Chapter 5: Project Scheduling Models

Network Models for project Scheduling

Introduction

- *Network models consists of a set of circles, or nodes, and lines, which are referred to as either arcs or branches, that connect some nodes to other nodes*
- ***Networks** are important tools of project management*
- *Not only can networks be used to model a wide variety of problems, they can often solved more easily than other models of the same problem, and they present models in a **visual format***

NETWORK TECHNIQUES

PERT

CPM

***PERT*(Program Evaluation and Review Technique)**

- Developed by the US Navy with Booz Hamilton Lockheed on the Polaris Missile/Submarine program 1958

***CPM* (Critical Path Method)**

-Developed by El Du Pont for Chemical Plant Shutdown Project- about same time as PERT

- ✓ ***Similarity***: *Both use same calculations, almost similar*
- ✓ ***Difference***: *Main difference is* ***probabilistic and deterministic in time estimation***

PERT and CPM

- *PERT* and *CPM* are the two most ***widely used techniques for planning and coordinating large-scale projects***
- *By using PERT and CPM, managers are able to obtain:*

1. *A graphical display of project activities*
2. *An estimate of how long the project will take*
3. *An indication of which activities are the **most critical** to timely completion of the project*
4. *An indication of how long any **activity can be delayed** with out lengthening the project*

PERT and CPM

- ***PERT** and **CPM** are best applied in **Project Scheduling***
- *"A project is a series of activities directed to accomplishment of a desired objective"*
- *"Schedule converts action plan into operating time table"*

CPM

In **CPM** activities are shown as a **network of precedence relationships** using activity-on-node network construction

– Single estimate of activity time

– Deterministic activity times

USED IN : Production management - for the jobs of repetitive in nature where the activity time estimates can be predicted with considerable certainty due to the existence of past experience.

PERT

*In **PERT** activities are shown as a network of precedence relationships using activity-on-arrow network construction*

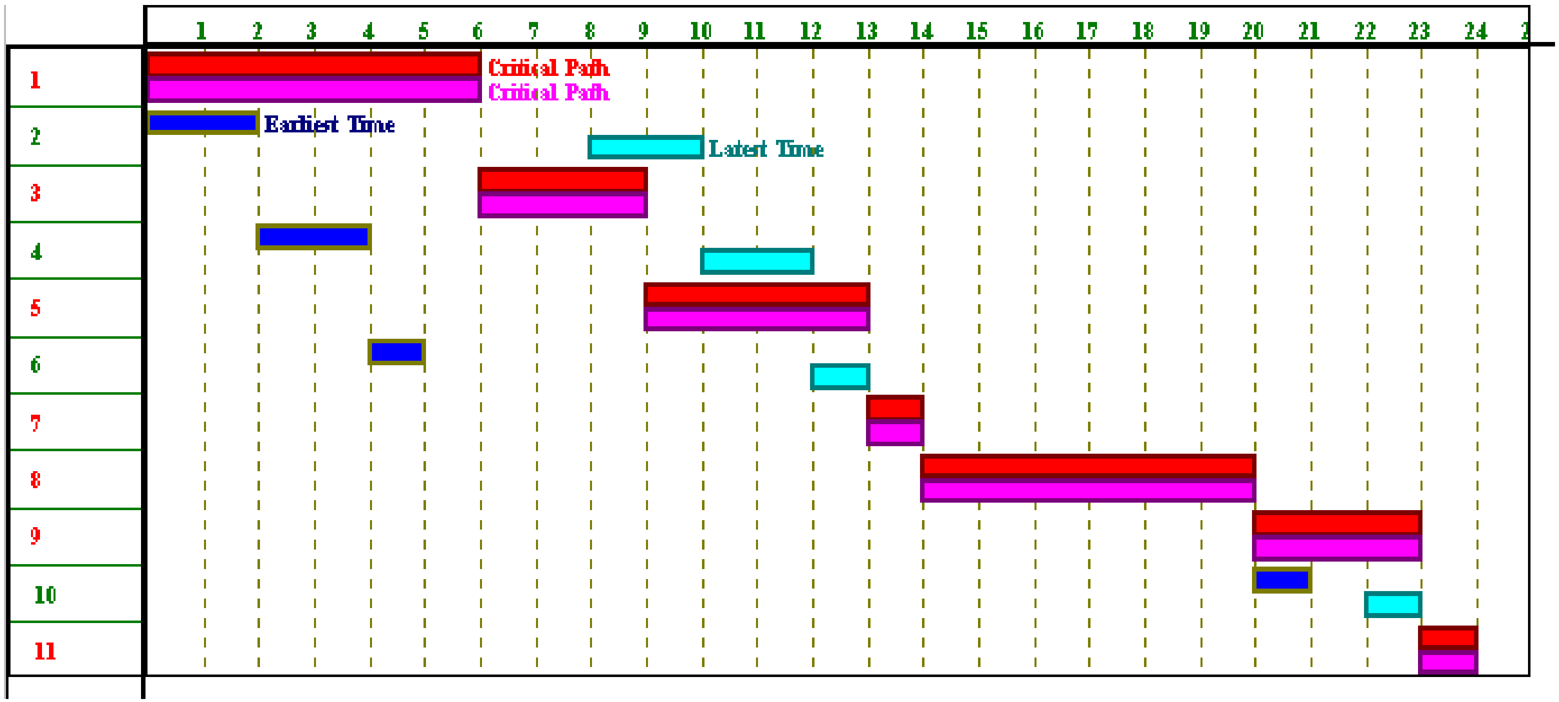
– *Probabilistic activity times*

❖ ***Used in : Project management -*** *for non-repetitive jobs (research and development work), where the time and cost estimates tend to be quite uncertain. This technique uses probabilistic time estimates.*

Gantt Chart

- *The Gantt Chart is a popular tool for planning and scheduling simple projects*
- *It enables managers to initially schedule project activities and, then, to monitor progress over time by comparing planned progress to actual progress*
- *Even though Gantt Chart is simple to use, it may delay the project completion time as activities could not start until the preceding activity was completed.*

Gantt Chart



Originated by H.L.Gantt in 1918

Advantages

- *Gantt charts are quite commonly used.*
- *They provide an easy graphical representation of when activities (might) take place.*

Limitations

- *Do not clearly indicate details regarding the progress of activities*
- *Do not give a clear indication of interrelationship between the separate activities*

- ***Some objectives of project scheduling include:***
  - *Completing the project as early as possible by determining an earliest start and finish time for each of the activities*
  - *Determining the likelihood a project that will be completed within a certain time period*
  - *Finding a minimum cost schedule that completes the project by a certain date*
  - *Finding a minimum time to complete a project within budget restrictions*
  - *Investigating the results of possible delays in one or more of an activity's completion time*
  - *Evaluating the costs and benefits of reducing the time of performing one or more of the activities*

Example of Simple Network – Survey

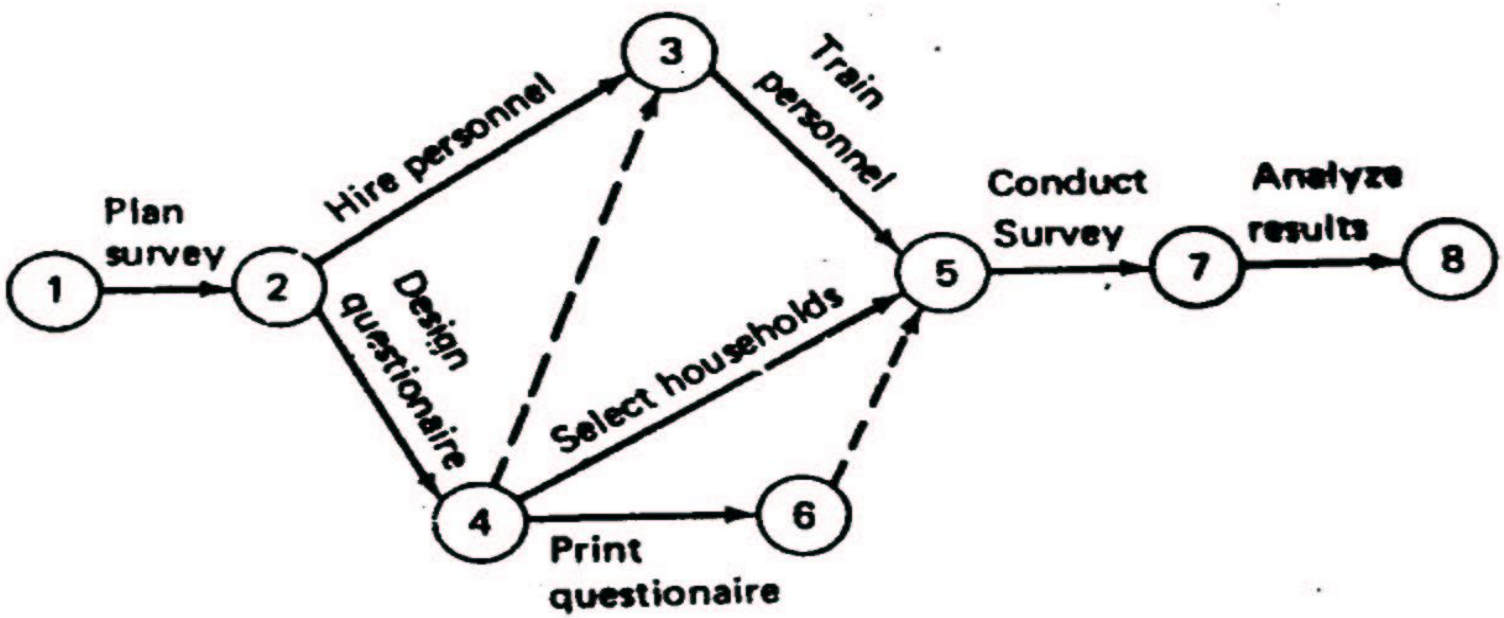


Figure 2-18

Example of Network – More Complex

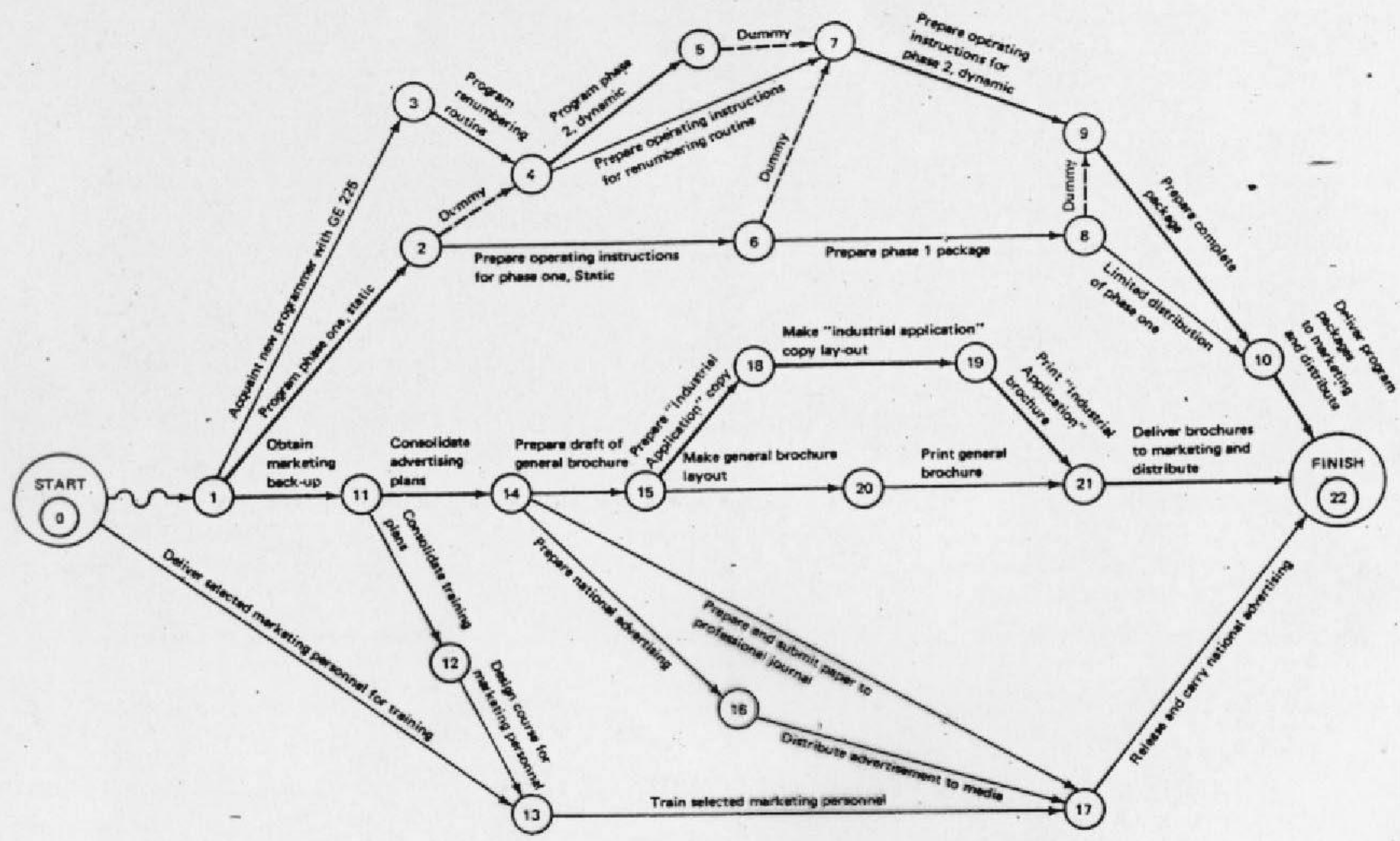
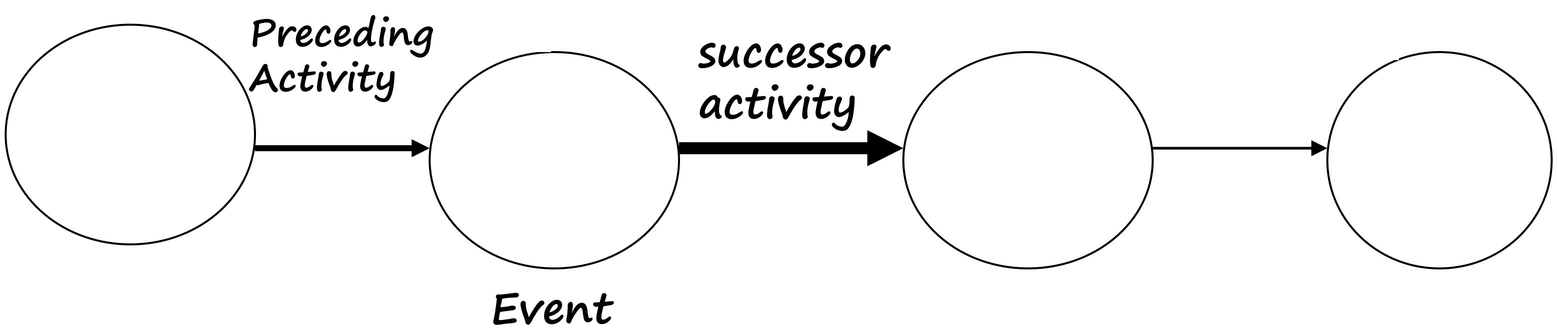


Figure 2-19 Network for the development and marketing of a new computer program.

Definition of terms in a network

- *Activity* : any portions of project (tasks) which required by project, uses up resource and consumes time – may involve labor, paper work, contractual negotiations, machinery operations
- *Event* : beginning or ending points of one or more activities, instantaneous point in time, also called 'nodes'
- *Network* : Combination of all project activities and the events



Emphasis on Logic in Network Construction

- *Construction of network should be based on logical or technical dependencies among activities*
- *Example - before activity 'Approve Drawing' can be started the activity 'Prepare Drawing' must be completed*
- *Common error – build network on the basis of time logic (a feeling for proper sequence) see example below*

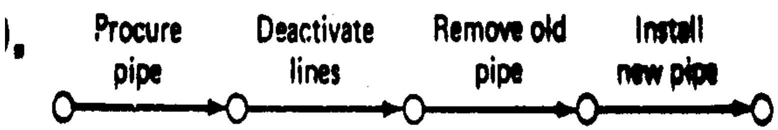
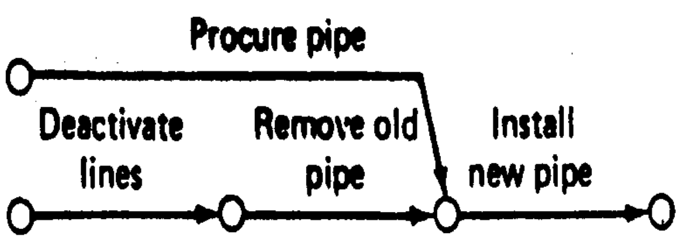


Figure 2-5a

WRONG X



CORRECT ✓

Figure 2-5b

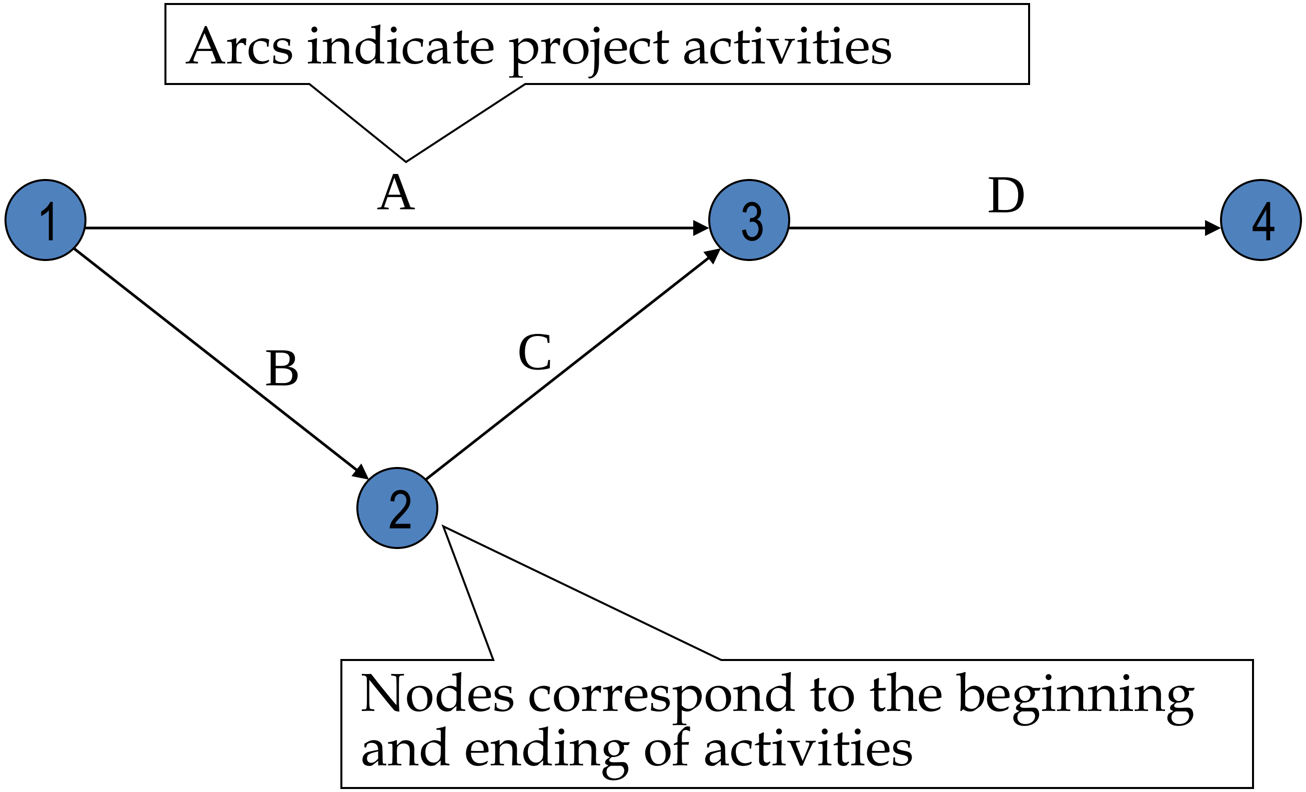
Example 1- A simple network

Consider the list of four activities for making a simple product:

Activity	Description	Immediate predecessors
A	Buy Plastic Body	-
B	Design Component	-
C	Make Component	B
D	Assemble product	A,C

Immediate predecessors *for a particular activity are the activities that, when completed, enable the start of the activity in question.*

Network of Four Activities



The above graphical representation is referred to as the PERT/CPM network

Sequence of activities

- *One can start work on activities A and B anytime, since neither of these activities depends upon the completion of prior activities.*
- *Activity C cannot be started until activity B has been completed*
- *Activity D cannot be started until both activities A and C have been completed.*

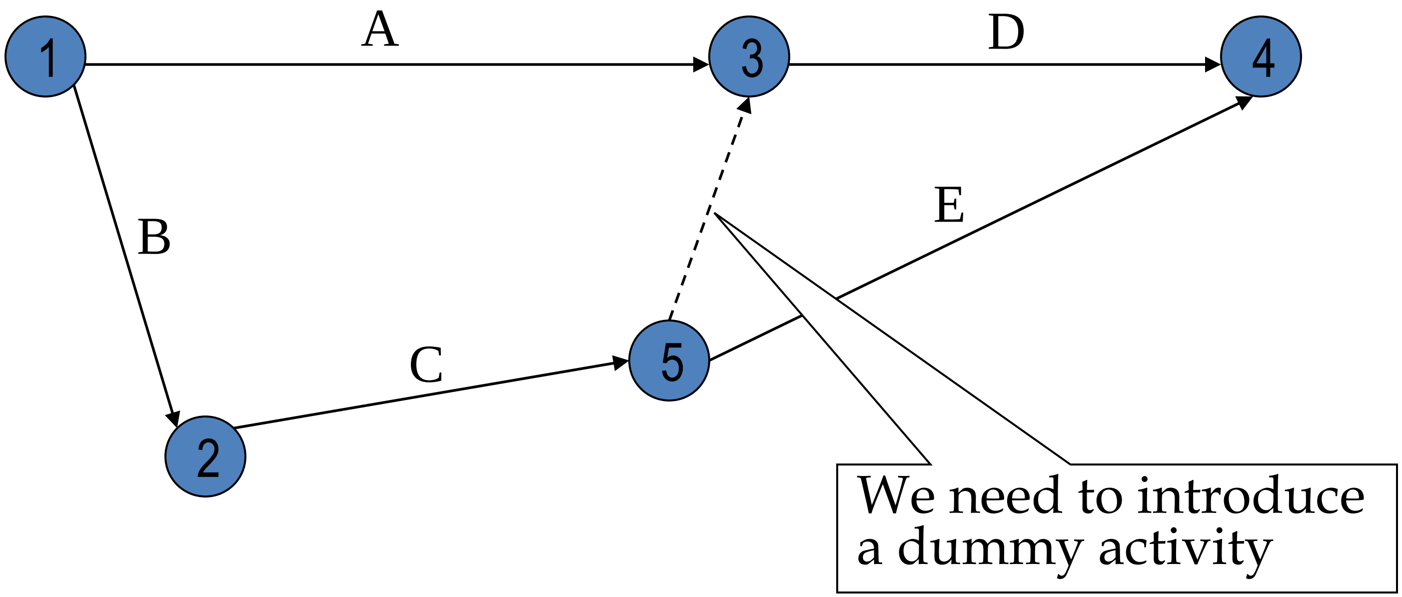
Example 2

Develop the network for a project with following activities and immediate predecessors:

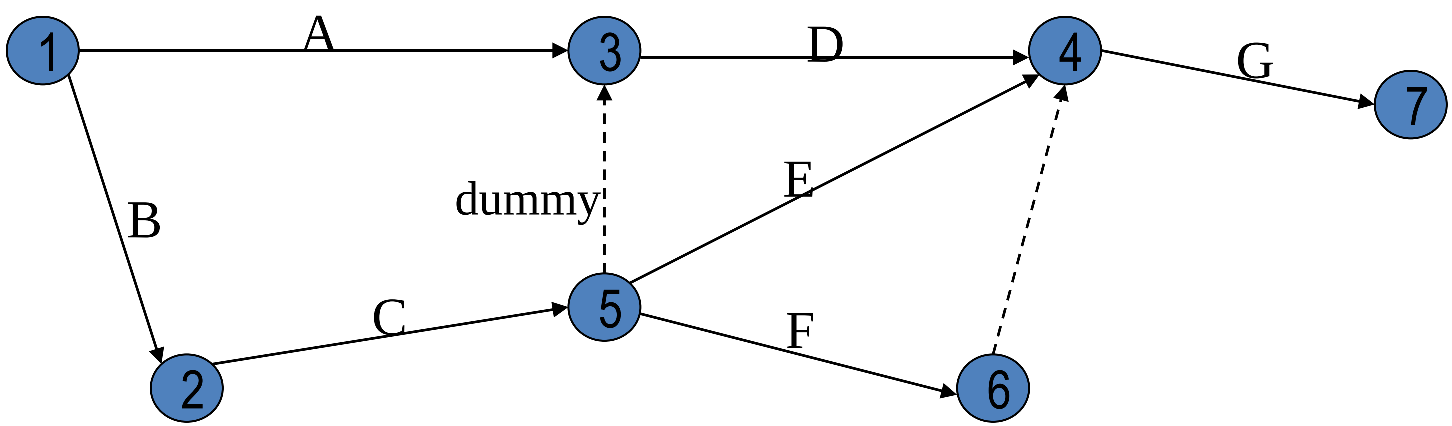
Activity	**Immediate predecessors**
A	-
B	-
C	B
D	A, C
E	C
F	C
G	D,E,F

Class Activity: Try to do network for the first five (A,B,C,D,E) activities

Network of first five activities



Network of all the Seven Activities



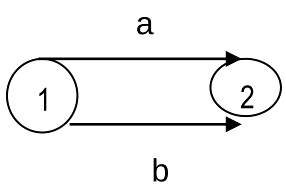
- *Note how the network correctly identifies D, E, and F as the immediate predecessors for activity G.*

- ***Dummy activities*** *is used to identify precedence relationships correctly and to eliminate possible confusion of two or more activities having the same starting and ending nodes*

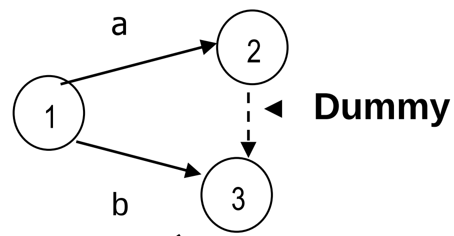
- *Dummy activities have no resources (time, labor, machinery, etc) – purpose is to preserve logic of the network*

Examples of the use of dummy activity

Network concurrent activities

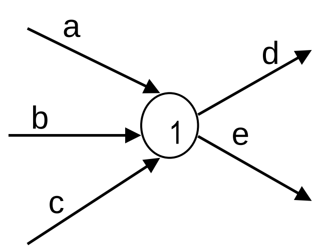


WRONG X

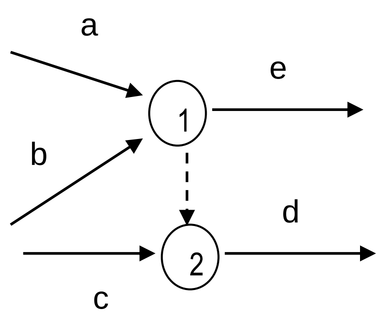


RIGHT ✓

Activity c not required for e



WRONG X



RIGHT ✓

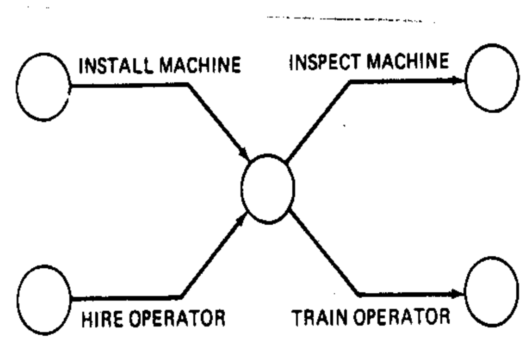


Figure 2-16

WRONG X

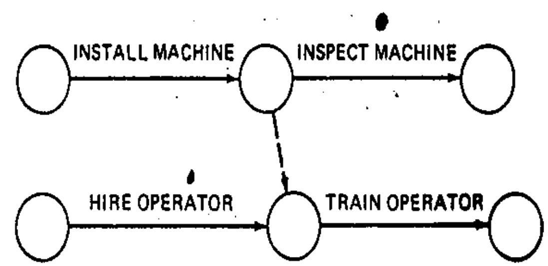
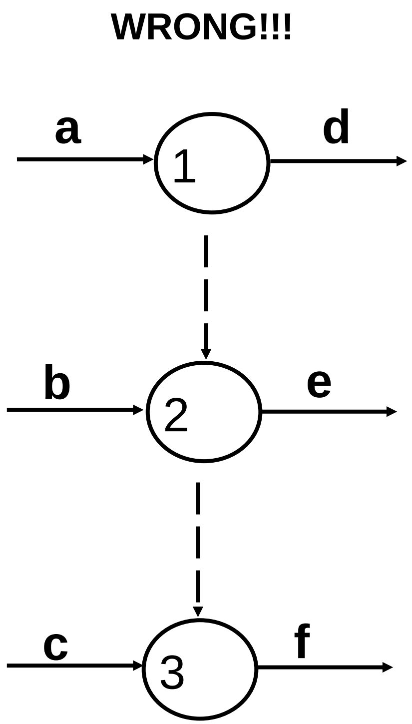
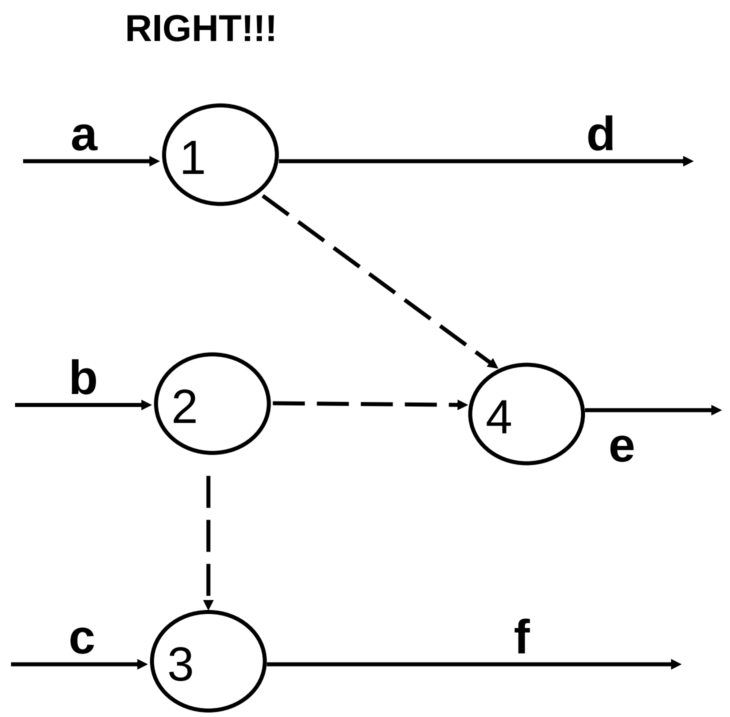


Figure 2-17

RIGHT ✓





a precedes d

a and b precede e,

b and c precede f (a does not precede f)

Scheduling with activity time

Activity	Immediate predecessors	Completion Time (week)
A	-	5
B	-	6
C	A	4
D	A	3
E	A	1
F	E	4
G	D,F	14
H	B,C	12
I	G,H	2
	Total ……	**51**

This information indicates that the total time required to complete activities is 51 weeks. However, we can see from the network that several of the activities can be conducted simultaneously (A and B, for example).

Earliest Start (ES) and Earliest Finish time (EF)

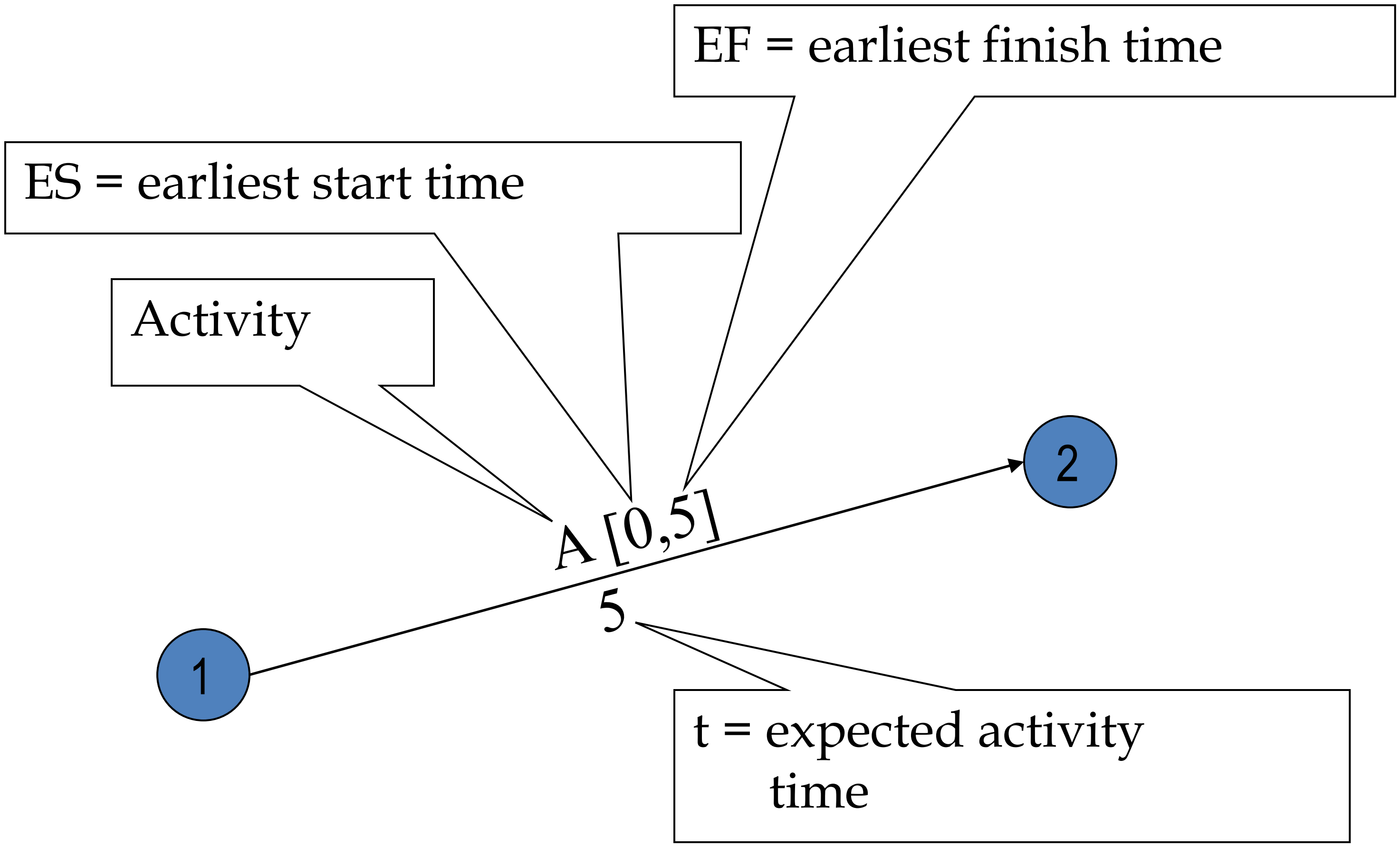
- *We are interested in the* ***longest path*** *through the network, i.e.,* ***the critical path.***

- *Starting at the network's origin* ***(node 1)*** *and using a starting time of* ***0****, we compute an* ***earliest start (ES)*** *and* ***earliest finish (EF)*** *time for each activity in the network (will be determined by forward pass calculation)*

- *The expression* ***EF = ES + t*** *can be used to find the earliest finish time for a given activity.*
- *For example, for activity A, ES = 0 and t = 5; thus the earliest finish time for activity A is*

$$EF = 0 + 5 = 5$$

Arc with ES and EF time



Scheduling with activity time

Activity	Immediate predecessors	Completion Time (week)
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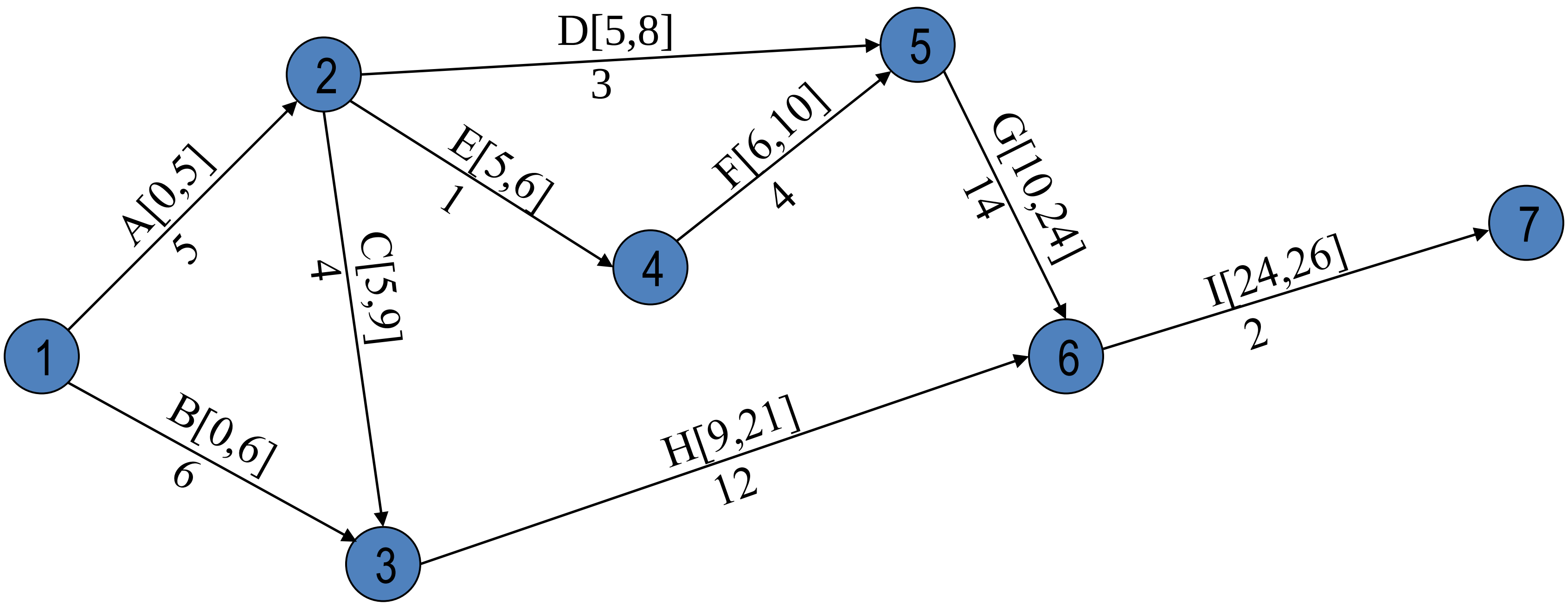
	Total ……	**51**

This information indicates that the total time required to complete activities is 51 weeks. However, we can see from the network that several of the activities can be conducted simultaneously (A and B, for example).

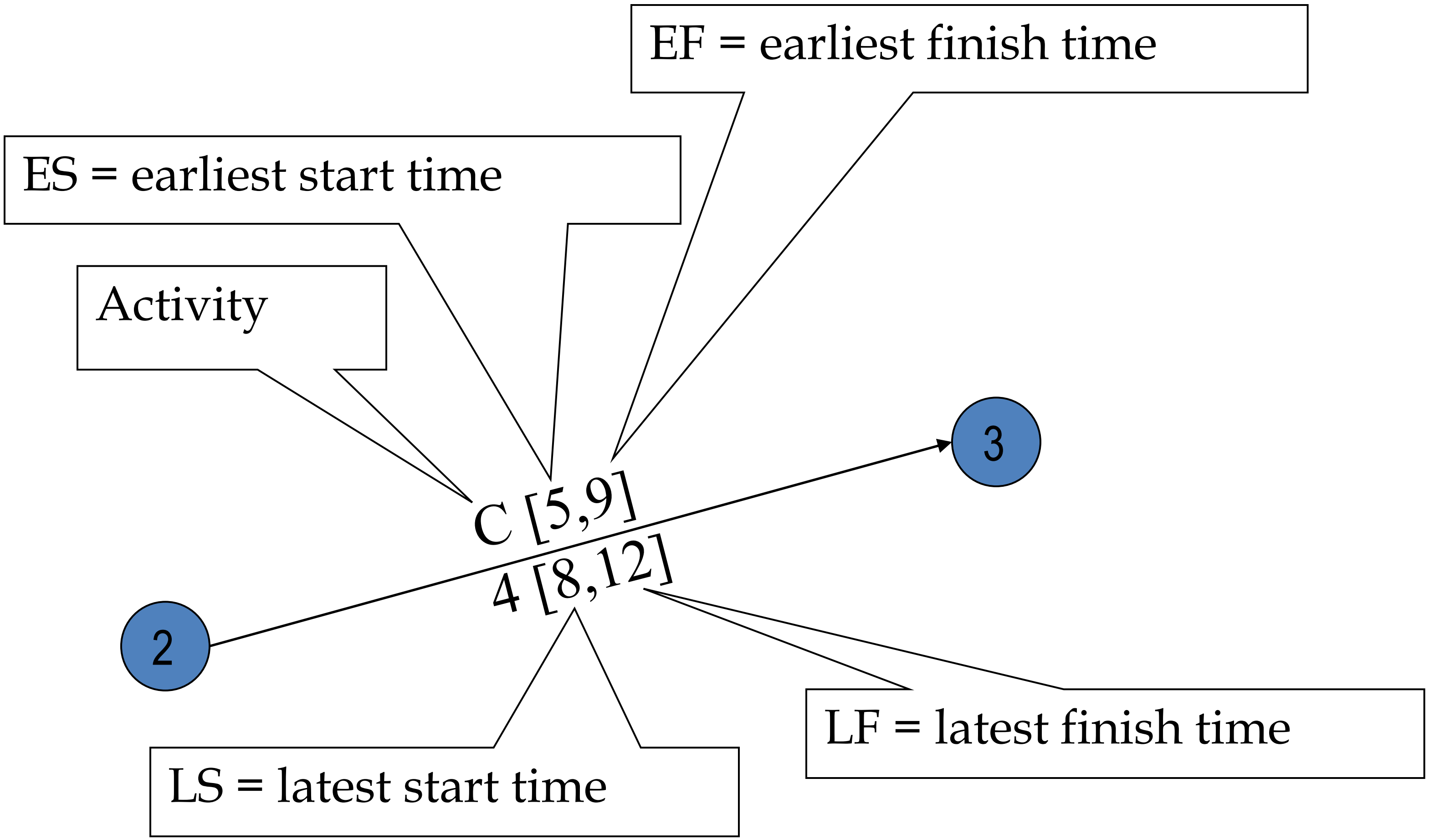
Network with ES & EF time



<u>Earliest start time rule</u>:

The earliest start time for an activity leaving a particular node is equal to the ***largest*** *of the earliest finish times for all activities entering the node.*

Activity, Duration, ES, EF, LS, LF

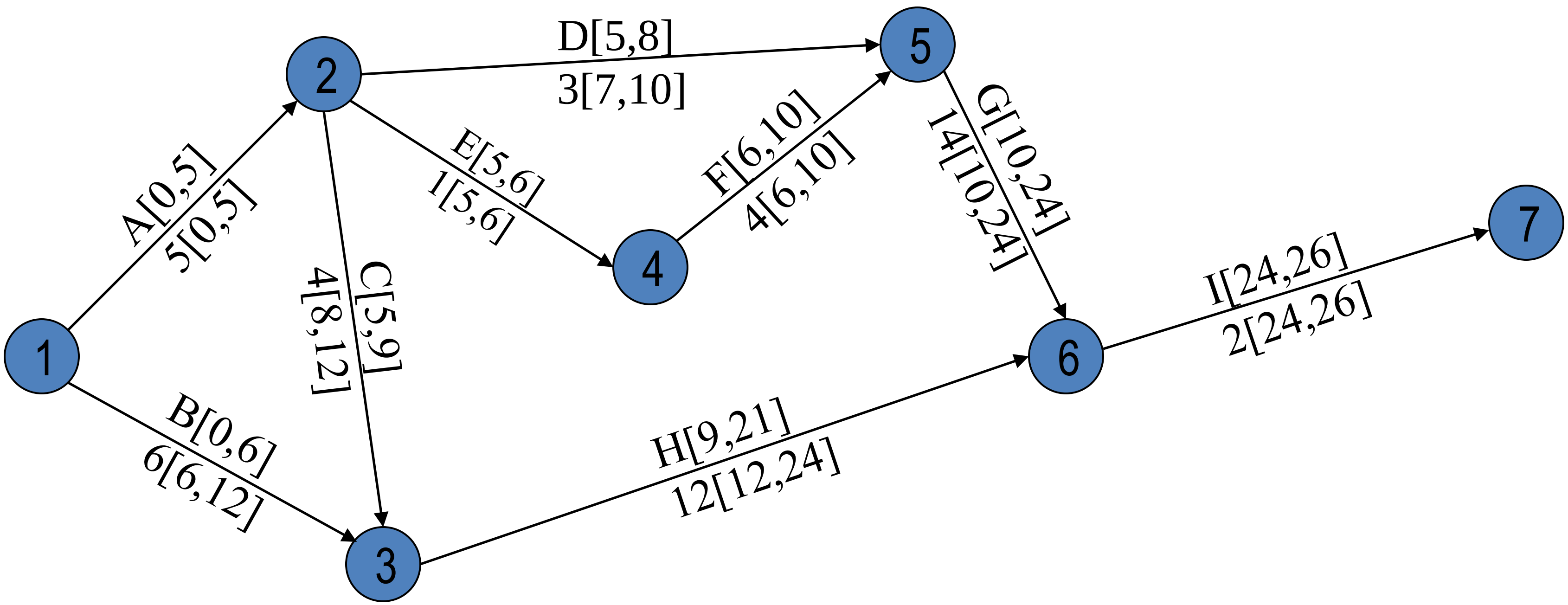


Latest Start (LS) and Latest Finish (LF) time

- *To find the critical path we need a **backward pass calculation**.*
- *Starting at the completion point (node 7) and using **a latest finish time (LF)** of 26 for activity I, we trace back through the network computing a **latest start (LS)** and latest finish time for each activity*
- *The expression **LS = LF – t** can be used to calculate latest start time for each activity.*
- *For example, for activity I, LF = 26 and t = 2, thus the latest start time for activity I is:*

LS = 26 – 2 = 24

Network with LS & LF time



<u>Latest finish time rule</u>:

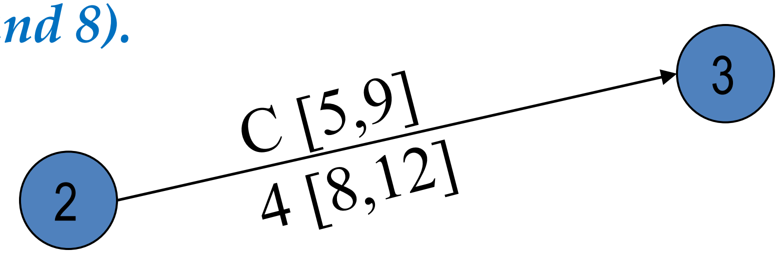
The latest finish time for an activity entering a particular node is equal to the ***smallest*** *of the latest start times for all activities leaving the node.*

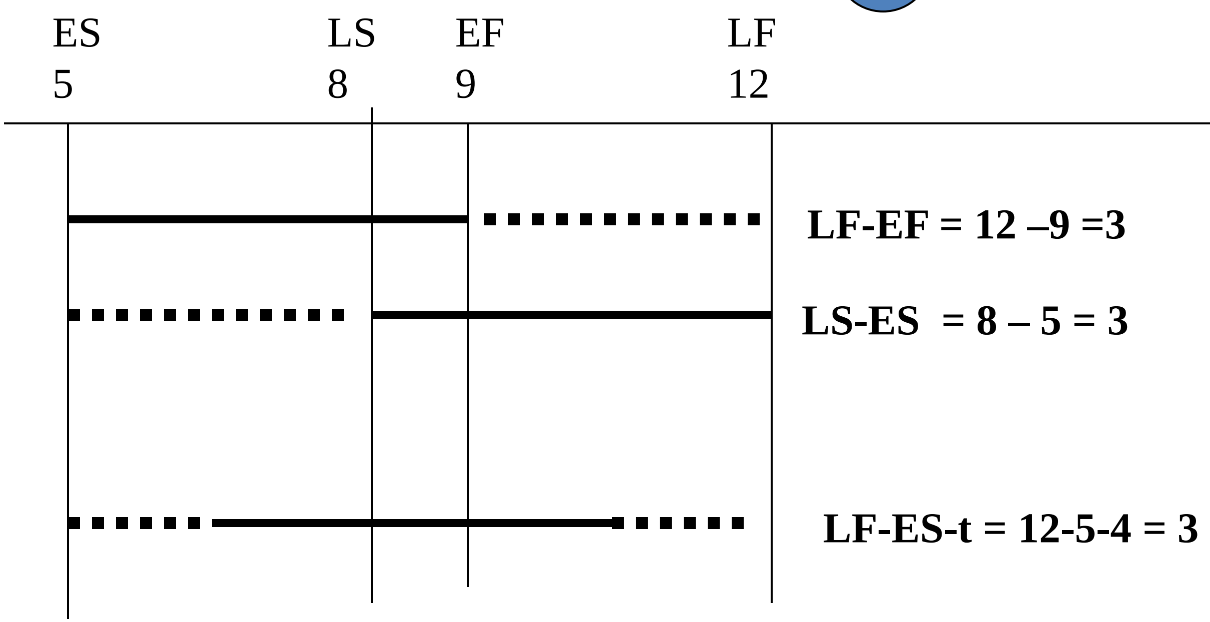
Slack or Free Time or Float

Slack/Free Time/Flot *is the length of time an activity can be delayed without affecting the completion date for the entire project.*

For example, slack for C = 3 weeks, i.e Activity C can be delayed up to 3 weeks

(start anywhere between weeks 5 and 8).





Summary of Slack Times

- *Activity start time and completion time may be delayed by deliberate reasons as well as by unforeseen reasons.*
- *Some of these delays may affect the overall completion date.*
- *The effects of these delays can be determined by the **slack time**, for each activity.*

Slack time for an activity = LS-ES or LF-EF

The Critical Path

*The activities with **0 slack time** form at least one **Critical Path** of connected activities, each of which is an immediate predecessor for another activity on the path from the beginning (time = 0) to the end (the completion time of the project).*

- *Critical activities must be rigidly scheduled.*
    - *Any delay in a critical activity will delay the entire project.*
- *The critical path is the longest in the network*

Sum of the completion times of activities on a critical path
=
Project completion time

Activity schedule for our example

Activity	Earliest start (ES)	Latest start (LS)	Earliest finish (EF)	Latest finish (LF)	Slack (LS-ES)	Critical path
A	0	0	5	5	0	Yes
B	0	6	6	12	6	
C	5	8	9	12	3	
D	5	7	8	10	2	
E	5	5	6	6	0	Yes
F	6	6	10	10	0	Yes
G	10	10	24	24	0	Yes
H	9	12	21	24	3	
I	24	24	26	26	0	Yes

Last EF= The project duration

Important Questions

- ***What is the total time to complete the project?***
  - *26 weeks if the individual activities are completed on schedule(Last EF).*
- ***What are the scheduled start and completion times for each activity?***
  - *ES, EF, LS, LF are given for each activity.*
- ***What activities are critical and must be completed as scheduled in order to keep the project on time?***
  - *Critical path activities: A, E, F, G, and I.*
- ***How long can non-critical activities be delayed before they cause a delay in the project's completion time***
  - *Slack time available for all activities are given.*

Importance of Float (Slack) and Critical Path

1. *Slack or Float shows how much allowance each activity has, i.e how long it can be delayed without affecting completion date of project*
2. *Critical path is a sequence of activities from start to finish with zero slack. Critical activities are activities on the critical path.*
3. *Critical path identifies the minimum time to complete project*
4. *If any activity on the critical path is* ***shortened or extended****, project time will be shortened or extended accordingly*

Importance of Float (Slack) and Critical Path

5. *So, a lot of effort should be put in trying to control activities along this path, so that project can meet due date. If any activity is* ***lengthened****, be aware that project will not meet deadline and some action needs to be taken.*
6. *If can spend resources to speed up some activity, do so only for* ***critical activities****.*
7. *Don't waste resources on non-critical activity, it will not shorten the project time.*
8. *If resources can be saved by lengthening some activities, do so for non-critical activities, up to limit of float.*
9. *Total Float belongs to the path*

Example (CPM)

- *Assume that ABC Computers manufactures computers.*
- *It is about to design, manufacture, and market a new model computer (a project).*
- *In broad terms, the three major tasks to perform are to:*
  - *Design and manufacture the computer*
  - *Train staff and vendor representatives on the features and use of the computer*
  - *Advertise the computer*

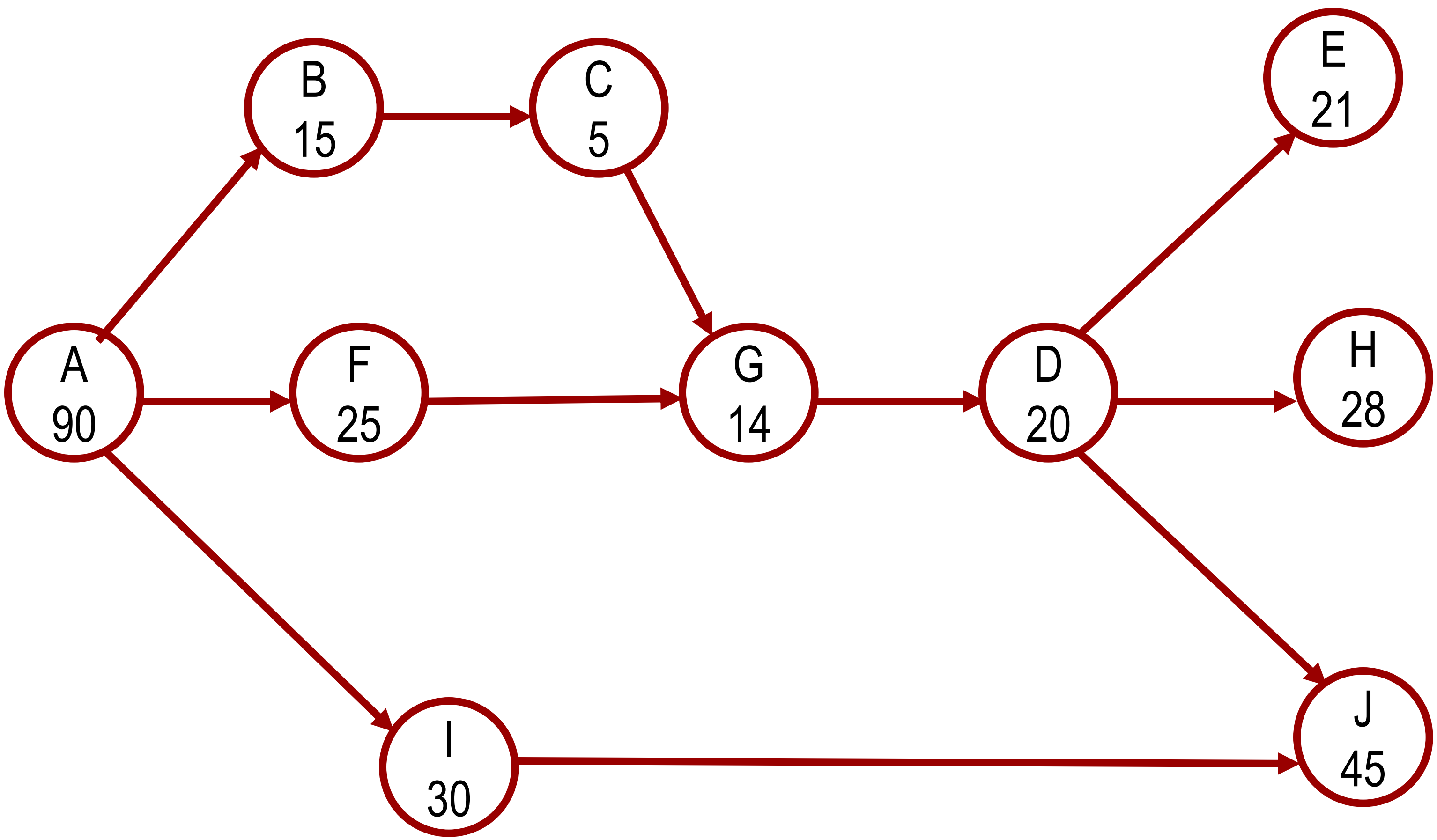
Detailed Activities

	Activity	Description
Manufacturing activities	A	Prototype model design
	B	Purchase of materials
	C	Manufacture of prototype model
	D	Revision of design
	E	Initial production run
Training activities	F	Staff training
	G	Staff input on prototype models
	H	Sales training
Advertising activities	I	Pre-production advertising campaign
	J	Post-redesign advertising campaign

Precedence Relations

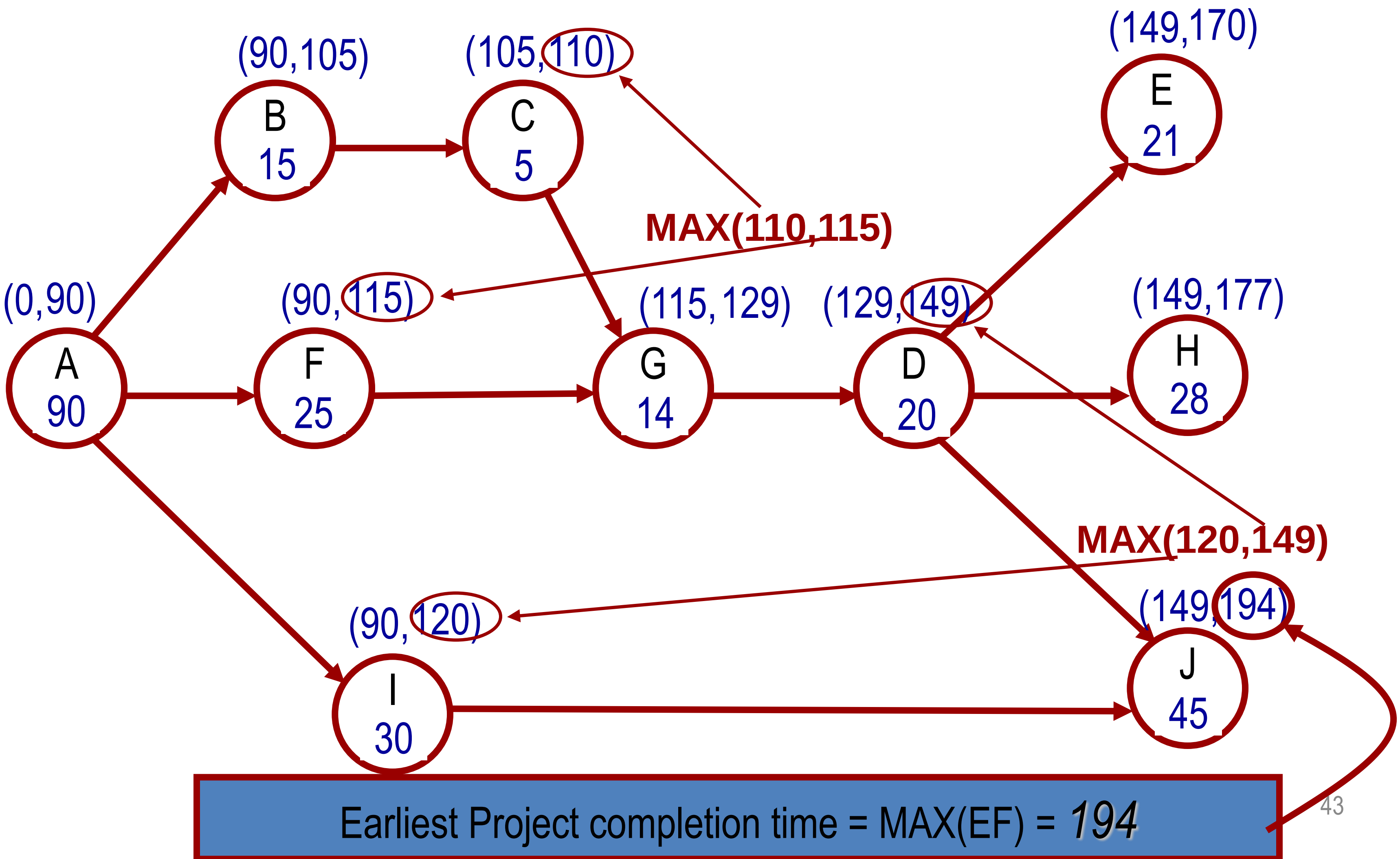
Activity	Starts after	Immediate Predecessor's)	Completion Days
A-Prototype Design		*NONE*	90
B-Purchase Materials	Starts After	A-Prototype Design	15
C-Manufacture Prototypes	Starts After	B-Purchase Materials	5
D-Design Revision	Starts After	G-Staff Input	20
E-Initial Production Run	Starts After	D-Design Revision	21
F-Staff Training	Starts After	A-Prototype Design	25
G-Staff Input	Starts After	C-Manufacture Prototypes ***and*** F-Staff Training	14
H-Sales Training	Starts After	D-Design Revision	28
I-Pre-Production Advertising	Starts After	A-Prototype Design	30
J-Post Redesign Advertising	Starts After	D-Design Revsion ***and*** I-Pre-Production Advertising	45

The CPM Network



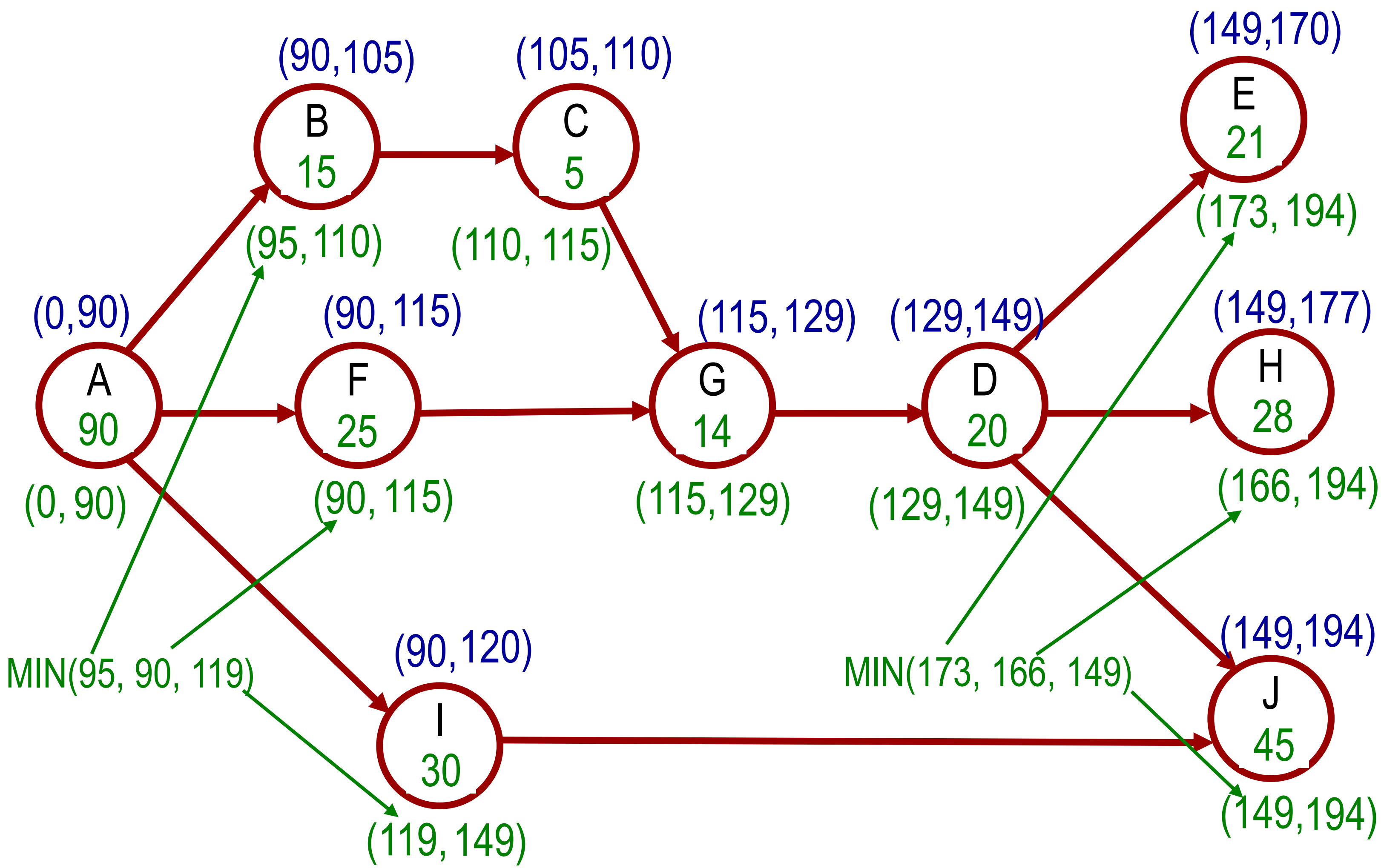
Earliest Start and Finish Times

- *We enter these as (ES,EF) above each node.*



Latest Start and Finish Times

- We enter these as (LS,LF) below each node.



Slack Time Calculations

- *Slack time = LS - ES*

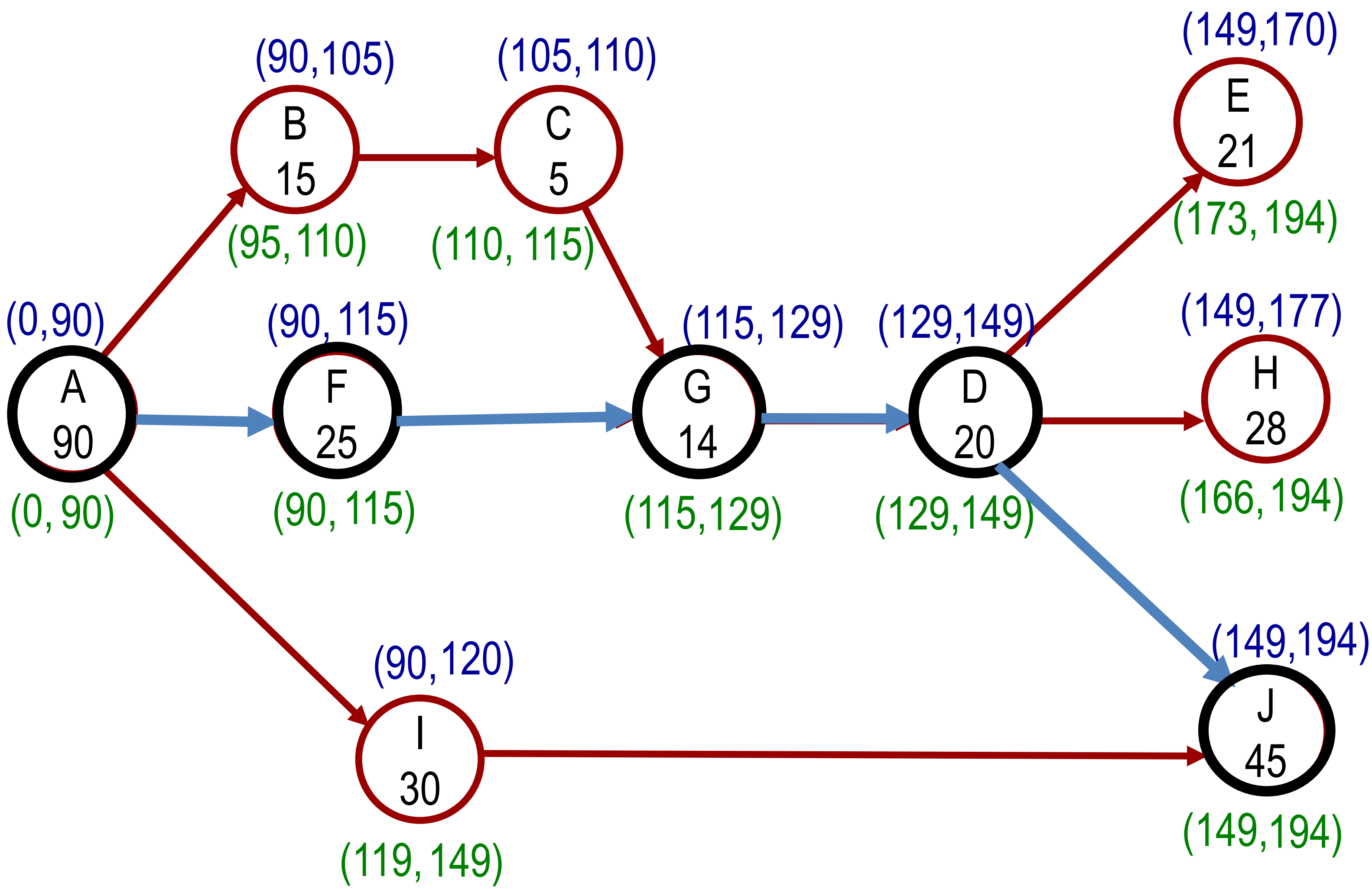
Activity	LS	-	ES	=	SLACK
A	0	-	0	=	0
B	95	-	90	=	5
C	110	-	105	=	5
D	129	-	129	=	0
E	173	-	149	=	24
F	90	-	90	=	0
G	115	-	115	=	0
H	166	-	149	=	17
I	119	-	90	=	29
J	149	-	149	=	0

Critical Activities

Critical Path

A → F → G → D → J

The Critical Path



Possible Delays

- *There could be a delay in just one activity.*
  - *Any delay more than the slack time for the activity will delay the entire project by the difference between the activity delay and the slack time*
- *There could be delays in more than one activity.*
  - *If activities are on different paths or on the same path but separated by a critical activity, each of the delays is evaluated separately. The project delay = max (these delays – corresponding slack).*
  - *Activities on the same path which are not separated by a critical activity share the slack. Both will have the same value for the slack and any combined delays in these activities that exceed this common slack results in a project delay equal to (total activity delay) – (common slack).*
  - *Usually with multiple delays the model is simply re-solved!*

Examples of Activity Delays

- *Activity G is delayed 5 days*
  - *G is on the critical path (has 0 slack) so the project will be delayed 5 days.*
- *Activity E is delayed 15 days*
  - *E has 24 days of slack so the project will not be delayed*
- *Activity B is delayed 15 days*
  - *B has 5 days of slack so the project will be delayed 10 days*
- *Activity E is delayed 30 days and Activity I is delayed 30 days*
  - *E and I are on different paths. E has 24 days of slack which could cause a 30-24 = 6 day delay; I has 29 days of slack which could cause 30-29 = 1 day delay. The project is delayed by the MAX(6,1) = 6 days.*

Examples of Activity Delays

- *Activity B is delayed 4 days and Activity E is delayed 4 days*
  - *B and E are on the same path but are separated by critical activities (G and D). This is the same as the case above. B has 5 days slack so delaying it 4 days would not delay the project; E has 24 days of slack so a 4 day delay will not delay the project – Net effect– No delay.*
- *Activity B is delayed 4 days and Activity C is delayed 4 days*
  - *B and C are on the same path with no critical activity in between. They share the same 5 days of slack. So sense both are delayed 4 days for a total of 8 days, the project is delayed 8 – 5 = 3 days.*

PERT For Dealing With Uncertainty

- *So far, times were estimated with relative certainty/confidence*
- *For many situations, however, this is not possible, e.g Research, development, new products and projects etc.*
- *In PERT we use 3 time estimates:*

  m= most likely time estimate, mode.

  a = optimistic time estimate, and

  b = pessimistic time estimate

Expected Value (TE) = $(a + 4m + b)/6$

Variance (V) = $((b - a)/6)^2$

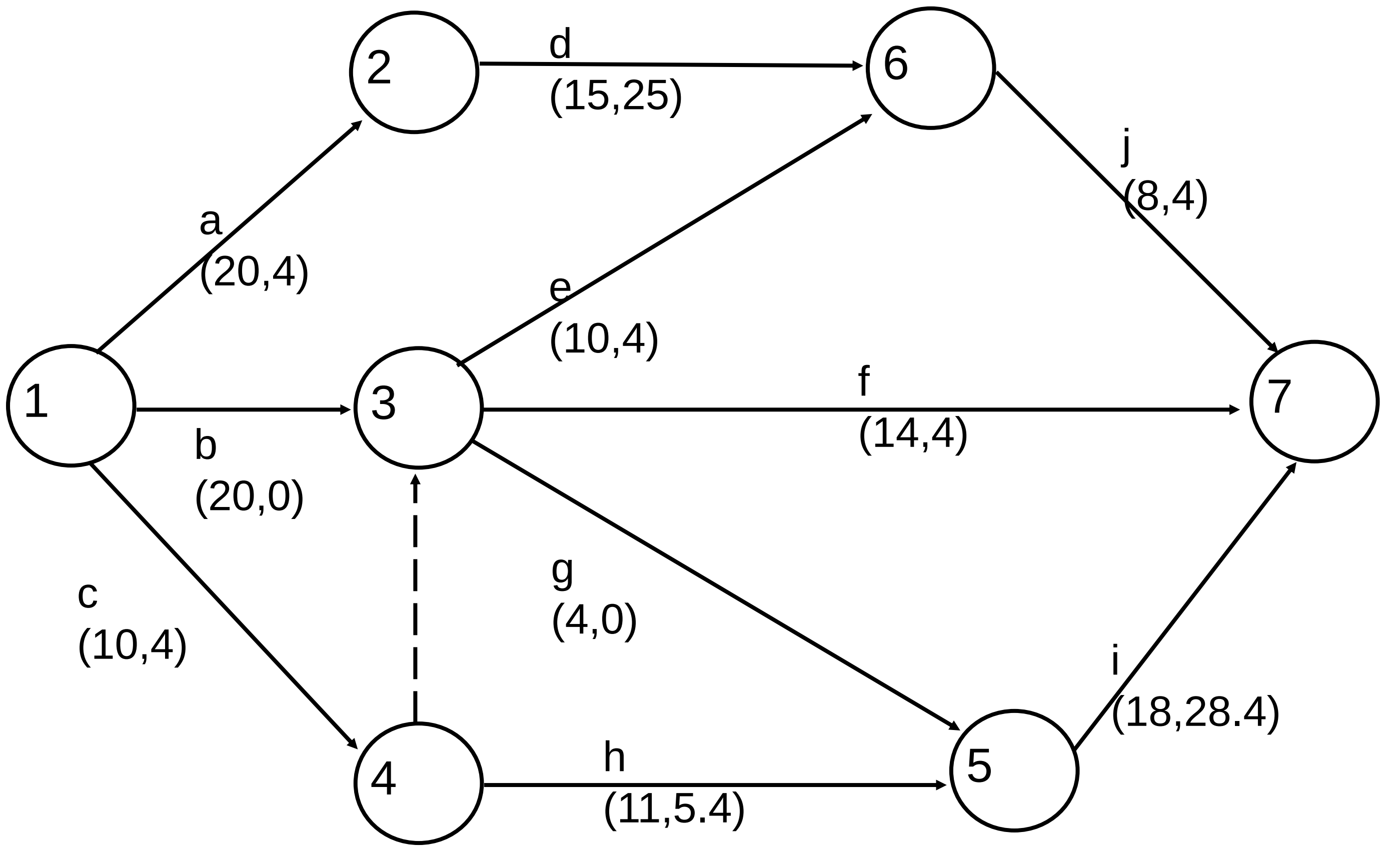
Std Deviation (δ) = SQRT (V)

Example (PERT)

Precedences and Project Activity Times

Activity	Immediate Predecessor	Optimistic Time	Most Likely Time	Pessimistic Time	EXP TE	Var V	S.Dev σ
a	-	10	22	22	20	4	2
b	-	20	20	20	20	0	0
c	-	4	10	16	10	4	2
d	a	2	14	32	15	25	5
e	b,c	8	8	20	10	4	2
f	b,c	8	14	20	14	4	2
g	b,c	4	4	4	4	0	0
h	c	2	12	16	11	5.4	2.32
I	g,h	6	16	38	18	28.4	5.33
j	d,e	2	8	14	8	4	2

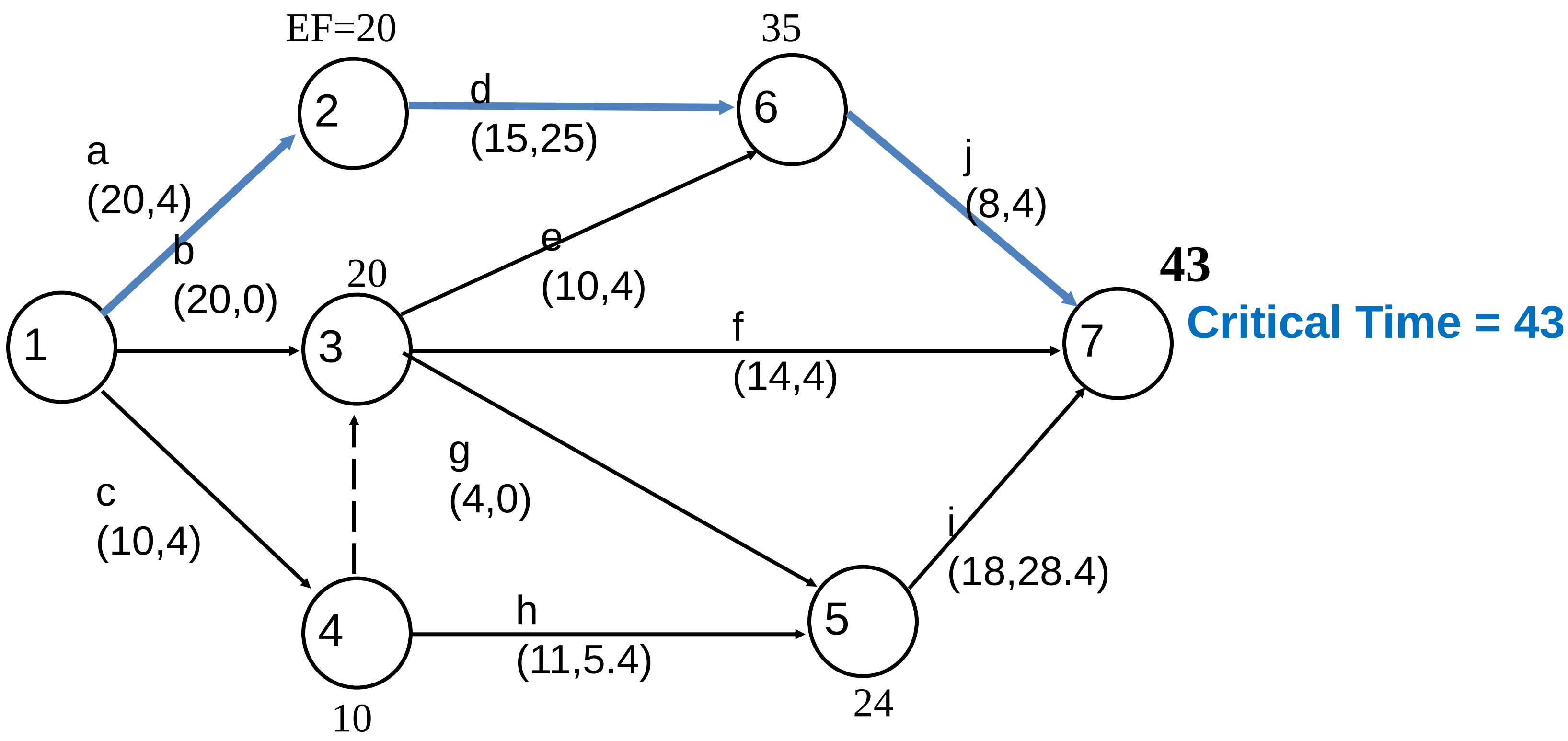
The complete PERT network



Critical Path Analysis (PERT)

Activity	LS	ES	Slacks	Critical ?
a	**0**	**0**	**0**	**Yes**
b	1	0	1	
c	4	0	4	
d	**20**	**20**	**0**	**Yes**
e	25	20	5	
f	29	20	9	
g	21	20	1	
h	14	10	4	
i	25	24	1	
j	**35**	**35**	**0**	**Yes**

The complete PERT Network



Assume, the Project Manager promised to complete the project in 50 days.

Question: *What are the chances of meeting that deadline? (Determine **P**)*

Calculate Z, where

$$Z = (D-S) / \sqrt{V}$$

Example,

$D = 50$ *(specified date);*

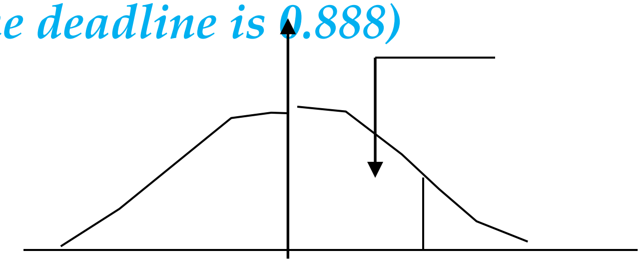
$S = 20+15+8 = 43$ *(Scheduled date);*

$V = (4+25+4) = 33$ *(Variance of the critical path);*

$Z = (50 - 43) / 5.745$

$= 1.22$ *standard deviations.*

The probability value of Z = 1.22, is 0.888 (Table value);i.e., the chance of meeting the deadline is 0.888)



Question: What deadline are you 95% sure of meeting? (Determine D)

$D = S+Z\sqrt{V}$……….. (From previous formula)

Z value associated with 0.95 is 1.645 (Table value)

$D=S + 5.745\,(1.645)$

$= 43 + 9.45$

$= 52.45$ days

- ***Thus, there is a 95 percent chance of finishing the project by 52.45 days.***

Comparison Between CPM and PERT

	CPM	*PERT*
1	*Uses network, calculate float or slack, identify critical path and activities, guides to monitor and controlling project*	*Same as CPM*
2	*Uses one value of activity time*	*Requires **3 estimates** of activity time* *Calculates mean and variance of time*
3	*Used where times can be estimated with confidence, familiar activities*	*Used where times cannot be estimated with confidence.* *Unfamiliar or new activities*
4	*Minimizing cost is more important*	*Meeting time target or estimating percent completion is more important*
5	*Example: construction projects, building one off machines, ships, etc*	*Example: Involving new activities or products, research and development etc*

Benefits of CPM / PERT Network

- ***Consistent framework for planning, scheduling, monitoring, and controlling project.***
- *Shows interdependence of all tasks, work packages, and work units.*
- *Helps proper communications between departments and functions.*
- *Determines expected project completion date.*
- *Identifies so-called **critical activities**, which can delay the project completion time.*

Benefits of CPM / PERT Network

- *Identified activities with **slacks** that can be delayed for specified periods without penalty, or from which resources may be temporarily borrowed*
- *Determines the **dates** on which tasks may be started or must be started if the project is to stay in schedule.*
- *Shows which tasks must be coordinated to avoid resource or timing conflicts.*
- *Shows which tasks may run in parallel to meet project completion date*

THANK YOU!