goes from a low level to where $F(X_t)$ reaches a maximum sustainable yield (MSY) at X_{MSY}, and then $F(X_t)$ declines as X_t goes from X_{MSY} to K. Let Y_t denote the rate of harvest, also measured in metric tons, and assume that net growth occurs before harvest. Then, the change in the fish stock, going from period t to period $t + 1$, is the difference $X_{t+1} - X_t$ and is given by the difference equation

$$X_{t+1} - X_t = F(X_t) - Y_t \qquad (1.1)$$

Note, if harvest exceeds net growth $[Y_t > F(X_t)]$, the fish stock declines $(X_{t+1} - X_t < 0)$, and if harvest is less than net growth $[Y_t < F(X_t)]$, the fish stock increases $(X_{t+1} - X_t > 0)$.

During period t, harvest, Y_t, flows to the economy, where it yields a net benefit to various firms and individuals. The stock left in the ocean forms the inventory at the beginning of the next period: i.e., X_{t+1}. This future stock also conveys a benefit to the economy, because it provides the basis for future growth, and it is often the case that larger stocks will lower the cost of future harvest. Thus, implicit in the harvest decision is a balancing of current net benefit from Y_t and future benefit that a slightly larger X_{t+1} would provide the economy.

On the left-hand side (LHS) of Figure 1.1 we show an equation describing the dynamics of a nonrenewable resource. The stock of extractable ore in period t is denoted by R_t and the current rate of extraction by q_t. With no growth or renewal the stock in period $t + 1$ is simply the stock in period t less the amount extracted in period t, so $R_{t+1} = R_t - q_t$. The amount extracted also flows into the economy, where it generates net benefits, but in contrast to harvest from the fish stock, consumption of the nonrenewable resource generates a residual waste, αq_t, propor-



Key

R_t= the stock of the nonrenewable resource in year t

q_t = the rate of production from nonrenewable resource in year t

αq_t = the flow of waste from q_t, $1 > \alpha > 0$

Z_t = the stock of accumulated waste

γZ_t = the rate of waste decomposition, $1 > \gamma > 0$

X_t = stock of fish in year t

Y_t = harvest of fish in year t

$F(X_t)$ = a net growth function

Figure 1.1. Renewable, Nonrenewable, and Environmental Resources

tional to the rate of extraction ($1 > \alpha > 0$). For example, if R_t were a deposit of coal (measured in metric tons) and q_t were the number of tons extracted and burned in period t, then αq_t might be the tons of CO_2 or SO_2 emerging from the smokestacks of utilities or foundries.

This residual waste can accumulate as a stock pollutant, denoted Z_t. If the rate at which the pollutant is generated, αq_t, exceeds the rate at which it is assimilated (or decomposed), $-\gamma Z_t$, the stock pollutant will increase, ($Z_{t+1} - Z_t > 0$), whereas if the rate of generation is less than assimilation, then the stock will decrease. The parameter γ is called the assimilation or degradation coefficient, where $1 > \gamma > 0$. Not shown in Figure 1.1 are the consequences of different levels of Z_t. Presumably there would be some social or external cost imposed on the economy (society). This is sometimes represented through a damage function, $D(Z_t)$. Damage functions will be discussed in greater detail in Chapter 6.

If the economy is represented by the box in Figure 1.1, then the natural environment, surrounding the economy, can be thought of as providing a flow of renewable and nonrenewable resources, and also various media for the disposal of unwanted (negatively valued) wastes. Missing from Figure 1.1, however, is one additional service, usually referred to as *amenity value*. A wilderness, a pristine stretch of beach, or a lake with “swimmable” water quality provides individuals in the economy with places for observation of flora and fauna, relaxation, and recreation that are fundamentally different from comparable services provided at a city zoo, an exclusive beach hotel, or a backyard swimming pool. The amenity value provided by various natural environments may critically depend on the location and rate of resource extraction and waste disposal. Thus, the optimal rates of harvest, extraction, and disposal should take into account any reduction in amenity values. In general, current net benefit from, say, Y_t or q_t, must be balanced with the discounted future costs from reduced resource stocks, X_{t+1} and R_{t+1}, and any reduction in amenity values caused by harvest, extraction, or disposal of associated wastes.

1.1 Discounting

When attempting to determine the optimal allocation of natural resources over time one immediately confronts the issue of “time preference.” Most individuals exhibit a preference for receiving benefits now, as opposed to receiving the same level of benefits at a later date. Such individuals are said to have a positive time preference. In order to induce these individuals to save (thus providing funds for investment), an interest payment or premium, over and above the amount borrowed, must be offered. A society composed of individuals with positive time preferences will typically develop "markets for loanable funds" (capital markets) where the interest rates which emerge are like prices and reflect, in part, society's underlying time preference.

An individual with a positive time preference will discount the value of a note or contract which promises to pay a fixed amount of money at some future date. For example, a bond which promises to pay \$10,000 10 years from now is not worth \$10,000 today in a society of individuals with positive time preferences. Suppose you own such a bond. What could you get for it if you wished to sell it today? The answer will depend on the credit rating (trustworthiness) of the government or corporation promising to make the payment, the expectation of inflation, and the taxes that would be paid on the interest income. Suppose the payment will be made with certainty, there is no expectation of inflation, and there is no tax on earned interest. Then, the bond payment would be discounted by a rate that would approximate society's "pure" rate of time preference. We will denote this rate by the symbol δ, and simply refer to it as the *discount rate*. The risk of default (nonpayment), the expectation of inflation, or the presence of taxes on earned interest would raise private market rates of interest above the discount rate. (Why?)

If the discount rate were 3%, so $\delta = 0.03$, then the "discount factor" is defined as $\rho = 1/(1+\delta) = 1/(1+0.03) \approx 0.97$. The present value of a \$10,000 payment made 10 years from now would be $\$10{,}000/(1+\delta)^{10} = \$10{,}000\rho^{10} \approx \$7{,}441$. This should be the amount of money you would get for your bond if you wished to sell it today. Note that the amount \$7,441 is also the amount you would need to invest at a rate of 3%, compounded annually, to have \$10,000 10 years from now.

The present-value calculation for a single payment can be generalized to a future stream of payments in a straightforward fashion. Let N_t denote a payment made in year t. Suppose these payments are made over the horizon $t = 0, 1, 2, \ldots, T$, where $t = 0$ is the current year (period) and $t = T$ is the last year (or terminal period). The present value of this stream of payments can be calculated by adding up the present value of each individual payment. We can represent this calculation mathematically as

$$N = \sum_{t=0}^{t=T} \rho^t N_t \tag{1.2}$$

Suppose that $N_0 = 0$ and $N_t = A$ for $t = 1, 2, \ldots, \infty$. In this case we have a bond which promises to pay A dollars every year, from next year until the end of time. Such a bond is called a perpetuity, and with $1 > \rho > 0$, when $\delta > 0$, equation (1.2) becomes an infinite geometric progression which converges to $N = A/\delta$. This special result might be used to approximate the value of certain long-lived projects or the decision to preserve a natural environment for all future generations. For example, if a proposed park were estimated to provide $A = \$10$ million in annual net benefits into the indefinite future, it would have a present value of \$500 million at $\delta = 0.02$.

The preceding examples presume that time can be partitioned into discrete periods (for example, years). In some resource allocation problems, it is useful to treat time as a continuous variable, where the future horizon becomes the interval $T \geq t \geq 0$. Recall the formula for compound interest. It says that if A dollars is put in the bank at interest rate δ, and compounded m times over a horizon of length T, then the value at the end of the horizon will be given by

$$V(T) = A(1+\delta/m)^{mT} = A\left[(1+\delta/m)^{m/\delta}\right]^{\delta T} = A\left[(1+1/n)^{n}\right]^{\delta T} \tag{1.3}$$

where $n = m/\delta$. If interest is compounded continuously, both m and n tend to infinity and $[1 + 1/n]^n$ tends to e, the base of the natural logarithm. This implies $V(T) = A\ e^{\delta T}$. Note that $A = V(T)e^{-\delta T}$ becomes the present value of a promise to pay $V(T)$ at $t = T$ (from the perspective of $t = 0$). Thus, the continuous-time discount factor for a payment at instant t is $e^{-\delta t,}$ and the present value of a continuous stream of payments $N(t)$ is calculated as

$$N = \int_0^T N(t)e^{-\delta t}dt \tag{1.4}$$

If $N(t) = A$ (a constant) and if $T \to \infty$, equation (1.4) can be integrated directly to yield $N = A/\delta$, which is interpreted as the present value of an asset which pays A dollars in each and every instant into the indefinite future.

Our discussion of discounting and present value has focused on the mathematics of making present-value calculations. The practice of discounting has an important ethical dimension, particularly with regard to the way resources are harvested over time, the evaluation of investments or policies to protect the environment, and more generally the way the current generation weights the welfare and options of future generations.

In financial markets the practice of discounting might be justified by society's positive time preference and by the economy's need to allocate scarce investment funds to firms which have expected returns that equal or exceed the appropriate rate of discount. To ignore the time preferences of individuals and to replace competitive capital markets by the decisions of some savings/investment czar would likely lead to inefficiencies, a reduction in the output and wealth generated by the economy, and the oppression of what many individuals regard as a fundamental economic right. The commodity prices and interest rates which emerge from competitive markets are highly efficient in allocating resources toward those economic activities which are demanded by the individuals with purchasing power.

Although the efficiency of competitive markets in determining the allocation of labor and capital is widely accepted, there remain questions about discounting and the appropriate rate of discount when allocating natural resources over time or investing in environmental quality. Basically the interest rates that emerge from capital markets reflect society's underlying rate of discount, the riskiness of a particular asset or portfolio, and the prospect of general inflation. These factors, as already noted, tend to raise market rates of interest above the discount rate.

Estimates of the discount rate in the United States have ranged between 2% and 5%. This rate will vary across cultures at a point in time and within a culture over time. A society's discount rate would in theory reflect its collective "sense of immediacy" and its general level of development. A society where time is of the essence or where a large fraction of the populace is on the brink of starvation would presumably have a higher rate of discount.

As we will see in subsequent chapters, higher discount rates tend to favor more rapid depletion of nonrenewable resources and lower stock levels for renewable resources. High discount rates can make investments to improve or protect environmental quality unattractive when compared to alternative investments in the private sector. High rates of discount will greatly reduce the value of harvesting decisions or investments that have a preponderance of their benefits in the distant future. Recall that a single payment of $10,000 in 10 years had a present value of $7,441 at $\delta = 0.03$. If the discount rate increases to $\delta = 0.10$, its present value drops to $3,855. If the payment of $10,000 would not be made until 100 years into the future, it would have a present value of only $520 at $\delta = 0.03$ and the minuscule value of $0.72 (72 cents) if $\delta = 0.10$.

The exponential nature of discounting has the effect of weighting near-term benefits much more heavily than benefits in the distant future. If 75 years were the life span of a single generation, and if that generation had absolute discretion over resource use and a discount rate of $\delta = 0.10$, then the weight attached to the welfare of the next generation would be similarly minuscule. Such a situation could lead the current generation to throw one long, extravagant, resource-depleting party that left subsequent generations with an impoverished inventory of natural resources, a polluted environment, and very few options to change their economic destiny.

There are some who would view the current mélange of resource and environmental problems as being precisely the result of tyrannical and selfish decisions by recent generations. Such a characterization would not be fair or accurate. Although many renewable resources have been mismanaged (such as marine fisheries and tropical rain forest), and various nonrenewable resources may have been depleted too rapidly (oil reserves in the United States), the process, though nonoptimal, has generated both physical and human capital in the form of buildings, a housing stock, highways and public infrastructure, modern agriculture, and the advancement of science and technology. These also benefit and have expanded the choices open to future generations. Further, any single generation is usually closely "linked" to the two generations which preceded it and the two generations which will follow. The current generation has historically made sacrifices in their immediate well-being to provide for parents, children, and grandchildren. Although intergenerational altruism may not be obvious in the functioning of financial markets, it is more obvious in the way we have collectively tried to regulate the use of natural resources and the quality of the environment. Our policies have not always been effective, but their motivation seems to derive from a sincere concern for future generations.

Determining the "best" endowment of human and natural capital to leave future generations is made difficult because we do not know what they will need or want. Some recommend that if we err, we should err on the side of leaving more natural resources and undisturbed natural environments. By saving them now we derive certain amenity benefits and preserve the options to harvest or develop in the future.

The process of discounting, to the extent that it reflects a stable time preference across a succession of generations is probably appropriate when managing natural resources and environmental quality for the maximum benefit of an ongoing society. Improving the well-being of the current generation is a part of an ongoing process seeking to improve the human condition. And when measured in terms of infant mortality, caloric intake, and life expectancy, successive generations have been made better off.

Nothing in the preceeding discussion helps us in determining the precise rate of discount which should be used for a particular natural resource or environmental project. In the analysis in future chapters we will explore the sensitivity of harvest and extraction rates, forest rotations, and rates of waste disposal to different rates of discount. This will enable us to get a numerical feel for the significance of discounting.

1.2 A Discrete-Time Extension of the Method of Lagrange Multipliers

In subsequent chapters we will encounter many problems where we wish to maximize some measure of economic value subject to resource dynamics. Such problems can often be viewed as special cases of a more general dynamic optimization problem. The method of Lagrange multipliers is a technique for solving constrained optimization problems. It is regularly used to solve static allocation problems, but it can be extended to solve dynamic problems as well. We will work through the mathematics of a general problem in this section. In Chapter 2 we will show how numerical problems can be posed and solved using Excel's Solver. Chapters 3–8 will examine how these problems arise when seeking to maximize the net value from renewable and nonrenewable resources, the control of stock pollutants, risky investment, and the selection of activities which might promote sustainable development.

Let X_t denote a physical measure of the size or amount of some resource in period t. In a fishery X_t might represent the number of metric tons of some (commercially valued) species. In a forest it may represent the volume of standing (merchantable) timber.

Let Y_t denote the level of harvest, measured in the same units as X_t. For renewable resources we will frequently assume that resource dynamics can be represented by the first-order difference equation (1.1). In that equation $X_{t+1} - X_t = F(X_t) - Y_t$, where $F(X_t)$ was the net growth function for the resource. It assumed that the net growth from period t to period $t + 1$ was a function of resource abundance in period t. We will assume that the net growth function has continuous first- and second-order derivatives. The current resource stock is represented by the initial condition, X_0, denoting the stock at $t = 0$.

The net benefits from resource abundance and harvest in period t are denoted by π_t and given by the function $\pi_t = \pi(X_t, Y_t)$, which is also assumed to have continuous first- and second-order derivatives. Higher levels of harvest of the resource stock will normally yield higher net benefits. The resource stock, X_t, may enter the net benefit function because a larger stock conveys cost savings during search and harvest, or because an intrinsic value is placed on the resource itself.

It is common practice to compare different harvest strategies, say $Y_{1,t}$ to $Y_{2,t}$, by computing the present value of the net benefits that they produce. Note from equation (1.1) that different harvest strategies will result in different time-paths for the resource stock, X_t. Suppose $Y_{1,t}$ results in $X_{1,t}$ and $Y_{2,t}$ results in $X_{2,t}$, and we wish to calculate present value over the horizon $t = 0, 1, 2, \ldots, T$. As in the previous section we will denote the discount factor by $\rho = 1/(1+\delta)$, where δ is called the periodic rate of discount. In this problem we will assume a constant, time-invariant rate of discount, which implies that the discount factor is also time-invariant. It is not difficult to allow for changes in the discount rate over time. You would, however, need to be able to predict the future values for this rate.

The present value comparison for the preceding two harvest strategies would require a comparison of

$$\sum_{t=0}^{T} \rho^t \pi(X_{1,t}, Y_{1,t}) \quad \text{with} \quad \sum_{t=0}^{T} \rho^t \pi(X_{2,t}, Y_{2,t})$$

In the first summation we are calculating the present value of the harvest schedule $Y_{1,t}$ and the resulting biomass levels, $X_{1,t}$. We would want to know if this summation is greater than, less than, or equal to the present value calculation for the second harvest schedule, $Y_{2,t}$, which results in $X_{2,t}$.

Frequently we will seek the "best" harvest policy: that is, a harvest strategy that maximizes the present value of net benefits. Candidate harvest strategies must also satisfy equation (1.1) describing resource dynamics. Mathematically we wish to find the harvest schedule, Y_t, which will

$$\begin{aligned} &\text{Maximize} \quad \pi = \sum_{t=0}^{T} \rho^t \pi(X_t, Y_t) \\ &\text{Subject to} \quad X_{t+1} - X_t = F(X_t) - Y_t \\ &\qquad\qquad\quad X_0 \text{ Given} \end{aligned}$$

Thus, the objective is to maximize π, the present value of net benefits, subject to the equation describing resource dynamics and the initial condition, X_0.

There are likely to be an infinite number of feasible harvest strategies. How can we find the optimal Y_t? Will it be unique? If $T \to \infty$, will it ever be the case, after some transition period, that the level of harvest and the resource stock are unchanging through time and the system attains a "steady state"? These are difficult but important questions. Let's take them one at a time.

Recall from calculus that when seeking the extremum (maximum, minimum, or inflection point) of a single variable function, a necessary condition requires that the first derivative of the function, when evaluated at a candidate extremum, be equal to zero. Our optimization problem is more complex because we have to determine the $T+1$ values for Y_t which maximize π, and we have constraints in the form of our first-order difference equation and the initial condition X_0. We can, however, follow a similar procedure after forming the appropriate Lagrangian expression for our problem. This is done by introducing a set of new variables, denoted λ_t, called Lagrange multipliers. In general, every variable defined by a difference equation will have an associated Lagrange multiplier. This means that X_t will be associated with λ_t, X_{t+1} will be associated with λ_{t+1}, and so on. It will turn out that the new variables, λ_t, will have an important economic interpretation. They are also called "shadow prices" because their value indicates the marginal value of an incremental increase in X_t in period t.

We form the Lagrangian expression by writing the difference equation in implicit form, $X_t + F(X_t) - Y_t - X_{t+1} = 0$, premultiplying it by $\rho^{t+1}\lambda_{t+1,}$ and then adding all such products to the objective function. The Lagrangian expression for our problem takes the form

$$L = \sum_{t=0}^{T} \rho^t \{\pi(X_t, Y_t) + \rho\lambda_{t+1}[X_t + F(X_t) - Y_t - X_{t+1}]\} \quad (1.5)$$

The rationale behind writing the Lagrangian this way is as follows: Since the Lagrange multipliers are interpreted as shadow prices which measure the value of an additional unit of the resource, we can think of the difference equation, written implicitly, as defining the level of X_{t+1} that will be available in period $t + 1$. The value of an additional (marginal) unit of X_{t+1} in period $t + 1$ is λ_{t+1}. This value is discounted one period, by ρ, to put it on the same present-value basis as the net benefits in period t. Thus, the expression in the curly brackets, $\{\bullet\}$, is the sum of net benefits in period t and the discounted value of the resource stock (biomass) in period $t + 1$. This sum is then discounted back to the present by ρ^t and similar expressions are summed over all periods.

After forming the Lagrangian expression we proceed to take a series of first-order partial derivatives and set them equal to zero. Collectively they define the first-order necessary conditions, analogous to the first-order condition for a single-variable function. They will be used in solving for the optimal levels of Y_t, X_t, and λ_t in transition and, if $T \to \infty$, at a steady state, if one exists. For our problem the necessary conditions require

$$\frac{\partial L}{\partial Y_t} = \rho^t \{\partial\pi(\bullet)/\partial Y_t - \rho\lambda_{t+1}\} = 0 \quad (1.6)$$

$$\frac{\partial L}{\partial X_t} = \rho^t \{\partial\pi(\bullet)/\partial X_t + \rho\lambda_{t+1}[1 + F'(\bullet)]\} - \rho^t\lambda_t = 0 \quad (1.7)$$

$$\frac{\partial L}{\partial[\rho\lambda_{t+1}]} = \rho^t \{X_t + F(X_t) - Y_t - X_{t+1}\} = 0 \quad (1.8)$$

The partial of the Lagrangian with respect to X_t may seem a bit puzzling. When we examine the Lagrangian and the representative term in period t, we observe X_t as an argument of the net benefit function $\pi(X_t, Y_t)$, by itself, and as the sole argument in the net growth function, $F(X_t)$. These partials appear in the brackets $\{\bullet\}$ in equation (1.7). Where did the last term, $-\rho^t\lambda_t$, come from? If we think of the Lagrangian as a long sum of expressions, and if we wish to take the partial with respect to X_t, we need to find all the terms involving X_t. When we back up one period, from t to $t-1$, most of the terms are subscripted $t-1$, with the notable exception of the last term, which becomes $-\rho^t\lambda_t X_t$, with partial derivative $-\rho^t\lambda_t$.

In addition to equations (1.6)–(1.8), which hold for $t = 0, 1, \ldots, \mathrm{T}$, there are two boundary conditions. The first is simply the initial condition that X_0 is known and given. To make things more concrete, suppose $X_0 = A$, where A is a known, positive constant. The second boundary condition for this problem is a condition on λ_{T+1}. Recall that the Lagrange multipliers were to be interpreted as shadow prices. Thus, λ_{T+1} would be the marginal value of one more unit of X_{T+1}. Let's suppose we are free to choose λ_{T+1} as some nonnegative number B, so that $\lambda_{T+1} = B \geq 0$. Then, along with $X_0 = A$ and $\lambda_{T+1} = B$, equations (1.6)–(1.8) can be thought of as a system of $(3T + 5)$ equations in $(3T + 5)$ unknowns. The unknowns are the optimal values for Y_t, $t = 0, 1, \ldots, T$, X_t, $t = 0, 1, \ldots, T + 1$, and λ_t, $t = 0, 1, \ldots, T + 1$.

Equations (1.6)–(1.8) are likely to be nonlinear; this means there could be more than one solution. It is also possible that there could be no solution in the sense that there is no set of values Y_t, X_t, λ_t which simultaneously solve (1.6)–(1.8) and the boundary conditions. It is possible to impose some curvature assumptions on $\pi(\bullet)$ and $F(\bullet)$ which will guarantee a unique solution for $A > 0$ and $B \geq 0$. The details of these conditions are a bit technical and need not concern us here. Of concern is the economic interpretation of equations (1.6)–(1.8).

We can simplify and rewrite the first-order conditions to facilitate their interpretation.

$$\partial\pi(\bullet)/\partial Y_t = \rho\lambda_{t+1} \tag{1.9}$$

$$\lambda_t = \partial\pi(\bullet)/\partial X_t + \rho\lambda_{t+1}[1 + F'(\bullet)] \tag{1.10}$$

$$X_{t+1} = X_t + F(X_t) - Y_t \tag{1.11}$$

The LHS of equation (1.9) is the marginal net benefit of an additional unit of the resource harvested in period t. For a harvest strategy to be optimal this marginal net benefit must equal the opportunity cost, also called *user cost*. User cost is represented by the term $\rho\lambda_{t+1}$, equal to the discounted value of an additional unit of the resource in period $t + 1$.

Thus equation (1.9) requires that we account for two types of costs, the standard marginal cost of harvest in the current period (which has already been accounted for in $\partial\pi(\bullet)/\partial Y_t$) and the future cost that results from the decision to harvest an additional unit of the resource today, which is $\rho\lambda_{t+1}$. In some problems we may see this condition written $p = \partial C(\bullet)/\partial Y_t + \rho\lambda_{t+1}$, implying that price today should equal marginal cost ($\partial C(\bullet)/\partial Y_t$) plus user cost, $\rho\lambda_{t+1}$,

On the LHS of equation (1.10) we have λ_t, the value of an additional unit of the resource, in situ, in period t. When a resource is optimally managed, the marginal value of an additional unit of the resource in period t equals the current period marginal net benefit, $\partial\pi(\bullet)/\partial X_t$, plus the marginal benefit that an unharvested unit will convey in the next period, $\rho\lambda_{t+1}[1 + F'(\bullet)]$. Note that this last term is the discounted value of the marginal unit itself plus its marginal growth.

Equation (1.11) is simply a rewrite of equation (1.1), but now obtained from the partial of the Lagrangian with respect to $\rho\lambda_{t+1}$. This should occur in general: that is, the partial of the Lagrangian with respect to a discounted multiplier should yield the difference equation for the associated state variable, in this case the resource stock.

What if $T \to \infty$? In this case we have an infinite-horizon problem. Equations (1.6)–(1.8) become an infinitely large system of equations in an infinite number of unknowns, a potentially daunting problem. Under certain conditions such problems will have a transitional period, say for $\tau \geq t \geq 0$, where Y_t, X_t, and λ_t are changing, followed by a period $\infty > t > \tau$, where Y_t, X_t, and λ_t are unchanging. In this infinitely long latter period the variables or "system" is said to have reached a steady state because $X_{t+1} = X_t = X^*$, $Y_{t+1} = Y_t = Y^*$, and $\lambda_{t+1} = \lambda_t = \lambda^*$. The triple $[X^*, Y^*, \lambda^*]$ is called a steady-state optimum.

It is often possible to solve for the steady-state optimum by evaluating the first-order necessary conditions when X_t, Y_t, and λ_t are unchanging. In steady state we can dispense with all the time subscripts in equations (1.6)–(1.8), which simply become three equations in three unknowns, X^*, Y^*, and λ^*, and may be written as

$$\rho\lambda = \partial\pi(\bullet)/\partial Y \tag{1.12}$$

$$\rho\lambda[1 + F'(X) - (1+\delta)] = -\partial\pi(\bullet)/\partial X \tag{1.13}$$

$$Y = F(X) \tag{1.14}$$

Equation (1.13) requires a little bit of algebra and use of the definition $\rho = 1/(1 + \delta)$. It can be further manipulated to yield

$$-\rho\lambda[\delta - F'(X)] = -\partial\pi(\bullet)/\partial X \tag{1.15}$$

Multiplying both sides by −1, substituting (1.12) into (1.15), and isolating δ on the RHS yields

$$F'(X)+\frac{\partial\pi(\bullet)/\partial X}{\partial\pi(\bullet)/\partial Y}=\delta \tag{1.16}$$

Equation (1.16) has been called the "fundamental equation of renewable resources." Along with equation (1.14) it will define the optimal steady-state values for X and Y.

Equation (1.16) has an interesting economic interpretation. On the LHS, the term $F'(X)$ my be interpreted as the marginal net growth rate. The second term, called the "marginal stock effect," measures the marginal value of the stock relative to the marginal value of harvest. The two terms on the LHS sum to what might be interpreted as the resource's internal rate of return. Equation (1.16) thus requires that the optimal steady-state values of X and Y cause the resource's internal rate of return to equal the rate of discount, δ, which presumably equals the rate of return on investments elsewhere in the economy. From this capital-theoretic point of view, the renewable resource is viewed as an asset, which under optimal management will yield a rate of return comparable to that of other capital assets. Are all renewable resources capable of yielding an internal rate of return equal to the rate of discount? We will revisit this question in Chapter 3.

Equation (1.14) results when equation (1.1) is evaluated at steady state. It has an obvious and compelling logic. At the bioeconomic optimum, and in fact at any sustainable equilibrium, harvest must equal net growth. If this were not the case, if net growth exceeded harvest or if harvest exceeded net growth, the resource stock would be changing and we could not, by definition, be at a steady-state equilibrium. Thus $Y = F(X)$ at any sustainable equilibrium, including the bioeconomic optimum.

Equation (1.16), by the implicit function theorem, will imply a curve in $X - Y$ space. Under a plausible set of curvature assumptions for $F(X)$ and $\pi(X,Y)$, the slope of this curve will be positive. Its exact shape and placement in $X - Y$ space will depend on all the bioeconomic parameters in the functions $F(X)$ and $\pi(X,Y)$, and on the discount rate δ.

Several possible curves (for different underlying parameters) are labeled ϕ_1, ϕ_2, and ϕ_3 in Figure 1.2. The net growth function is assumed to take a logistic form where $Y = F(X) = rX(1 - X/K)$. The intersection of $F(X)$ and a particular $\phi(X)$ would represent the solution of equations

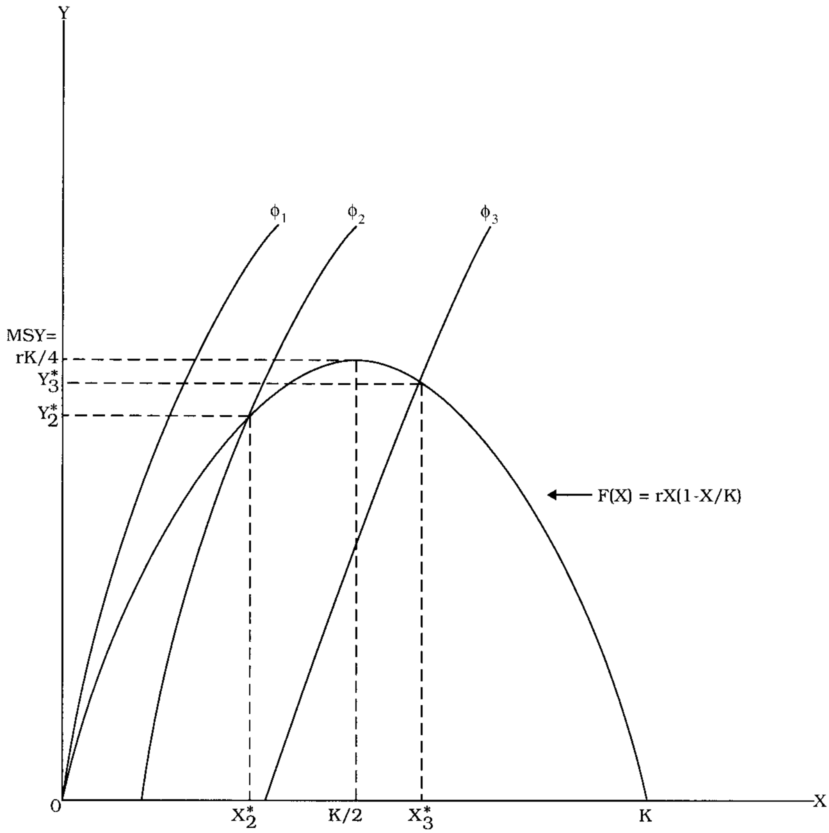


Figure 1.2. Maximum Sustainable Yield (MSY) and Three Bioeconomic Optima

(1.14) and (1.16) and therefore depict the steady-state bioeconomic optimum.

Figure 1.2 shows four equilibria: three bioeconomic optima and maximum sustainable yield (MSY). The bioeconomic optimum at the intersection of ϕ_1 and $F(X)$ would imply that extinction is optimal! Such an equilibrium might result if a slow-growing resource were confronted by a high rate of discount and if harvesting costs for the last members of the species were less than their market price.

The intersection of $F(X)$ and ϕ_2 implies an optimal resource stock of X^*_2, positive, but less than $K/2$, which supports $MSY = rK/4$. This would be the case if the marginal stock effect is less than the discount rate. (Look at equation [1.16] and see if you can figure out why this is true.)

The curve ϕ_3 implies a large marginal stock effect, greater in magnitude than the discount rate, δ. This would occur if smaller fishable stocks significantly increased cost. In such a case it is optimal to maintain a large stock at the bioeconomic optimum, even greater than the maximum sustainable yield stock, $K/2$. The conclusion to be drawn from Figure 1.2 is that the optimal stock, from a bioeconomic perspective, may be less than or greater than the stock level supporting maximum sustainable yield. Its precise location will depend on the forms for $\pi(X,Y)$ and $F(X)$ and the relevant bioeconomic parameters.

In our discussion of the infinite-horizon problem we mentioned that for certain problems the dynamics of the system has two stages, a transitional stage, where the variables are changing, and a steady state, where the variables are unchanging. Equations (1.14) and (1.16), when plotted in $X - Y$ space, would define the steady-state values X^* and Y^*. A possible transition (approach) to X^* from $X_0 < X^*$ is shown in Figure 1.3. This might be the approach and steady state in a single-species fishery where open access or mismanagement allowed the stock to be overfished to a suboptimal level. By restricting harvest to a level less than net growth $[Y_t < F(X_t)]$, the fish stock would grow, reaching X^* at $t = \tau$.

Although the general problem has the virtue of providing some broad and important insights into resource management from an economic perspective, its presentation has been tedious and abstract. In the next chapter we will solve some numerical problems using Excel's Solver. These numerical problems, and the problems found elsewhere in this book, will, it is hoped, make the basic concepts and the economic approach introduced in this chapter more operational, and thus more meaningful.

1.3 Questions and Exercises

Q1.1 What is the central subject in the field of resource economics?

Q1.2 What is the economic distinction between renewable and nonrenewable resources?

Q1.3 What is meant by the term user cost? If user cost increases, what happens to the level of harvest or extraction today?

E1.1 Suppose the dynamics of a fish stock are given by the difference equation (written in "iterative" form) $X_{t+1} = X_t + rX_t(1 - X_t/K) - Y_t$, where

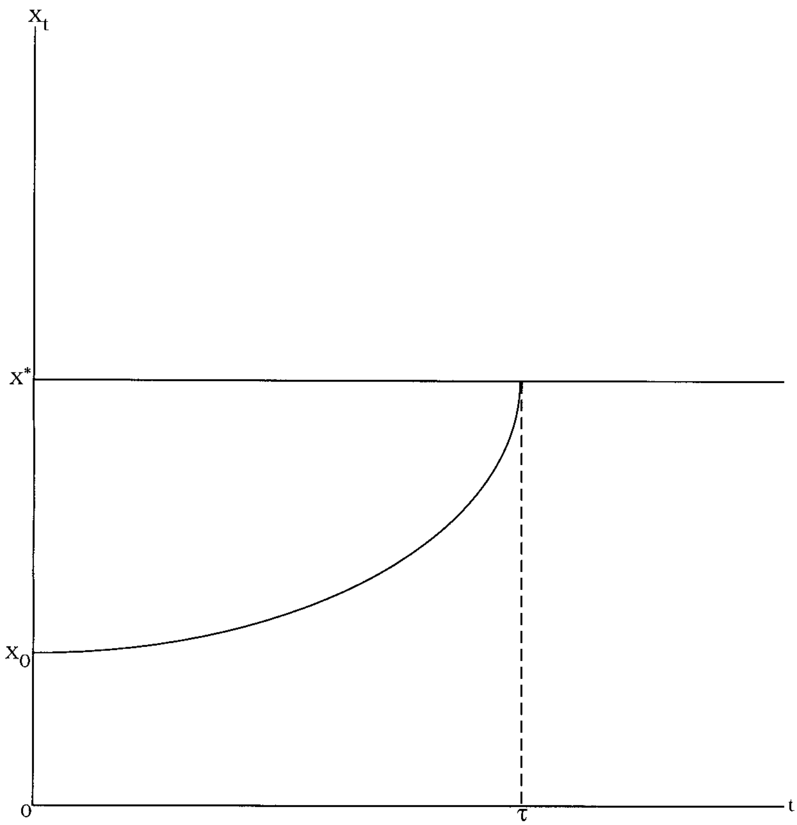


Figure 1.3. An Approach to the Steady-State Optimum X^*

$X_0 = 0.1$, $r = 0.5$, and $K = 1$. Management authorities regard the stock as being dangerously depleted and have imposed a 10-year moratorium on harvesting ($Y_t = 0$ for $t = 0, 1, 2, \ldots, 9$). What happens to X_t during the moratorium? Plot the time path for X_t ($t = 0, 1, 2, \ldots, 9$) in $t - X$ space. (Hint: Set up an Excel Spreadsheet.)

E1.2 After the moratorium the management authorities are planning to allow fishing for 10 years at a harvest rate of $Y_t = 0.125$ (for $t = 10, 11, \ldots, 19$). Suppose the net benefit from harvest is given by $\pi_t = pY_t - cY_t/X_t$, where $p = 2$, $c = 0.5$, and $\delta = 0.05$. What is the present value of net benefits of the 10-year moratorium followed by 10 years of fishing at $Y_t = 0.125$? (Hint: Modify the Excel Spreadsheet of **E1.1**.)

E1.3 In steady state the fishery will yield $Y = F(X) = rX(1 - X/K)$ and an annual net benefit of $\pi(X, Y) = (p - c/X)Y$. Take the derivatives $F'(X)$, $\partial\pi(\bullet)/\partial X$, and $\partial\pi(\bullet)/\partial Y$, substitute them and $Y = rX(1 - X/K)$ into equation (1.16), and simplify the LHS. We will make use of the result in Chapter 3.