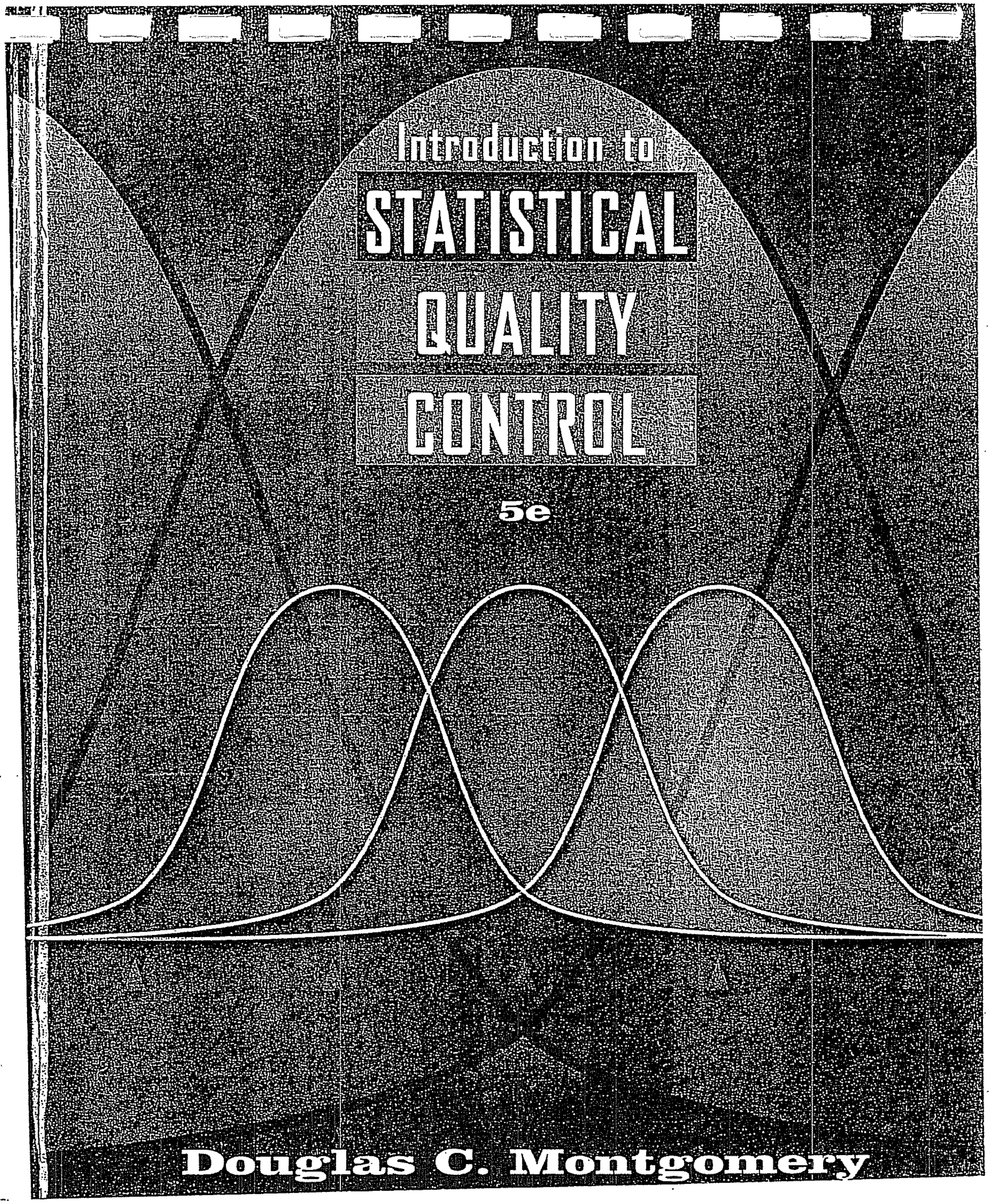


Introduction to STATISTICAL QUALITY CONTROL

5e

Douglas C. Montgomery

SPC
Calculations for Control Limits

Notation:
UCL—Upper Control Limit
LCL—Lower Control Limit
CL—Center Line
n—Sample Size
PCR—Process Capability Ratio
$\hat{\sigma}$—Process Standard Deviation
$\bar{x}$—Average of Measurements
$\bar{\bar{x}}$—Average of Averages
R—Range
$\bar{R}$—Average of Ranges
USL—Upper Specification Limit
LSL—Lower Specification Limit

Variables Data ($\bar{x}$ and R Control Charts)

$\bar{x}$ Control Chart

$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$

$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$

$\text{CL} = \bar{\bar{x}}$

R Control Chart

$\text{UCL} = \bar{R}\,D_4$

$\text{LCL} = \bar{R}\,D_3$

$\text{CL} = \bar{R}$

n	A_2	D_3	D_4	d_2
2	1.880	0.000	3.267	1.128
3	1.023	0.000	2.574	1.693
4	0.729	0.000	2.282	2.059
5	0.577	0.000	2.114	2.326
6	0.483	0.000	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

Capability Study

$C_p = (\text{USL} - \text{LSL})/(6\hat{\sigma})$; where $\hat{\sigma} = \bar{R}/d_2$

Attribute Data (p, np, c, and u Control Charts)

noncomformities

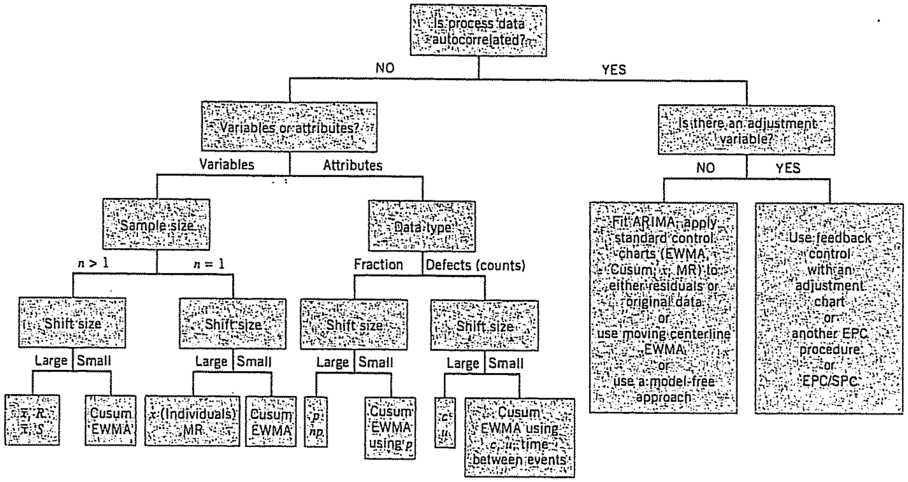
defective

Control Chart Formulas

Defects

	p (fraction)	np (number of nonconforming)	c (count of nonconformances)	u (count of nonconformances/unit)
CL	$\bar{p}$	$n\bar{p}$	$\bar{c}$	$\bar{u}$
UCL	$\bar{p}+3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}+3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}+3\sqrt{\bar{c}}$	$\bar{u}+3\sqrt{\frac{\bar{u}}{n}}$
LCL	$\bar{p}-3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}-3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}-3\sqrt{\bar{c}}$	$\bar{u}-3\sqrt{\frac{\bar{u}}{n}}$
Notes	If n varies, use $\bar{n}$ or individual n_i	n must be a constant	n must be a constant	If n varies, use $\bar{n}$ or individnal n_i

Guide to Univariate Process Monitoring and Control



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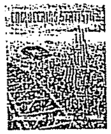

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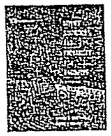

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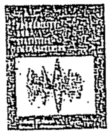

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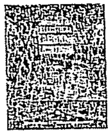

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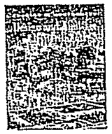

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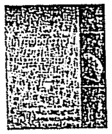

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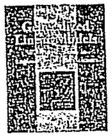

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Introduction to Statistical Quality Control

Fifth Edition

Douglas C. Montgomery
Arizona State University

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Preface

INTRODUCTION

This book is about the use of modern statistical methods for quality control and improvement. It provides comprehensive coverage of the subject from basic principles to state-of-the-art concepts and applications. The objective is to give the reader a sound understanding of the principles and the basis for applying them in a variety of situations. Although statistical techniques are emphasized throughout, the book has a strong engineering and management orientation. Extensive knowledge of statistics is not a prerequisite for using this book. Readers whose background includes a basic course in statistical methods will find much of the material in this book easily accessible.

AUDIENCE

The book is an outgrowth of over 30 years of teaching, research, and consulting in the application of statistical methods for industrial problems. It is designed as a textbook for students enrolled in colleges and universities, who are studying engineering, statistics, management, and related fields and are taking a first course in statistical quality control. The basic quality-control course is often taught at the junior or senior level. All of the standard topics for this course are covered in detail. Some more advanced material is also available in the book, and this could be used with advanced undergraduates who have had some previous exposure to the basics or in a course aimed at graduate students. I have also used the text materials extensively in programs for professional practitioners, including quality and reliability engineers, manufacturing and development engineers, product designers, managers, procurement specialists, marketing personnel, technicians and laboratory analysts, inspectors, and operators. Many professionals have also used the material for self-study.

CHAPTER ORGANIZATION AND TOPICAL COVERAGE

Chapter 1 is an introduction to the philosophy and basic concepts of quality improvement. It notes that quality has become a major business strategy and that organizations that successfully improve quality can increase their productivity, enhance their market penetration, and achieve greater profitability and a strong competitive advantage. Some of the managerial and implementation aspects of quality improvement are included.

Following the introductory chapter, the book is divided into five parts. Part I is a description of statistical methods useful in quality improvement. Topics covered include sampling and descriptive statistics, the basic notions of probability and probability distributions, point and interval estimation of parameters, and statistical hypothesis testing. These topics are usually covered in a basic course in statistical methods; however, their presentation in this text is from the quality-engineering viewpoint. My experience has been that even readers with a strong statistical background will find the approach to this material useful and somewhat different from a standard statistics textbook.

Part II contains four chapters covering the basic methods of statistical process control (SPC) and methods for process capability analysis. Even though several SPC problem-solving tools are discussed (including Pareto charts, cause-and-effect diagrams, for example), the primary focus in this section is on the Shewhart control chart. The Shewhart control chart is certainly not new, but its use in modern-day business and industry is of tremendous value.

There are four chapters in Part III that present some more advanced SPC methods. Included are the cumulative sum and exponentially weighted moving average control charts (Chapter 8), several important univariate control charts such as procedures for short production runs, autocorrelated data, and multiple stream processes (Chapter 9), multivariate process monitoring and control (Chapter 10), and feedback adjustment techniques (Chapter 11). Some of this material is at a higher level than Part II, but much of it is accessible by advanced undergraduates or first-year graduate students. This material forms the basis of a second course in statistical quality control and improvement for this audience.

Part IV contains two chapters that show how statistically designed experiments can be used for process design, development, and improvement. Chapter 12 presents the fundamental concepts of designed experiments and introduces the reader to factorial and fractional factorial designs, with particular emphasis on the two-level system of designs. These designs are used extensively in industry for factor screening and process characterization. Although the treatment of the subject is not extensive and is no substitute for a formal course in experimental design, it will enable the reader to appreciate more sophisticated examples of experimental design. Chapter 13 introduces response surface methods and designs, illustrates evolutionary operation (EVOP) for process monitoring, and shows how statistically designed experiments can be used for process robustness studies. Chapters 12 and 13 emphasize the important interrelationship between statistical process control and experimental design for process improvement.

Two chapters deal with acceptance sampling in Part V. The focus is on lot-by-lot acceptance sampling, although there is some discussion of continuous sampling and MIL STD 1235C in Chapter 14. Other sampling topics presented include various aspects of the design of acceptance-sampling plans, a discussion of MIL STD 105E, MIL STD 414 (and their civilian counterparts, ANSI/ASQC Z1.4 and ANSI/ASQC Z1.9), and other techniques such as chain sampling and skip-lot sampling.

Throughout the book, guidelines are given for selecting the proper type of statistical technique to use in a wide variety of situations. Additionally extensive references to journal articles and other technical literature should assist the reader in applying the methods described.

SUPPORTING TEXT MATERIALS

Computer Software

The computer plays an important role in a modern quality-control course. This edition of the book uses Minitab as the primary illustrative software package. Instructors may order this book with a student version of Minitab included. I strongly recommend that the course have a meaningful computing component. To request this book with Minitab included, contact your local Wiley representative. You can find your local Wiley representative by going to www.wiley.com and clicking on the tab for "Who's My Rep?".

Supplemental Text Material

I have written a set of supplemental material to augment many of the chapters in the book. The supplemental material contains topics that I could not easily fit into that chapter without seriously disrupting the flow. The topics are shown in the Table of Contents for the book and in the individual chapter outlines. Some of this material consists of proofs or derivations, new topics of a (sometimes) more advanced nature, supporting details concerning remarks or concepts presented in the text, and answers to frequently asked questions. The supplemental material provides an interesting set of accompanying readings for anyone curious about the field. It is available on the World Wide Web page that supports the book, located at www.wiley.com/college/montgomery.

Student Resource Manual

The text contains answers to most of the odd-numbered exercises. A Student Resource Manual is available from John Wiley & Sons that presents comprehensive annotated solutions to these same odd-numbered problems. This is an excellent study aid that many text users will find extremely helpful. The Student Resource Manual may be ordered in a set with the text or purchased separately. Contact your local Wiley representative to request the set for your bookstore or purchase the Student Resource Manual from the Wiley website.

Instructor's Materials

The instructor's section of the textbook website contains the following:

1. Solutions to the text problems
2. The supplemental text material described above
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The instructor's section is for instructor use only and is password-protected. Visit the Instructor Companion Site portion of the website, located at www.wiley.com/college/montgomery, to register for a password.

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KNOWLEDGMENTS

Many people have generously contributed their time and knowledge of statistics and quality improvement to this book. I would like to thank Dr. Bill Woodall, Dr. Doug Hawkins, Dr. Joe Sullivan, Dr. George Runger, Dr. Bert Keats, Dr. Bob Hogg, Mr. Eric Ziegel, Dr. Joe Pignatiello, Dr. John Ramberg, Dr. Ernie Saniga, Dr. Enrique Del Castillo, Dr. Sarah Streett, and Dr. Jim Alloway for their thorough and insightful comments on this and previous editions. They generously shared many of their ideas and teaching experiences with me, leading to substantial improvements in the book.

Over the years since the first edition was published, I have received assistance and ideas from a great many other people. A complete list of colleagues with whom I have interacted would be impossible to enumerate. However, some of the major contributors and their professional affiliations are as follows: Dr. Mary R. Anderson-Rowland, Dr. Dwayne A. Rollier, and Dr. Norma F. Hubele, Arizona State University; Mr. Seymour M. Selig, formerly of the Office of Naval Research; Dr. Lynwood A. Johnson, Dr. Russell G. Heikes, Dr. David E. Fyffe, and Dr. H. M. Wadsworth, Jr., Georgia Institute of Technology; Dr. Sharad Prabhu and Dr. Robert Rodriguez, SAS Institute; Dr. Richard L. Storch and Dr. Christina M. Mastrangelo, University of Washington; Dr. Cynthia A. Lowry, formerly of Texas Christian University; Dr. Smiley Cheng, Dr. John Brewster, Dr. Brian Macpherson, and Dr. Fred Spiring, the University of Manitoba; Dr. Joseph D. Moder, University of Miami; Dr. Frank B. Alt, University of Maryland; Dr. Kenneth E. Case, Oklahoma State University; Dr. Daniel R. McCarville, Dr. Lisa Custer, Dr. Pat Spagon, and Mr. Robert Stuart, all formerly of Motorola; Dr. Richard Post, Intel Corporation; Dr. Dale Sevier, Hybritech; Mr. John A. Butora, Mr. Leon V. Mason, Mr. Lloyd K. Collins, Mr. Dana D. Lesher, Mr. Roy E. Dent, Mr. Mark Fazey, Ms. Kathy Schuster, Mr. Dan Fritze, Dr. J. S. Gardiner, Mr. Ariel Rosentrater, Mr. Lolly Marwah, Mr. Ed Schleicher, Mr. Amiin Weiner, and Ms. Elaine Baechtle, IBM; Mr. Thomas C. Bingham, Mr. K. Dick Vaughn, Mr. Robert LeDoux, Mr. John Black, Mr. Jack Wires, Dr. Julian Anderson, Mr. Richard Alkire, and Mr. Chase Nielsen, The Boeing Company; Ms. Karen Madison, Mr. Don Walton, and Mr. Mike Goza, Alcoa; Mr. Harry Peterson-Nedry, Ridgecrest Vineyards and The Chehalem Group; Dr. Russell A. Boyles, formerly of Precision Castparts Corporation; Dr. Sadre Khalessi and Mr. Franz Wagner, Signetics Corporation; Mr. Larry Newton and Mr. C. T. Howlett, Georgia Pacific Corporation; Mr. Robert V. Baxley, Monsanto Chemicals; Dr. Craig Fox, Dr. Thomas L. Sadosky, Mr. James F. Walker, and Mr. John Belvins, The Coca-Cola Company; Mr. Bill Wagner and Mr. Al Pariseau, Litton Industries; Mr. John M. Fluke, Jr., John Fluke Manufacturing Company; Dr. Paul Tobias, formerly of IBM and Semitech; Dr. William DuMouchel and Ms. Janet Olson, BBN Software Products Corporation. I would also like to acknowledge the many contributions of my late partner in Statistical Productivity Consultants, Mr. Sumner S. Averett. All of these individuals and many others have contributed to my knowledge of the quality improvement field.

The editorial and production staff at Wiley, particularly Ms. Charity Robey and Mr. Wayne Anderson, with whom I worked for many years, and Ms. Jenny Welter; they have had much patience with me over the years and have contributed greatly towards the success of this book. Dr. Cheryl L. Jennings made many valuable contributions by her careful checking of the manuscript and proof materials, and in preparing the solutions. I also thank Dr. Gary Hogg, Chair of the Department of Industrial Engineering at Arizona State University, for his support and for providing a terrific environment in which to teach and conduct research.

I thank the various professional societies and publishers who have given permission to reproduce their materials in my text. Permission credit is acknowledged at appropriate places in this book.

I am also indebted to the many organizations that have sponsored my research and my graduate students for a number of years, including the member companies of the National Science Foundation/Industry/University Cooperative Research Center in Quality and Reliability Engineering at Arizona State University, the Office of Naval Research, the National Science Foundation, the Aluminum Company of America, and the IBM Corporation. Finally, I would like to thank the many users of the previous editions of this book, including students, practicing professionals, and my academic colleagues. Many of the changes and (hopefully) improvements in this edition of the book are the direct result of your feedback.

Douglas C. Montgomery
Tempe, Arizona

Contents

CHAPTER 1

PART I

CHAPTER 2

CHAPTER 3

PART II

CHAPTER 4

CHAPTER 5

CHAPTER 6

CHAPTER 7

PART III

CHAPTER 8

CHAPTER 10

[redacted]

approach is to compare current performance with past performance, so that, in effect

2

Modeling Process Quality

probability distribution

Table 2-1 Days to Pay Employee Health Insurance Claims

IMPORTANT TERMS AND CONCEPTS

duced by the process. It is impossible to know this exactly in a real production process.

[redacted]

The general situation is shown in Fig. 3-8. Population 1 has mean

A *P*-value could also be used for decision making in this example. The actual value

EXERCISES

Temperature

(a) Does the size of the orifice affect the

PART

if they behave in a systematic or

had been in a shutdown mode because of holidays. Apparently, when the process was

[redacted]

[illegible]

Note that all the cases in Fig. 5-3 assume that the process is centered at the midpoint

[illegible] 2) and to draw the center line and the upper control limit on the R chart

tures of two normal distributions. Their study indicates that, in most cases, samples of size

[redacted]

5-3 CONTROL CHARTS FOR $\bar{x}$ AND s

[redacted]

Size of Shift	β	ARL_1

[redacted]

bility is often undefined; and (2) more imagination may be required to select the proper

[redacted]

[illegible] 5.14 The table shown presents 20 subgroups

(c) Estimate the standard deviation of

(b) If it is desired that the probability of

UCL = 104

(a) What are your conclusions regardof the can, any package that weighs less

and the average of these m individual sample fractions nonconforming is

[redacted]

[redacted]

These probabilities (α, β) can be calculated directly

where $\langle n\text{LCL}\rangle$ denotes the smallest integer greater than or equal to nLCL and [illegible]

Determining Which Characteristics to Control

tailor or customize the system for each application, although this installation activ

EXERCISES

number of nonconformities

performance, illustrating graphical methods, as well as

$$C\,(q)=\frac{\mathrm{USL}-\mathrm{LSL}}{} \qquad (7\text{-}11)$$

$\bar{R}/d$ to estimate σ. This fur-

when the process is normally distributed and in control, $\hat{P}_p$ is

measured, and some of the variability will result from the measurement system that is

7.22 Consider a simplified version of equa-

7.44 Find the sample size required to con-

PART

[redacted]

$$V[y(x, z)] = \sigma^2(10.81 - 9.06x + 8.31x)^2 + \sigma^2$$

model is

PART

The Dodge–Romig AOQL plans are designed so that the average total inspection f

the area under the standard normal curve below Z_{LSL}. (Actually, using Q_{LSL} =

(a) Sketch the OC curve for this sam-

these containers has been delivered to a

15-10 Consider a single-sampling plan with $n =$

15-13 A chain-sampling plan is used for the

Appendix I Summary of Common Probability Distributions Often Used in Statistical Quality Control

Appendix IV Percentage Points of the t Distribution[a]

33 49 13 .76 .10 .72 .47 .29 .16 .06 .97 .90 .85 .80 .76 .72 .69 .66 .63 .61 1.59 1.57 1.55 1.53 1.52 1.50 1.49 1.48 1.47 1.46 1.38 1.29 1.19 1.00

"Quality Control Methods for Several Related Variables," *Technometrics*, Vol. 1(4),

Lieberman, G. J., and G. J. Resnikoff (1955). "Sampling Plans for Inspection by Variables," *Journal of*

"Statistical Process Control of Multiple Stream Processes."

Schmidt, S. R., and J. R. Boudot (1989). "A Monte Carlo Simulation Study Comparing Effectiveness of

Answers to Selected Exercises

Control limits for y_1 model MR chart: CL = 0.791, UCL = 2.586, LCL = 0

[illegible] and [illegible]

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