Mechanical Engineering Department

Fluid Mechanics

(MEng 2113)

Chapter 7

Compressible Flow

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Contents of the Chapter

- Introduction
- Thermodynamic Relations
- Stagnation Properties
- Speed of Sound and Mach number
- One Dimensional Isentropic flow: Effect of area changes of flow parameters
- Converging Nozzle: Effect of back pressure on flow parameters
- Converging-Diverging Nozzles
- Normal shock waves; Fanno and Rayleigh lines
- Oblique shock waves

Introduction

- Flows that involve significant changes in density are called **compressible flows**.
- Therefore, ρ(x, y, z) must now be treated as a field variable rather than simply a constant.
- Typically, significant density variations start to appear when the flow Mach number exceeds 0.3 or so. The effects become especially large when the Mach number approaches and exceeds unity.
- In this chapter we will consider flows that involve significant changes in density. Such flows are called ***compressible flows***, and they are frequently encountered in devices that involve the flow of gases at very high speeds such as flows in gas turbine engine components . Many aircraft fly fast enough to involve compressible flow.

Introduction

- Gas has large compressibility but when its velocity is low compared with the sonic velocity the change in density is small and it is then treated as an incompressible fluid.
- When a fluid moves at speeds comparable to its speed of sound, density changes become significant and the flow is termed compressible.
- Such flows are difficult to obtain in liquids, since high pressures of order 1000 atm are needed to generate sonic velocities. In gases, however, a pressure ratio of only 2:1 will likely cause sonic flow. Thus compressible gas flow is quite common, and this subject is often called ***gas dynamics.***

Thermodynamic Relations

Perfect gas

- A perfect gas is one whose individual molecules interact only via direct collisions, with no other intermolecular forces present.
- For such a perfect gas, p, ρ, and the temperature T are related by the following equation of state

$$p = \rho R T$$

- where R is the specific gas constant. For air, R =287J/kg-K°.
- It is convenient at this point to define the specific volume as the limiting volume per unit mass,

$$v \equiv \lim_{\Delta \mathcal{V} \to 0} \frac{\Delta \mathcal{V}}{\Delta m} = \frac{1}{\rho}$$

- which is merely the reciprocal of the density.

Thermodynamic Relations

- The equation of state can now be written as

$$p\upsilon = RT$$

- which is the more familiar thermodynamic form.
- Here R is the gas constant, and

$$R = \frac{R_0}{\mathcal{M}}$$

- where R_o is the universal gas constant (R_o = **8314J/(kg K))** and $\mathcal{M}$ is the molecular weight. For example, for air assuming $\mathcal{M}$ = 28.96, the gas constant is

$$R = \frac{8314}{28.96} = 287\,\mathrm{J/(kg\,K)} = 287\,\mathrm{m^2/(s^2\,K)}$$

Thermodynamic Relations

- Then, assuming internal energy and enthalpy per unit mass **e** and ***h*** respectively,

specific heat at constant volume: $c_v = \left(\frac{\partial e}{\partial T}\right)_v \quad de = c_v \, dT$

Specific heat at constant pressure: $c_p = \left(\frac{\partial h}{\partial T}\right)_p \quad dh = c_p \, dT$

- the specific enthalpy, denoted by h, and related to the other variables by

$$h = e + pv$$

- For a calorically perfect gas, which is an excellent model for air at moderate temperatures both *e* and *h* are directly proportional to the temperature.

Thermodynamic Relations

- Therefore we have

$$e = c_v T$$
$$h = c_p T$$

- where c_v *and* c_p are specific heats at constant volume and constant pressure, respectively.

$$h - e = pv = (c_p - c_v)T$$

- and comparing to the equation of state, we see that

$$c_p - c_v = R$$

- Defining the ratio of specific heats, $\gamma \equiv c_p/c_v$, we can with a bit of algebra write

$$c_v = \frac{1}{\gamma - 1} R$$
$$c_p = \frac{\gamma}{\gamma - 1} R$$

Thermodynamic Relations

- so that c_v and c_p can be replaced with the equivalent variables γ and R. For air, it is handy to remember that

$$\gamma = 1.4 \qquad \frac{1}{\gamma - 1} = 2.5 \qquad \frac{\gamma}{\gamma - 1} = 3.5 \qquad \text{(air)}$$

First Law of Thermodynamics

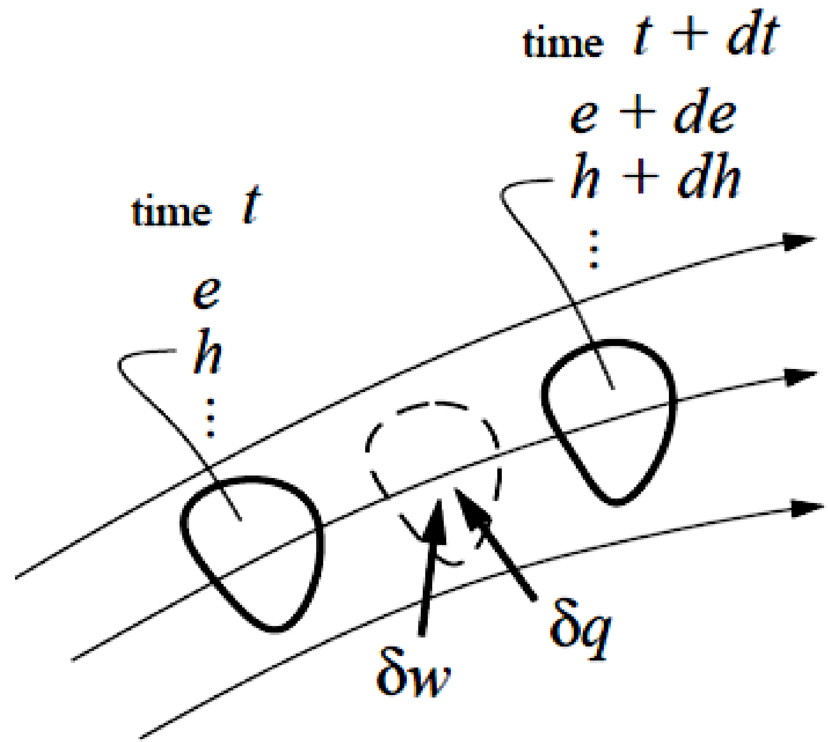
- Consider *a thermodynamic system* consisting of a small Lagrangian control volume (CV) moving with the flow.
- Over the short time interval *dt*, the CV undergoes a process where it receives work δw and heat δq from its surroundings, both per unit mass. This process results in changes in the state of the CV, described by the increments *de*, *dh*, *dp* . . .

Thermodynamic Relations

- The first law of thermodynamics for the process is

$$\delta q + \delta w = de$$

- This states that whatever energy is added to the system, whether by heat or by work, it must appear as an increase in the internal energy of the system.



1. *Adiabatic process*, where no heat is transferred, or $\delta q = 0$. This rules out heating of the CV via conduction though its boundary, or by combustion inside the CV.

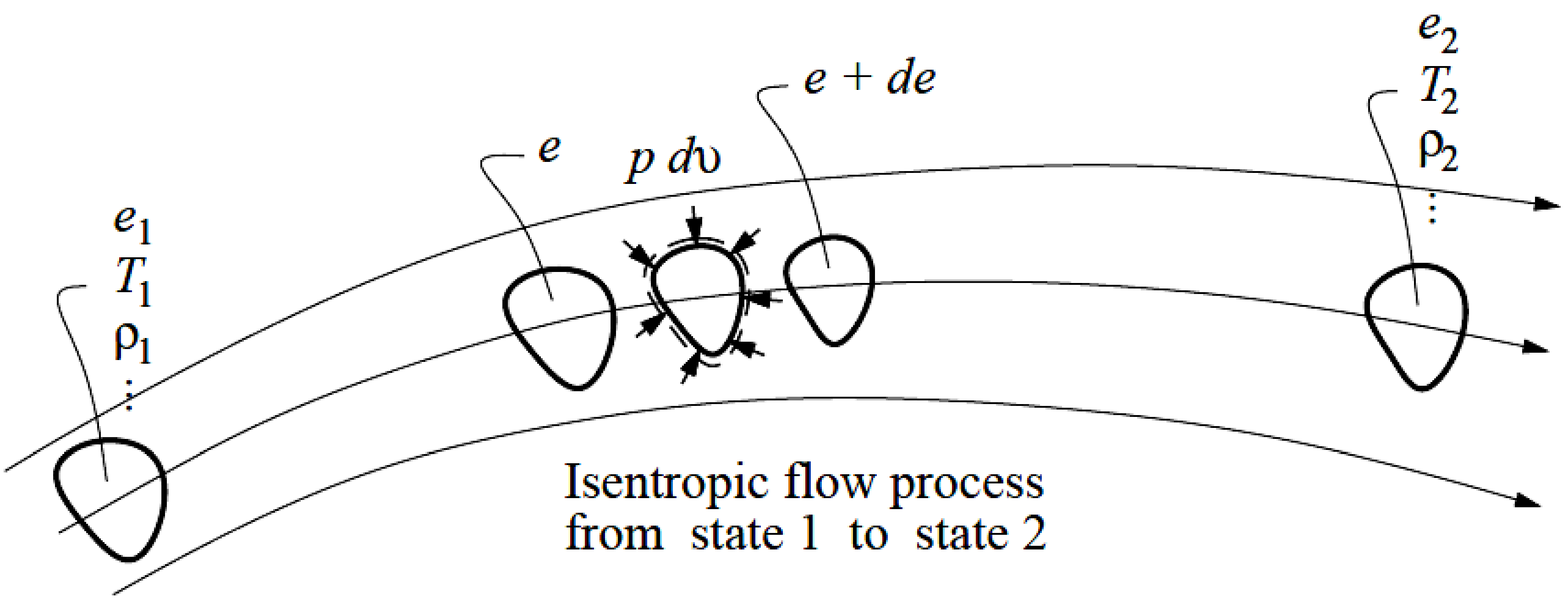
2. *Reversible process*, no dissipation occurs, implying that work must be only via volumetric compression, or $dw = -p\, dv$. This rules out work done by friction forces.

3. *Isentropic process*, which is both adiabatic and reversible, implying $-p\, dv = de$.

Thermodynamic Relations

Isentropic relations

- Aerodynamic flows are effectively inviscid outside of boundary layers. This implies they have negligible heat conduction and friction forces, and hence are isentropic.
- Therefore, along the pathline followed by the CV in the figure above, the isentropic version of the first law applies



Thermodynamic Relations

$$-p\,dv \;=\; de$$

- This relation can be integrated after a few substitutions. First we note that

$$dv \;=\; d\left(\frac{1}{\rho}\right) \;=\; -\frac{d\rho}{\rho^2}$$

- and with the perfect gas relation

$$de \;=\; c_v\,dT \;=\; \frac{1}{\gamma - 1}\,R\,dT$$

- the isentropic first law becomes

$$p\,\frac{d\rho}{\rho^2} \;=\; \frac{1}{\gamma - 1}\,R\,dT \;\rightarrow\; \begin{aligned} \frac{d\rho}{\rho} &= \frac{1}{\gamma - 1}\,\frac{\rho R}{p}\,dT \\ \frac{d\rho}{\rho} &= \frac{1}{\gamma - 1}\,\frac{dT}{T} \end{aligned}$$

Thermodynamic Relations

- The final form can now be integrated from any state 1 to any state 2 along the pathline.

$$\ln \rho = \frac{1}{\gamma - 1} \ln T + \text{const.}$$

$$\rho = \text{const.} \times T^{1/(\gamma-1)}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)}$$

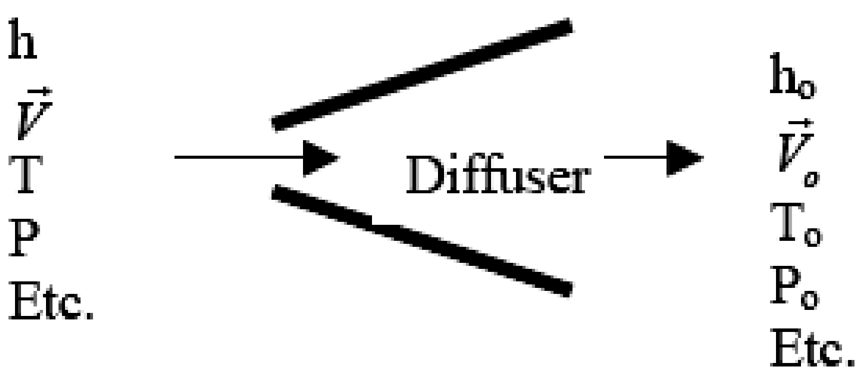
- From the equation of state we also have $\displaystyle \frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} \frac{T_1}{T_2}$

- Which gives the alternative isentropic relation

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

Stagnation Properties

- Consider a fluid flowing into a diffuser at a velocity $\vec{V}$, temperature T, pressure P, and enthalpy h, etc. Here the ordinary properties T, P, h, etc. are called the static properties; that is, they are measured relative to the flow at the flow velocity.
- If the diffuser is sufficiently long and the exit area is sufficiently large that the fluid is **brought to rest (zero velocity) at the diffuser exit** while no work or heat transfer is done. The resulting state is called the **stagnation state**.



Stagnation Properties

- We apply the first law per unit mass for one entrance, one exit, and neglect the potential energies. Let the inlet state be unsubscripted and the exit or stagnation state have the subscript o.

$$q_{net} + h + \frac{\vec{V}^2}{2} = w_{net} + h_o + \frac{\vec{V}_o^2}{2}$$

- Since the exit velocity, work, and heat transfer are zero,

$$h_o = h + \frac{\vec{V}^2}{2}$$

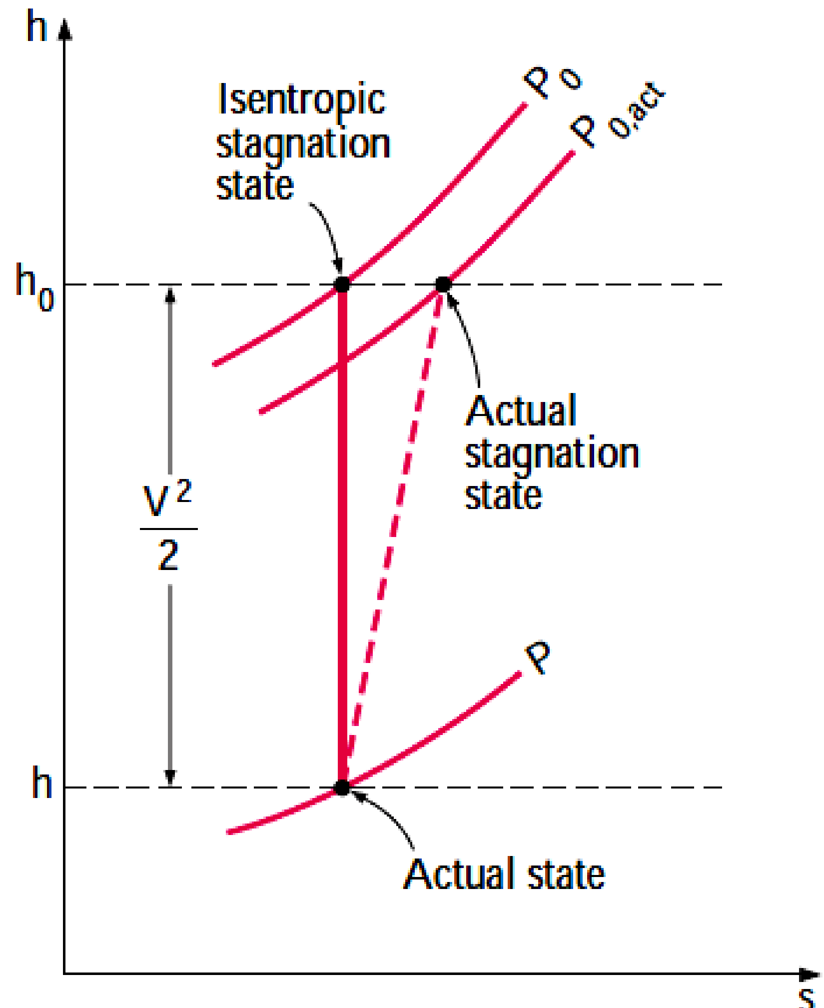
- The term h_o is called the **stagnation enthalpy** (some authors call this the total enthalpy).
- It is the enthalpy the fluid attains when brought to rest adiabatically while no work is done.

Stagnation Properties

- If, in addition, the process is also reversible, the process is isentropic, and the inlet and exit entropies are equal.

$$s_o = s$$

- The stagnation enthalpy and entropy define the stagnation state and the isentropic stagnation pressure, P_o.
- The actual stagnation pressure for irreversible flows will be somewhat less than the isentropic stagnation pressure as shown in the fig.



Example 1

- Steam at 400°C, 1.0 MPa, and 300 m/s flows through a pipe. Find the properties of the steam at the stagnation state.

Solution

- At T = 400°C and P = 1.0 MPa,

$$h = 3264.5 \text{ kJ/kg} \qquad s = 7.4670 \text{ kJ/kg·K}$$

Then

$$h_o = h + \frac{\vec{V}^2}{2}$$

$$= 3264.5\frac{kJ}{kg} + \frac{\left(300\frac{m}{s}\right)^2}{2}\frac{\frac{kJ}{kg}}{1000\frac{m^2}{s^2}}$$

$$= 3309.5\frac{kJ}{kg}$$

Example 1…. solution

and

$$s_o = s = 7.4670 \frac{kJ}{kg \cdot K}$$

$$h_o = h(P_o, s_o)$$

- We can find P_o by trial and error. The resulting stagnation properties are

$$P_o = 1.16\, MPa$$

$$T_o = 422.2^o C$$

$$\rho_o = \frac{1}{v_o} = 3.640 \frac{kg}{m^3}$$

Ideal Gas Result

Rewrite the equation defining the stagnation enthalpy as

$$h_o - h = \frac{\vec{V}^2}{2}$$

For ideal gases with constant specific heats, the enthalpy difference becomes

$$C_P(T_o - T) = \frac{\vec{V}^2}{2}$$

where T_o is defined as the stagnation temperature.

$$T_o - T = \frac{\vec{V}^2}{2C_P}$$

For the isentropic process, the stagnation pressure can be determined from

$$\frac{T_o}{T} = \left(\frac{P_o}{P}\right)^{(k-1)/k}$$

or

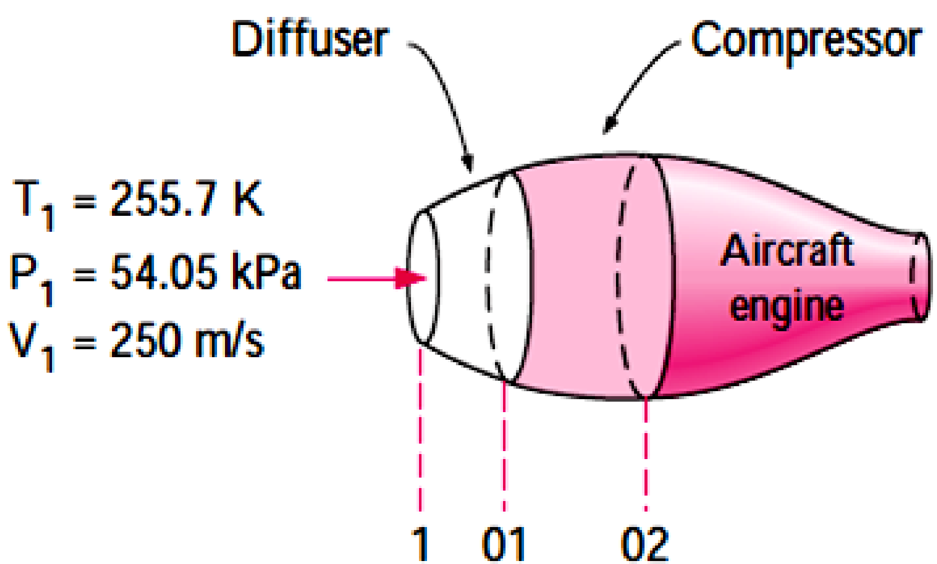
$$\frac{P_o}{P} = \left(\frac{T_o}{T}\right)^{k/(k-1)}$$

The ratio of the stagnation density to static density can be expressed as

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}$$

Example 2

- An aircraft is flying at a cruising speed of 250 m/s at an altitude of 5000 m where the atmospheric pressure is 54.05 kPa and the ambient air temperature is 255.7 K. The ambient air is first decelerated in a diffuser before it enters the compressor. Assuming both the diffuser and the compressor to be isentropic, determine (a) the stagnation pressure at the compressor inlet and (b) the required compressor work per unit mass if the stagnation pressure ratio of the compressor is 8.



Solution

- High-speed air enters the diffuser and the compressor of an aircraft. The stagnation pressure of the air and the compressor work input are to be determined.
- ***Assumptions*** 1 Both the diffuser and the compressor are isentropic. 2 Air is an ideal gas with constant specific heats at room temperature.
- ***Properties*** The constant-pressure specific heat c_p and the specific heat ratio k of air at room temperature are

$$c_p = 1.005 \text{ kJ/kg . K} \quad \text{and} \quad k = 1.4$$

- ***Analysis***

(a) the stagnation temperature T_{01} at the compressor inlet can be determined from

Solution

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 255.7\ \text{K} + \frac{(250\ \text{m/s})^2}{(2)(1.005\ \text{kJ/kg}\cdot\text{K})}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right)$$
$$= 286.8\ \text{K}$$

- Then

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (54.05\ \text{kPa})\left(\frac{286.8\ \text{K}}{255.7\ \text{K}}\right)^{1.4/(1.4-1)}$$
$$= \mathbf{80.77\ kPa}$$

- That is, the temperature of air would increase by 31.1◦C and the pressure by 26.72 kPa as air is decelerated from 250 m/s to zero velocity. These increases in the temperature and pressure of air are due to the conversion of the kinetic energy into enthalpy.

Solution

- To determine the compressor work, we need to know the stagnation temperature of air at the compressor exit T_{02}.

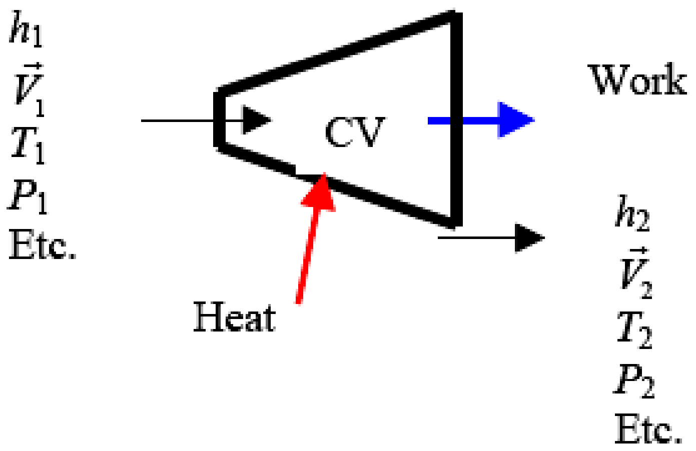
$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (286.8\text{ K})(8)^{(1.4-1)/1.4} = 519.5\text{ K}$$

- Disregarding potential energy changes and heat transfer, the compressor work per unit mass of air is determined from

$$\begin{aligned} w_{in} &= c_p(T_{02} - T_{01}) \\ &= (1.005\text{ kJ/kg}\cdot\text{K})(519.5\text{ K} - 286.8\text{ K}) \\ &= \mathbf{233.9\text{ kJ/kg}} \end{aligned}$$

Thus the work supplied to the compressor is 233.9 kJ/kg.

Conservation of Energy for Control Volumes Using Stagnation Properties



The steady-flow conservation of energy for the above figure is

$$\dot{Q}_{net} + \sum_{inlets} \dot{m}_i \left(h + \frac{\vec{V}^2}{2} + gz \right)_i = \dot{W}_{net} + \sum_{outlets} \dot{m}_e \left(h + \frac{\vec{V}^2}{2} + gz \right)_e$$

Since

$$h_o = h + \frac{\vec{V}^2}{2}$$

$$\dot{Q}_{net} + \sum_{inlets} \dot{m}_i \left(h_o + gz \right)_i = \dot{W}_{net} + \sum_{outlets} \dot{m}_e \left(h_o + gz \right)_e$$

For no heat transfer, one entrance, one exit, this reduces to

$$\dot{W}_{net} = \dot{m}\left((h_{o1} - h_{o2}) + g(z_1 - z_2)\right)$$

If we neglect the change in potential energy, this becomes

$$\dot{W}_{net} = \dot{m}\left(h_{o1} - h_{o2}\right)$$

For ideal gases with constant specific heats we write this as

$$\dot{W}_{net} = \dot{m}C_P\left(T_{o1} - T_{o2}\right)$$

Conservation of Energy for a Nozzle

We assume steady-flow, no heat transfer, no work, one entrance, and one exit and neglect elevation changes; then the conservation of energy becomes

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_{o1} = \dot{m}_2 h_{o2}$$

But $\dot{m}_1 = \dot{m}_2$ thus $h_{o1} = h_{o2}$

Thus the stagnation enthalpy remains constant throughout the nozzle. At any cross section in the nozzle, the stagnation enthalpy is the same as that at the entrance. For ideal gases this last result becomes

$$T_{o1} = T_{o2}$$

Thus the stagnation temperature remains constant through out the nozzle. At any cross section in the nozzle, the stagnation temperature is the same as that at the entrance.

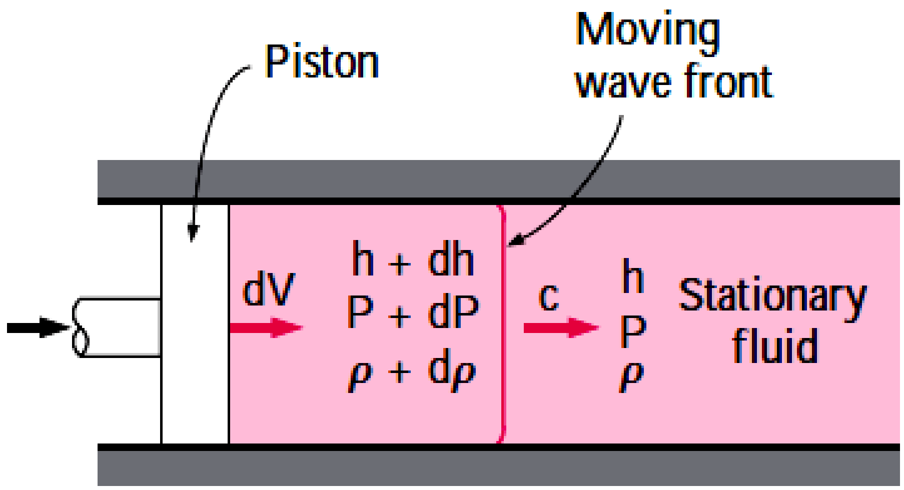
Assuming an isentropic process for flow through the nozzle, we can write for the entrance and exit states

$$\frac{P_{o2}}{P_{o1}} = \left(\frac{T_{o2}}{T_{o1}}\right)^{k/(k-1)}$$

So we see that the stagnation pressure is also constant through out the nozzle for isentropic flow.

Speed of Sound and Mach number

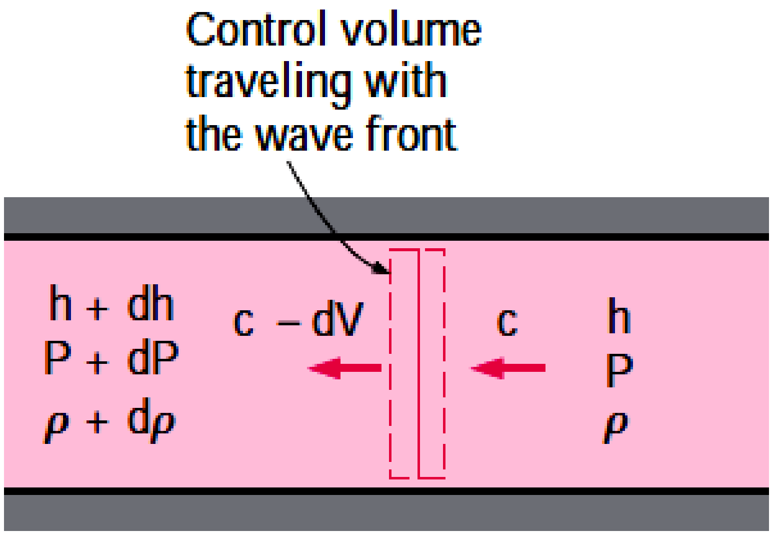
- An important parameter in the study of compressible flow is the **speed of sound** (or the **sonic speed**), which is the speed at which an infinitesimally small pressure wave travels through a medium.
- The pressure wave may be caused by a small disturbance, which creates a slight rise in local pressure.
- To obtain a relation for the speed of sound in a medium, consider a duct that is filled with a fluid at rest,



Speed of Sound and Mach number

- A piston fitted in the duct is now moved to the right with a constant incremental velocity dV, creating a sonic wave.
- The wave front moves to the right through the fluid at the speed of sound c and separates the moving fluid adjacent to the piston from the fluid still at rest.
- The fluid to the left of the wave front experiences an incremental change in its thermodynamic properties, while the fluid on the right of the wave front maintains its original thermodynamic properties, as shown in Fig.
- To simplify the analysis, consider a control volume that encloses the wave front and moves with it, as shown in the fig. below.
- To an observer traveling with the wave front, the fluid to the right will appear to be moving toward the wave front with a speed of c and the fluid to the left to be moving away from the wave front with a speed of c - dV.

Speed of Sound and Mach number



The mass balance for this single-stream, steady-flow process can be expressed as

$$\dot{m}_{right} = \dot{m}_{left}$$

or

$$\rho A c = (\rho + d\rho)A(c - dV)$$

Speed of Sound and Mach number

By canceling the cross-sectional (or flow) area A and neglecting the higher-order terms, this equation reduces to

$$c\,d\rho - \rho\,dV = 0 \qquad \text{(a)}$$

No heat or work crosses the boundaries of the control volume during this steady-flow process, and the potential energy change can be neglected. Then the steady-flow energy balance $e_{in} = e_{out}$ becomes

$$h + \frac{c^2}{2} = h + dh + \frac{(c - dV)^2}{2}$$

$$h + \frac{C^2}{2} = (h + dh) + \frac{(C^2 - 2Cd\vec{V} + d\vec{V}^2)}{2}$$

Cancel terms and neglect $d\vec{V}^2$; we have

$$dh - Cd\vec{V} = 0$$

Now, apply the conservation of mass or continuity equation

$$\dot{m} = \rho A \vec{V}$$

to the control volume.

$$\rho AC = (\rho + d\rho) A (C - d\vec{V})$$

$$\rho AC = A(\rho C - \rho d\vec{V} + C d\rho - d\rho d\vec{V})$$

Cancel terms and neglect the higher-order terms like $d\rho d\vec{V}$.

We have

$$C\, d\rho - \rho\, d\vec{V} = 0$$

Also, we consider the property relation $dh = T\, ds + v\, dP$

$$dh = T\, ds + \frac{1}{\rho} dP$$

Let's assume the process to be isentropic; then $ds = 0$ and

$$dh = \frac{1}{\rho} dP$$

Using the results of the first law

$$dh = \frac{1}{\rho} dP = C\, d\vec{V}$$

From the continuity equation

$$d\vec{V} = \frac{C\, d\rho}{\rho}$$

Now we have

$$\frac{1}{\rho} dP = C\left(\frac{C\, d\rho}{\rho}\right)$$

Thus
$$\frac{dP}{d\rho} = C^2$$

Since the process is assumed to be isentropic, the above becomes

$$\left(\frac{\partial P}{\partial \rho}\right)_s = C^2$$

By using thermodynamic property relations this can be written as

$$C^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

where k is the ratio of specific heats, $k = C_P/C_V$.

Ideal Gas Result

$$P = \rho RT$$

For ideal gases $\rightarrow$

$$\left(\frac{\partial P}{\partial \rho}\right)_T = RT$$

$$C^2 = kRT$$

$$C = \sqrt{kRT}$$

Example -3

Find the speed of sound in air at an altitude of 5000 m.

At 5000 m, T = 255.7 K.

$$C = \sqrt{1.4(0.287\frac{kJ}{kg \cdot K})(255.7K)\frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}}}$$

$$= 320.5\frac{m}{s}$$

Notice that the temperature used for the speed of sound is the static (normal) temperature.

Example -4

Find the speed of sound in steam where the pressure is 1 MPa and the temperature is 350^{o}C.

At P = 1 MPa, T = 350^{o}C,

$$C = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} = \sqrt{\left(\frac{\partial P}{\partial\left(\frac{1}{v}\right)}\right)_s}$$

Here, we approximate the partial derivative by perturbating the pressure about 1 MPa. Consider using $P \pm 0.025$ MPa at the entropy value s = 7.3011 kJ/kg· K, to find the corresponding specific volumes.

$$C = \sqrt{\frac{(1025-975)kPa}{\left(\frac{1}{10.2773}-\frac{1}{10.2882}\right)\frac{kg}{m^3}} \frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}} \frac{kJ}{m^3 kPa}}$$

$$= 605.5\frac{m}{s}$$

What is the speed of sound for steam at 350°C assuming ideal-gas behavior?

Assume k = 1.3, then

$$C = \sqrt{1.3(0.4615\frac{kJ}{kg \cdot K})(350+273)K\frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}}}$$

$$= 611.4\frac{m}{s}$$

Mach Number

The Mach number M is defined as $M = \frac{\vec{V}}{C}$

$M < 1$ flow is subsonic
$M = 1$ flow is sonic
$M > 1$ flow is supersonic

Example -5

In the air and steam examples above, find the Mach number if the air velocity is 250 m/s and the steam velocity is 300 m/s.

$$M_{air} = \frac{250\frac{m}{s}}{320.5\frac{m}{s}} = 0.780$$

$$M_{steam} = \frac{300\frac{m}{s}}{605.5\frac{m}{s}} = 0.495$$

The flow parameters T_o/T, P_o/P, ρ_o/ρ, etc. are related to the flow Mach number. Let's consider ideal gases, then

$$T_o = T + \frac{\vec{V}^2}{2C_P}$$

$$\frac{T_o}{T} = 1 + \frac{\vec{V}^2}{2C_P T}$$

but

$$C_P = \frac{k}{k-1} R \quad or \quad \frac{1}{C_P} = \frac{k-1}{kR}$$

$$\frac{T_o}{T} = 1 + \frac{\vec{V}^2}{2T} \frac{(k-1)}{kR}$$

and

$$C^2 = kRT$$

so

$$\frac{T_o}{T} = 1 + \frac{(k-1)}{2} \frac{\vec{V}^2}{C^2}$$

$$= 1 + \frac{(k-1)}{2} M^2$$

The pressure ratio is given by

$$\frac{P_o}{P} = \left(\frac{T_o}{T} \right)^{k/(k-1)}$$

$$= \left(1 + \frac{(k-1)}{2} M^2 \right)^{k/(k-1)}$$

We can show the density ratio to be

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{1/(k-1)}$$

$$= \left(1 + \frac{(k-1)}{2} M^2\right)^{1/(k-1)}$$

For the Mach number equal to 1, the sonic location, the static properties are denoted with a superscript "*". This condition, when $M = 1$, is called the sonic condition. When $M = 1$ and $k = 1.4$, the static-to-stagnation ratios are

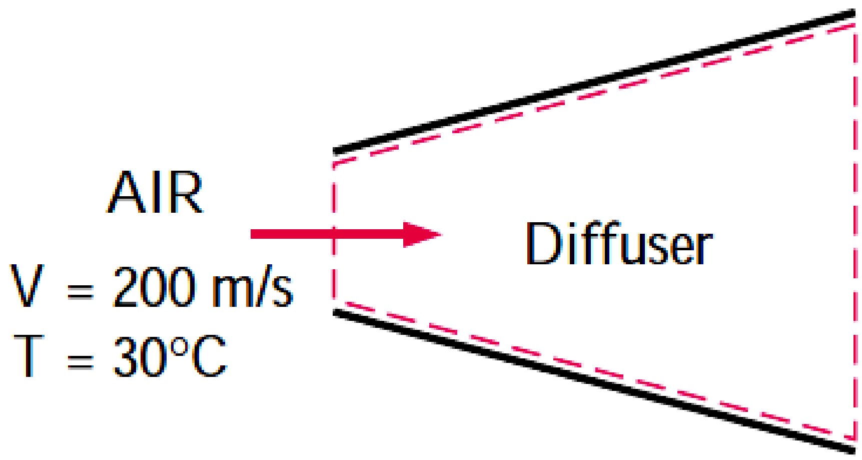
$$\frac{T^*}{T_o} = \frac{2}{k+1} = 0.83333$$

$$\frac{P^*}{P_o} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = 0.52828$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{k+1}\right)^{1/(k-1)} = 0.63394$$

Example. Mach Number of Air Entering a Diffuser

- Air enters a diffuser shown in Fig. with a velocity of 200 m/s. Determine (a) the speed of sound and (b) the Mach number at the diffuser inlet when the air temperature is 30°C.



SOLUTION Air enters a diffuser with a high velocity. The speed of sound and the Mach number are to be determined at the diffuser inlet.

Assumption Air at specified conditions behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kJ/kg · K, and its specific heat ratio at 30°C is 1.4.

Analysis We note that the speed of sound in a gas varies with temperature, which is given to be 30°C.

(*a*) The speed of sound in air at 30°C is determined from Eq. 12–11 to be

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(303 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{349 \text{ m/s}}$$

(*b*) Then the Mach number becomes

$$\text{Ma} = \frac{V}{c} = \frac{200 \text{ m/s}}{349 \text{ m/s}} = \mathbf{0.573}$$

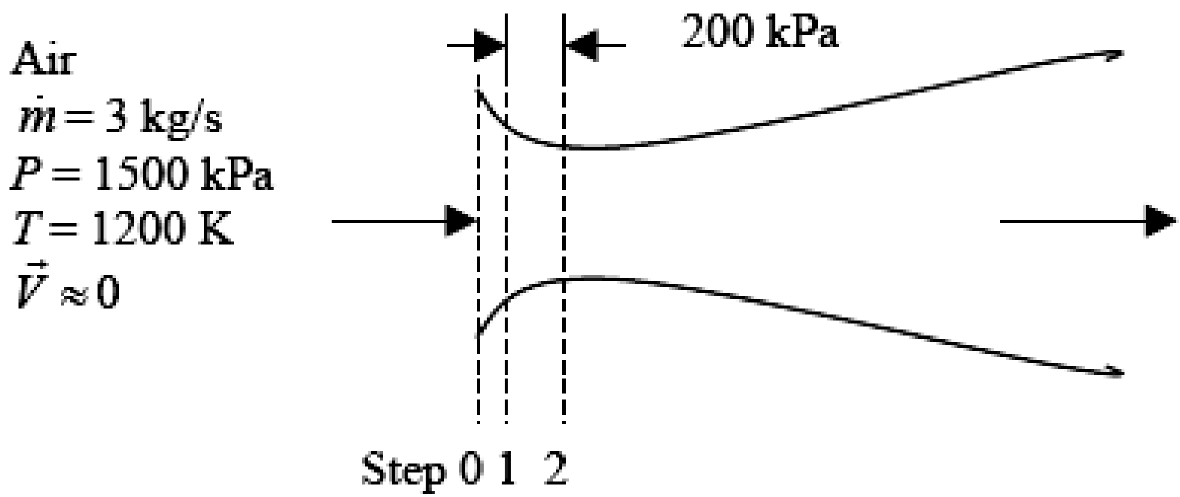
Discussion The flow at the diffuser inlet is subsonic since Ma < 1.

One-Dimensional Isentropic Flow

- During fluid flow through many devices such as nozzles, diffusers, and turbine blade passages, flow quantities vary primarily in the flow direction only, and the flow can be approximated as one-dimensional isentropic flow with good accuracy.

Effect of Area Changes on Flow Parameters

- Consider the isentropic steady flow of an ideal gas through the nozzle shown below.



Air flows steadily through a varying-cross-sectional-area duct such as a nozzle at a flow rate of 3 kg/s. The air enters the duct at a low velocity at a pressure of 1500 kPa and a temperature of 1200 K and it expands in the duct to a pressure of 100 kPa. The duct is designed so that the flow process is isentropic. Determine the pressure, temperature, velocity, flow area, speed of sound, and Mach number at each point along the duct axis that corresponds to a pressure drop of 200 kPa.

Since the inlet velocity is low, the stagnation properties equal the static properties.

$$T_o = T_1 = 1200\,K, \quad P_o = P_1 = 1500\,kPa$$

After the first 200 kPa pressure drop, we have

$$T = T_o\left(\frac{P}{P_o}\right)^{(k-1)/k} = 1200K\left(\frac{1300\,kPa}{1500\,kPa}\right)^{(1.4-1)/1.4}$$

$$= 1151.9\,K$$

$$\vec{V} = \sqrt{2C_P(T_0 - T)}$$

$$= \sqrt{2(1.005\frac{kJ}{kg\cdot K})(1200 - 1151.9)K\frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}}}$$

$$= 310.77\frac{m}{s}$$

$$\rho = \frac{P}{RT} = \frac{(1300kPa)}{(0.287\frac{kJ}{kg\cdot K})(1151.9K)}\frac{kJ}{m^3kPa}$$

$$= 3.932\frac{kg}{m^3}$$

$$A = \frac{\dot{m}}{\rho \vec{V}} = \frac{3\frac{kg}{s}}{(3.9322\frac{kg}{m^3})(310.77\frac{m}{s})} \frac{10^4 cm^2}{m^2}$$

$$= 24.55 cm^2$$

$$C = \sqrt{kRT} = \sqrt{1.4(0.287\frac{kJ}{kg \cdot K})(1151.9K)\frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}}}$$

$$= 680.33\frac{m}{s}$$

$$M = \frac{\vec{V}}{C} = \frac{310.77\frac{m}{s}}{680.33\frac{m}{s}} = 0.457$$

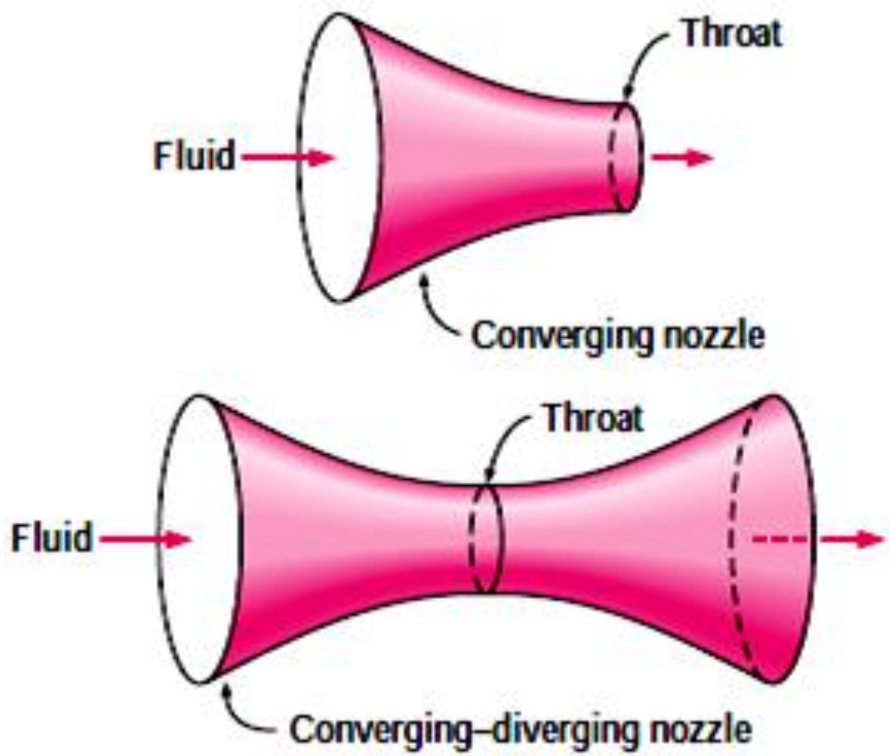
Now we tabulate the results for the other 200 kPa increments in the pressure until we reach 100 kPa.

Summary of Results for Nozzle Problem

Step	P kPa	T K	$\vec{V}$ m/s	ρ kg/m^3	C m/s	A cm^2	M
0	1500	1200	0	4.3554	694.38	∞	0
1	1300	1151.9	310.77	3.9322	680.33	24.55	0.457
2	1100	1098.2	452.15	3.4899	664.28	19.01	0.681
3	900	1037.0	572.18	3.0239	645.51	17.34	0.886
4	792.4	1000.0	633.88	2.7611	633.88	17.14	1.000
5	700	965.2	786.83	2.5270	622.75	17.28	1.103
6	500	876.7	805.90	1.9871	593.52	18.73	1.358
7	300	757.7	942.69	1.3796	551.75	23.07	1.709
8	100	553.6	1139.62	0.6294	471.61	41.82	2.416

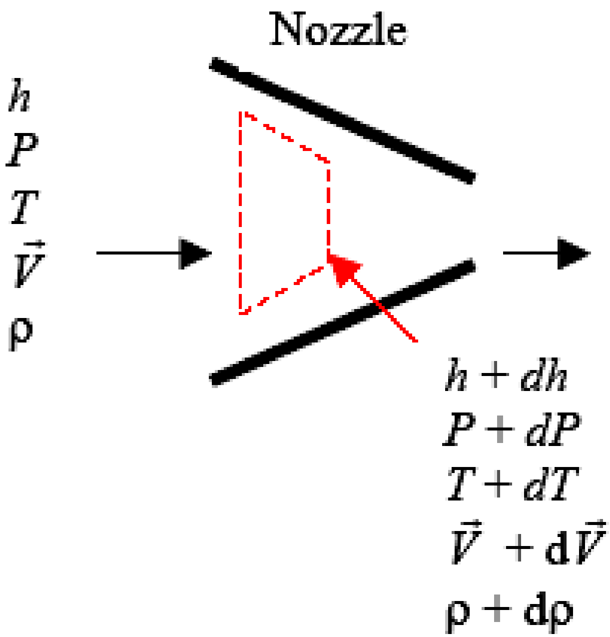
Note that at P = 797.42 kPa, M = 1.000, and this state is the critical state.

- We note from the Nozzle Example that the flow area decreases with decreasing pressure down to a critical-pressure value where the Mach number is unity, and then it begins to increase with further reductions in pressure.
- The Mach number is unity at the location of smallest flow area, called the **throat** .
- Note that the velocity of the fluid keeps increasing after passing the throat although the flow area increases rapidly in that region. This increase in velocity past the throat is due to the rapid decrease in the fluid density.



- The flow area of the duct considered in this example first decreases and then increases. Such ducts are called **converging–diverging nozzles.** These nozzles are used to accelerate gases to supersonic speeds and should not be confused with Venturi nozzles, which are used strictly for incompressible flow.

Now let's see why these relations work this way. Consider the nozzle and control volume shown below.



The first law for the control volume is

$$dh + \vec{V} d\vec{V} = 0$$

The continuity equation for the control volume $\dot{m} = \rho A \vec{V}$ yields

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{d\vec{V}}{\vec{V}} = 0$$

Also, we consider the property relation for an isentropic process

$$Tds = dh - \frac{dP}{\rho} = 0$$

and the Mach Number relation $\frac{dP}{d\rho} = C^2 = \frac{\vec{V}^2}{M^2}$

Putting these four relations together yields

$$\frac{dA}{A} = \frac{dP}{\rho\vec{V}^2}(1 - M^2)$$

Let's consider the implications of this equation for both nozzles and diffusers. A **nozzle** is a device that increases fluid velocity while causing its pressure to drop; thus, $d\vec{V} > 0$, $dP < 0$.

Nozzle Results $\frac{dA}{A} = \frac{dP}{\rho\vec{V}^2}(1 - M^2)$

Subsonic: $M < 1 \quad dP(1 - M^2) < 0 \quad dA < 0$

Sonic: $M = 1 \quad dP(1 - M^2) = 0 \quad dA = 0$

Supersonic: $M > 1 \quad dP(1 - M^2) > 0 \quad dA > 0$

To accelerate subsonic flow, the nozzle flow area must first decrease in the flow direction. The flow area reaches a minimum at the point where the Mach number is unity. To continue to accelerate the flow to supersonic conditions, the flow area must increase.

The minimum flow area is called the **throat** of the nozzle.

We are most familiar with the shape of a subsonic nozzle. That is, the flow area in a subsonic nozzle decreases in the flow direction.

A **diffuser** is a device that decreases fluid velocity while causing its pressure to rise; thus, $d\vec{V} < 0$, $dP > 0$.

Diffuser Results

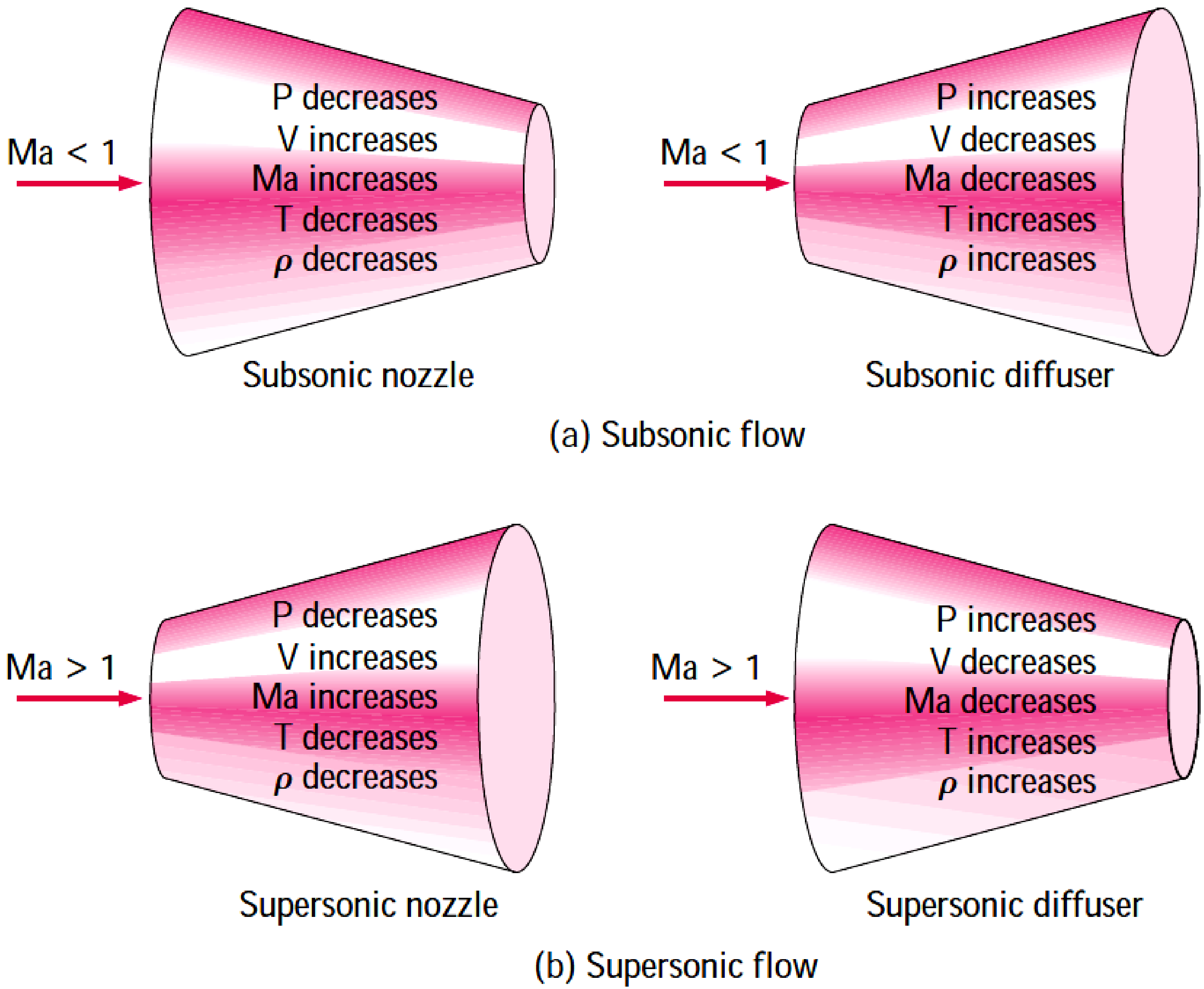
$$\frac{dA}{A} = \frac{dP}{\rho \vec{V}^2}(1 - M^2)$$

$$\text{Subsonic:}\ M < 1 \quad dP(1 - M^2) > 0 \quad dA > 0$$

$$\text{Sonic:}\ M = 1 \quad dP(1 - M^2) = 0 \quad dA = 0$$

$$\text{Supersonic:}\ M > 1 \quad dP(1 - M^2) < 0 \quad dA < 0$$

To diffuse supersonic flow, the diffuser flow area must first decrease in the flow direction. The flow area reaches a minimum at the point where the Mach number is unity. To continue to diffuse the flow to subsonic conditions, the flow area must increase. We are most familiar with the shape of a subsonic diffuser. That is the flow area in a subsonic diffuser increases in the flow direction.


Ma < 1
P decreases
V increases
Ma increases
T decreases
ρ decreases
Subsonic nozzle
Ma < 1
P increases
V decreases
Ma decreases
T increases
ρ increases
Subsonic diffuser
(a) Subsonic flow
Ma > 1
P decreases
V increases
Ma increases
T decreases
ρ decreases
Supersonic nozzle
Ma > 1
P increases
V decreases
Ma decreases
T increases
ρ increases
Supersonic diffuser
(b) Supersonic flow

Equation of Mass Flow Rate through a Nozzle

Let's obtain an expression for the flow rate through a converging nozzle at any location as a function of the pressure at that location. The mass flow rate is given by

$$\dot{m} = \rho A \vec{V}$$

The velocity of the flow is related to the static and stagnation enthalpies.

$$\vec{V} = \sqrt{2(h_o - h)} = \sqrt{2C_P(T_0 - T)} = \sqrt{2C_P T_0 \left(1 - \frac{T}{T_o}\right)}$$

and

$$\frac{T}{T_o} = \left(\frac{P}{P_o}\right)^{(k-1)/k}$$

$$\vec{V} = \sqrt{2C_P T_0 \left(1 - \left(\frac{P}{P_o}\right)^{(k-1)/k}\right)}$$

Write the mass flow rate as

$$\dot{m} = A\vec{V}\rho_o \frac{\rho}{\rho_o}$$

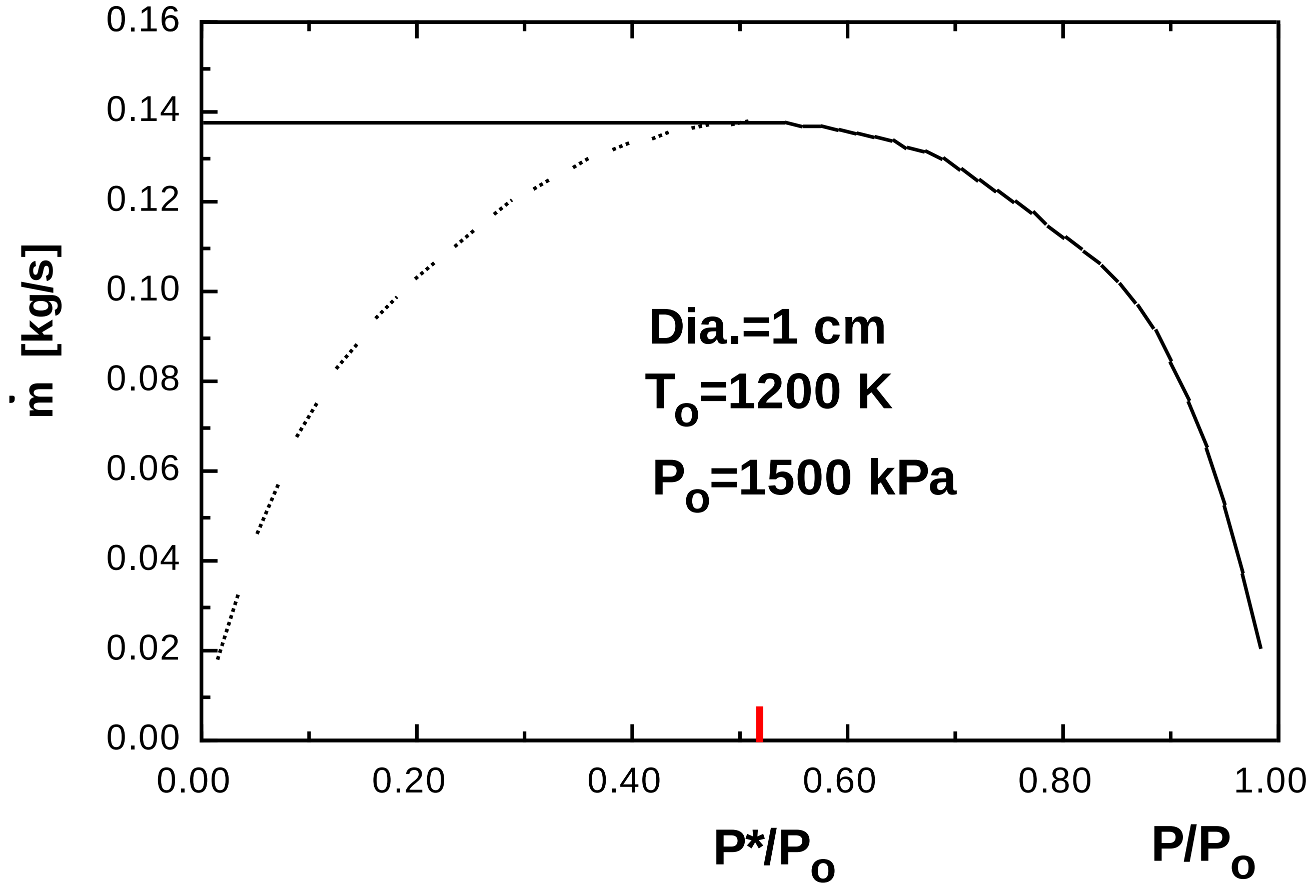
$$\frac{\rho}{\rho_o} = \left(\frac{P}{P_o}\right)^{1/k}$$

We note from the ideal-gas relations that

$$\rho_o = \frac{P_o}{RT_o}$$

$$\dot{m} = AP_o\sqrt{\frac{2k}{(k-1)RT_o}}\sqrt{\left(\frac{P}{P_o}\right)^{2/k} - \left(\frac{P}{P_o}\right)^{(k+1)/k}}$$

What pressure ratios make the mass flow rate zero?
Do these values make sense?
Now let's make a plot of mass flow rate versus the static-to-stagnation pressure ratio.



This plot shows there is a value of P/P_o that makes the mass flow rate a maximum. To find that mass flow rate, we note

$$\frac{d\dot{m}}{d\left(\frac{P}{P_o}\right)} = 0$$

The result is

$$\frac{P}{P_o} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = \frac{P^*}{P_o}$$

So the pressure ratio that makes the mass flow rate a maximum is the same pressure ratio at which the Mach number is unity at the flow cross-sectional area. This value of the pressure ratio is called the **critical pressure ratio** for nozzle flow. For pressure ratios less than the critical value, the nozzle is said to be **choked**. When the nozzle is choked, the mass flow rate is the maximum possible for the flow area, stagnation pressure, and stagnation temperature. Reducing the pressure ratio below the critical value will not increase the mass flow rate.

What is the expression for mass flow rate when the nozzle is choked?

Using

$$\frac{P_o}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)}$$

The mass flow rate becomes

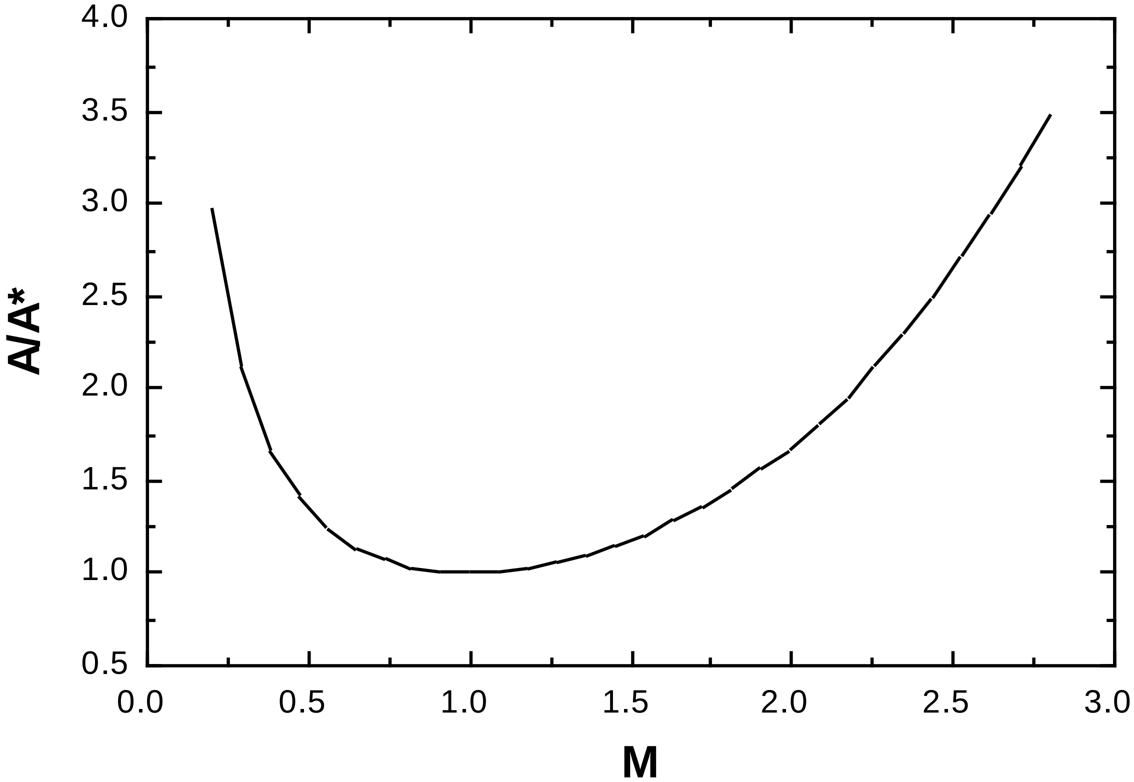
$$\dot{m} = AP_o \sqrt{\frac{k}{RT_o}} \left[\frac{M}{\left(1 + \frac{k-1}{2} M^2\right)^{(k+1)/[2(k-1)]}} \right]$$

When the Mach number is unity, $M = 1$, $A = A^*$

$$\dot{m} = A^* P_o \sqrt{\frac{k}{RT_o}} \left(\frac{2}{k+1}\right)^{(k+1)/[2(k-1)]}$$

Taking the ratio of the last two results gives the ratio of the area of the flow A at a given Mach number to the area where the Mach number is unity, A^*.

Then $$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}M^2\right)\right]^{(k+1)/[2(k-1)]}$$

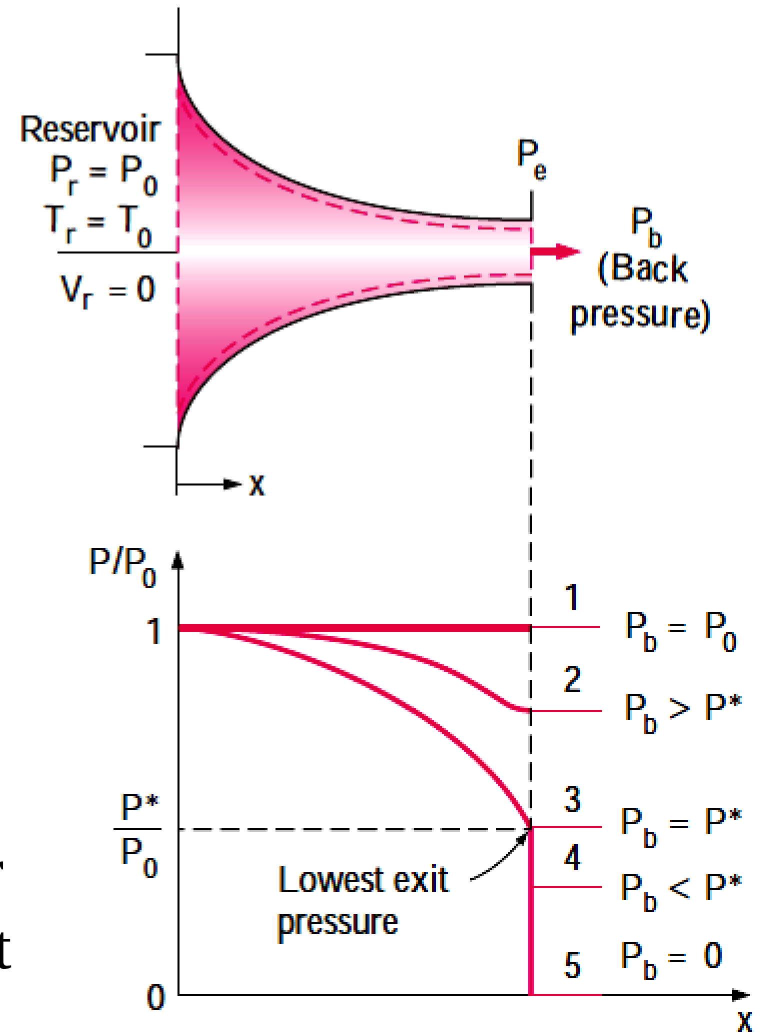


From the above plot we note that for each *A*/*A** there are two values of *M*: one for subsonic flow at that area ratio and one for supersonic flow at that area ratio. The area ratio is unity when the Mach number is equal to one.

Effect of Back Pressure on Flow through a Converging Nozzle

Consider the converging nozzle shown below. The flow is supplied by a reservoir at pressure P_r and temperature T_r. The reservoir is large enough that the velocity in the reservoir is zero.

Let's plot the ratio P/P_o along the length of the nozzle, the mass flow rate through the nozzle, and the exit plane pressure P_e as the back pressure P_b is varied. Let's consider isentropic flow so that P_o is constant throughout the nozzle.



Effect of Back Pressure on Flow through a Converging Nozzle

- Now we begin to reduce the back pressure and observe the resulting effects on the pressure distribution along the length of the nozzle, as shown in the Fig. above.
- If the back pressure P_b is equal to P_1, which is equal to P_r, there is no flow and the pressure distribution is uniform along the nozzle.
- When the back pressure is reduced to P_2, the exit plane pressure P_e also drops to P_2. This causes the pressure along the nozzle to decrease in the flow direction.
- When the back pressure is reduced to P3 (= P*, which is the pressure required to increase the fluid velocity to the speed of sound at the exit plane or throat), the mass flow reaches a maximum value and the flow is said to be **choked.**

Effect of Back Pressure on Flow through a Converging Nozzle

- Further reduction of the back pressure to level P_4 or below does not result in additional changes in the pressure distribution, or anything else along the nozzle length.
- Under steady-flow conditions, the mass flow rate through the nozzle is constant and can be expressed as

$$\dot{m} = \rho A V = \left(\frac{P}{RT}\right) A\left(Ma\sqrt{kRT}\right) = PAMa\sqrt{\frac{k}{RT}}$$

- Solving for *T* from $\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)Ma^2$ and for P from

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)Ma^2\right]^{k/(k-1)}$$

and substituting

$$\dot{m} = \frac{AMaP_0\sqrt{k/(RT_0)}}{[1 + (k-1)Ma^2/2]^{(k+1)/[2(k-1)]}}$$

Effect of Back Pressure on Flow through a Converging Nozzle

- Thus the mass flow rate of a particular fluid through a nozzle is a function of the stagnation properties of the fluid, the flow area, and the Mach number.
- The maximum mass flow rate can be determined by differentiating the above equation with respect to Ma and setting the result equal to zero. It yields Ma = 1.
- Since the only location in a nozzle where the Mach number can be unity is the location of minimum flow area (the throat), the mass flow rate through a nozzle is a maximum when Ma = 1 at the throat. Denoting this area by *A**, we obtain an expression for the maximum mass flow rate by substituting Ma = 1

$$\dot{m}_{max} = A^* P_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1} \right)^{(k+1)/[2(k-1)]}$$

Effect of Back Pressure on Flow through a Converging Nozzle

- A relation for the variation of flow area A through the nozzle relative to throat area A* can be obtained by combining equations f or $\dot{m}$ and $\dot{m}_{max}$ for the same mass flow rate and stagnation properties of a particular fluid. This yields

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}Ma^2\right)\right]^{(k+1)/[2(k-1)]}$$

- Another parameter sometimes used in the analysis of one-dimensional isentropic flow of ideal gases is Ma*, which is the ratio of the local velocity to the speed of sound at the throat:

$$Ma^* = \frac{V}{c^*}$$

It can also be expressed as

$$Ma^* = \frac{V}{c}\frac{c}{c^*} = \frac{Ma\, c}{c^*} = \frac{Ma\sqrt{kRT}}{\sqrt{kRT^*}} = Ma\sqrt{\frac{T}{T^*}}$$

Effect of Back Pressure on Flow through a Converging Nozzle

- where Ma is the local Mach number, T is the local temperature, and T* is the critical temperature.
- Solving for T and for T* and substituting, we get

$$Ma^* = Ma\sqrt{\frac{k+1}{2+(k-1)Ma^2}}$$

- Note that the parameter Ma* differs from the Mach number Ma in that Ma* is the local velocity nondimensionalized with respect to the sonic velocity at the throat, whereas Ma is the local velocity nondimensionalized with respect to the local sonic velocity.

Table 1

$$Ma^* = Ma\sqrt{\frac{k+1}{2+(k-1)Ma^2}}$$

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}Ma^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}Ma^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}Ma^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}Ma^2\right)^{-1}$$

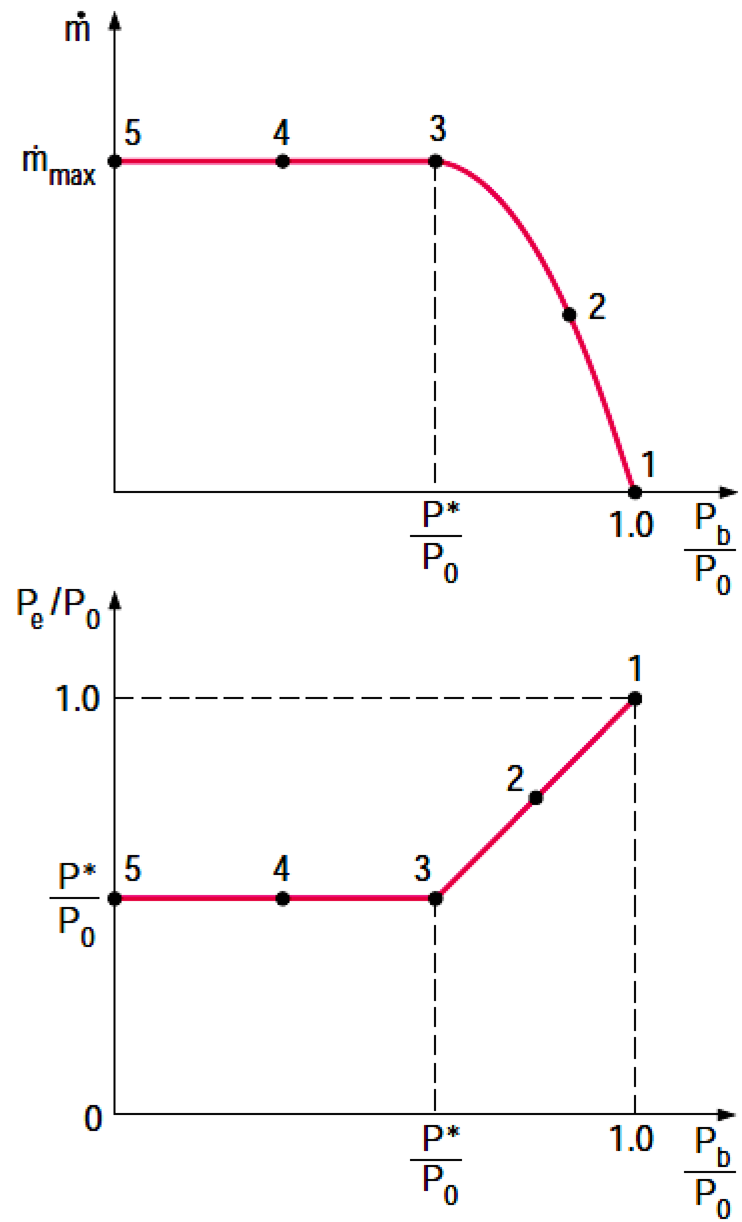
One-dimensional isentropic compressible flow functions for an ideal gas with $k = 1.4$

Ma	Ma*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
0	0	∞	1.0000	1.0000	1.0000
0.1	0.1094	5.8218	0.9930	0.9950	0.9980
0.2	0.2182	2.9635	0.9725	0.9803	0.9921
0.3	0.3257	2.0351	0.9395	0.9564	0.9823
0.4	0.4313	1.5901	0.8956	0.9243	0.9690
0.5	0.5345	1.3398	0.8430	0.8852	0.9524
0.6	0.6348	1.1882	0.7840	0.8405	0.9328
0.7	0.7318	1.0944	0.7209	0.7916	0.9107
0.8	0.8251	1.0382	0.6560	0.7400	0.8865
0.9	0.9146	1.0089	0.5913	0.6870	0.8606
1.0	1.0000	1.0000	0.5283	0.6339	0.8333
1.2	1.1583	1.0304	0.4124	0.5311	0.7764
1.4	1.2999	1.1149	0.3142	0.4374	0.7184
1.6	1.4254	1.2502	0.2353	0.3557	0.6614
1.8	1.5360	1.4390	0.1740	0.2868	0.6068
2.0	1.6330	1.6875	0.1278	0.2300	0.5556
2.2	1.7179	2.0050	0.0935	0.1841	0.5081
2.4	1.7922	2.4031	0.0684	0.1472	0.4647
2.6	1.8571	2.8960	0.0501	0.1179	0.4252
2.8	1.9140	3.5001	0.0368	0.0946	0.3894
3.0	1.9640	4.2346	0.0272	0.0760	0.3571
5.0	2.2361	25.000	0.0019	0.0113	0.1667
∝	2.2495	∝	0	0	0

Effect of Back Pressure on Flow through a Converging Nozzle

- A plot of $\dot{m}$ versus P_b/P_0 for a converging nozzle is shown in Fig. below.
- Notice that the mass flow rate increases with decreasing P_b/P_0, reaches a maximum at $P_b = P^*$, and remains constant for P_b/P_0 values less than this critical ratio. Also illustrated on this figure is the effect of back pressure on the nozzle exit pressure P_e. We observe that

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$



Effect of Back Pressure on Flow through a Converging Nozzle

- To summarize, for all back pressures lower than the critical pressure P*, the pressure at the exit plane of the converging nozzle P_e is equal to P*, the Mach number at the exit plane is unity, and the mass flow rate is the maximum (or choked) flow rate.
- Because the velocity of the flow is sonic at the throat for the maximum flow rate, a back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.

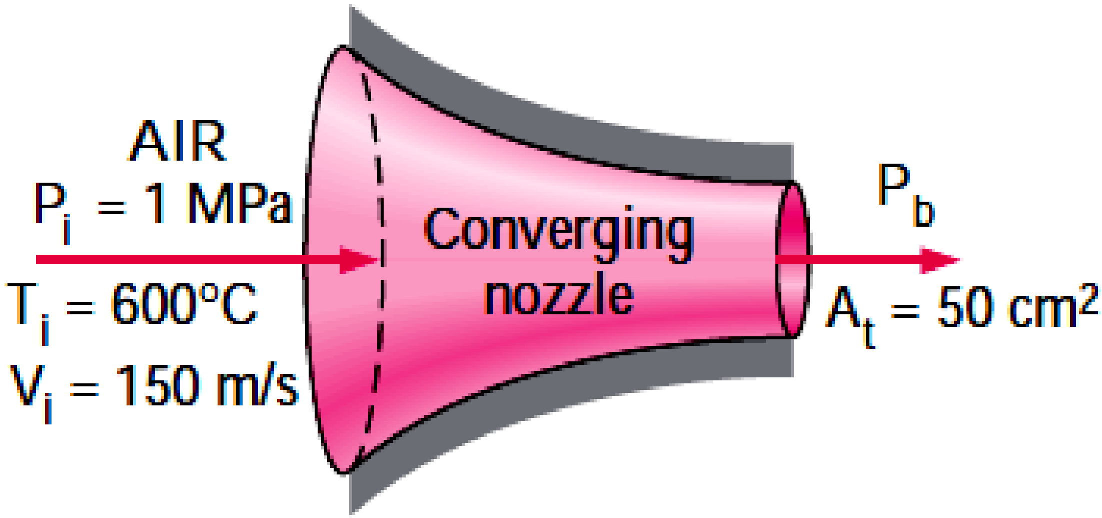
Effect of Back Pressure on Flow through a Converging Nozzle

1. $P_b = P_o$, $P_b/P_o = 1$. No flow occurs. $P_e = P_b$, $M_e=0$.
2. $P_b > P^*$ or $P^*/P_o < P_b/P_o < 1$. Flow begins to increase as the back pressure is lowered. $P_e = P_b$, $M_e < 1$.
3. $P_b = P^*$ or $P^*/P_o = P_b/P_o < 1$. Flow increases to the choked flow limit as the back pressure is lowered to the critical pressure. $P_e = P_b$, $M_e=1$.
4. $P_b < P^*$ or $P_b/P_o < P^*/P_o < 1$. Flow is still choked and does not increase as the back pressure is lowered below the critical pressure, pressure drop from P_e to P_b occurs outside the nozzle. $P_e = P^*$, $M_e=1$.
5. $P_b = 0$. Results are the same as for item 4.

Consider the converging-diverging nozzle shown below.

Example. Effect of Back Pressure on Mass Flow Rate

- Air at 1 MPa and 600◦C enters a converging nozzle, shown in Fig., with a velocity of 150 m/s. Determine the mass flow rate through the nozzle for a nozzle throat area of 50 cm2 when the back pressure is (a) 0.7 MPa and (b) 0.4 MPa.



SOLUTION Air enters a converging nozzle. The mass flow rate of air through the nozzle is to be determined for different back pressures.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The constant pressure specific heat and the specific heat ratio of air are $c_p = 1.005$ kJ/kg · K and k = 1.4.

Analysis We use the subscripts *i* and *t* to represent the properties at the nozzle inlet and the throat, respectively. The stagnation temperature and pressure at the nozzle inlet are determined from Eqs. 12–4 and 12–5:

$$T_{0i} = T_i + \frac{V_i^2}{2c_p} = 873\text{ K} + \frac{(150\text{ m/s}^2)^2}{2(1.005\text{ kJ/kg}\cdot\text{K})}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 884\text{ K}$$

$$P_{0i} = P_i\left(\frac{T_{0i}}{T_i}\right)^{k/(k-1)} = (1\text{ MPa})\left(\frac{884\text{ K}}{873\text{ K}}\right)^{1.4/(1.4-1)} = 1.045\text{ MPa}$$

These stagnation temperature and pressure values remain constant throughout the nozzle since the flow is assumed to be isentropic. That is,

$$T_0 = T_{0i} = 884\text{ K} \qquad \text{and} \qquad P_0 = P_{0i} = 1.045\text{ MPa}$$

The critical-pressure ratio is determined from Table 1 (or Eq. below)

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

to be $P^*/P_0 = 0.5283$.

(*a*) The back pressure ratio for this case is

$$\frac{P_b}{P_0} = \frac{0.7 \text{ MPa}}{1.045 \text{ MPa}} = 0.670$$

which is greater than the critical-pressure ratio, 0.5283. Thus the exit plane pressure (or throat pressure P_t) is equal to the back pressure in this case.

That is, $P_t = P_b = 0.7$ MPa, and $P_t/P_0 = 0.670$. Therefore, the flow is not choked. From Table 1 at $P_t/P_0 = 0.670$, we read $Ma_t = 0.778$ and $T_t/T_0 = 0.892$.

$$T_t = 0.892T_0 = 0.892(884 \text{ K}) = 788.5 \text{ K}$$

$$\rho_t = \frac{P_t}{RT_t} = \frac{700 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(788.5 \text{ K})} = 3.093 \text{ kg/m}^3$$

$$V_t = Ma_t c_t = Ma_t\sqrt{kRT_t}$$

$$= (0.778)\sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(788.5\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

$$= 437.9\text{ m/s}$$

Thus,

$$\dot{m} = \rho_t A_t V_t = (3.093\text{ kg/m}^3)(50\times10^{-4}\text{ m}^2)(437.9\text{ m/s}) = \mathbf{6.77\text{ kg/s}}$$

(*b*) The back pressure ratio for this case is

$$\frac{P_b}{P_0} = \frac{0.4\text{ MPa}}{1.045\text{ MPa}} = 0.383$$

which is less than the critical-pressure ratio, 0.5283. Therefore, sonic conditions exist at the exit plane (throat) of the nozzle, and Ma = 1. The flow is choked in this case, and the mass flow rate through the nozzle can be calculated from Eq. 12–25:

$$\dot{m} = A^*P_0\sqrt{\frac{k}{RT_0}\left(\frac{2}{k+1}\right)^{(k+1)/[2(k-1)]}}$$

$$= (50\times10^{-4}\text{ m}^2)(1045\text{ kPa})\sqrt{\frac{1.4}{(0.287\text{ kJ/kg}\cdot\text{K})(884\text{ K})}\left(\frac{2}{1.4+1}\right)^{2.4/0.8}}$$

$$= \mathbf{7.10\text{ kg/s}}$$

Converging–Diverging Nozzles

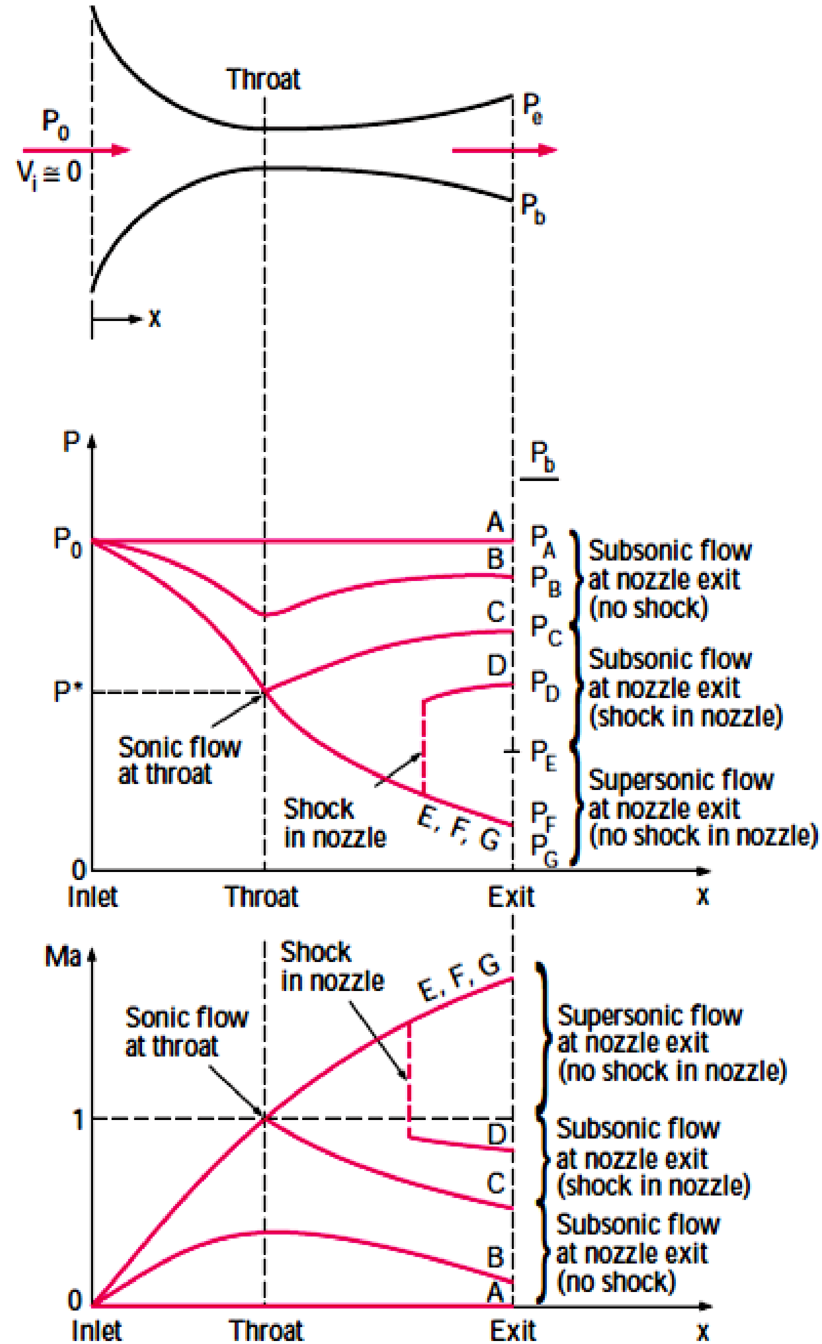
- When we think of nozzles, we ordinarily think of flow passages whose cross-sectional area decreases in the flow direction. However, the highest velocity to which a fluid can be accelerated in a converging nozzle is limited to the sonic velocity (Ma = 1), which occurs at the exit plane (throat) of the nozzle.
- Accelerating a fluid to supersonic velocities (Ma > 1) can be accomplished only by attaching a diverging flow section to the subsonic nozzle at the throat. The resulting combined flow section is a converging– diverging nozzle, which is standard equipment in supersonic aircraft and rocket propulsion.

Converging–Diverging Nozzles

- Forcing a fluid through a converging–diverging nozzle is no guarantee that the fluid will be accelerated to a supersonic velocity.
- In fact, the fluid may find itself decelerating in the diverging section instead of accelerating if the back pressure is not in the right range.
- The state of the nozzle flow is determined by the overall pressure ratio P_b/P_0. Therefore, for given inlet conditions, the flow through a converging–diverging nozzle is governed by the back pressure P_b.

Converging–Diverging Nozzles

- Consider the converging–diverging nozzle shown in Fig. A fluid enters the nozzle with a low velocity at stagnation pressure P0. When $P_b = P_0$ (case A), there is no flow through the nozzle.
- This is expected since the flow in a nozzle is driven by the pressure difference between the nozzle inlet and the exit.
- Now let us examine what happens as the back pressure is lowered.



Converging–Diverging Nozzles

1. When $P_0 > P_b > P_C$, the flow remains subsonic throughout the nozzle, and the mass flow is less than that for choked flow. The fluid velocity increases in the first (converging) section and reaches a maximum at the throat (but Ma < 1). However, most of the gain in velocity is lost in the second (diverging) section of the nozzle, which acts as a diffuser. The pressure decreases in the converging section, reaches a minimum at the throat, and increases at the expense of velocity in the diverging section.
2. When $P_b = P_C$, the throat pressure becomes P^* and the fluid achieves sonic velocity at the throat. But the diverging section of the nozzle still acts as a diffuser, slowing the fluid to subsonic velocities. The mass flow rate that was increasing with decreasing P_b also reaches its maximum value.

Recall that P^* is the lowest pressure that can be obtained at the throat, and the sonic velocity is the highest velocity that can be achieved with a converging nozzle. Thus, lowering P_b further has no influence on the fluid flow in the converging part of the nozzle or the mass flow rate through the nozzle. However, it does influence the character of the flow in the diverging section.

Converging–Diverging Nozzles

3. When $P_C > P_b > P_E$, the fluid that achieved a sonic velocity at the throat continues accelerating to supersonic velocities in the diverging section as the pressure decreases. This acceleration comes to a sudden stop, however, as a **normal shock** develops at a section between the throat and the exit plane, which causes a sudden drop in velocity to subsonic levels and a sudden increase in pressure.

- The fluid then continues to decelerate further in the remaining part of the converging–diverging nozzle. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The normal shock moves downstream away from the throat as P_b is decreased, and it approaches the nozzle exit plane as P_b approaches P_E.

Converging–Diverging Nozzles

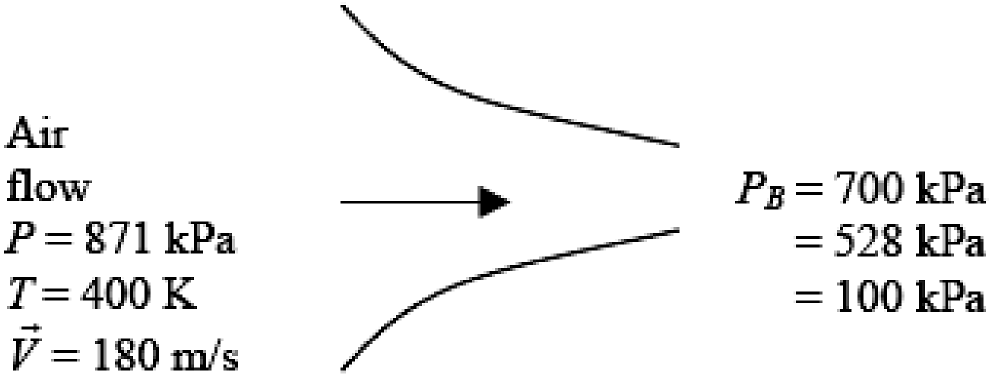
- When $P_b = P_E$, the normal shock forms at the exit plane of the nozzle. The flow is supersonic through the entire diverging section in this case, and it can be approximated as isentropic. However, the fluid velocity drops to subsonic levels just before leaving the nozzle as it crosses the normal shock.

4. When $P_E > P_b > 0$, the flow in the diverging section is supersonic, and the fluid expands to P_F at the nozzle exit with no normal shock forming within the nozzle. Thus, the flow through the nozzle can be approximated as isentropic.

- When $P_b = P_F$, no shocks occur within or outside the nozzle. When $P_b < P_F$, irreversible mixing and expansion waves occur downstream of the exit plane of the nozzle. When $P_b > P_F$, however, the pressure of the fluid increases from P_F to P_b irreversibly in the wake of the nozzle exit, creating what are called oblique shocks.

Example -7

Air leaves the turbine of a turbojet engine and enters a convergent nozzle at 400 K, 871 kPa, with a velocity of 180 m/s. The nozzle has an exit area of 730 cm^2. Determine the mass flow rate through the nozzle for back pressures of 700 kPa, 528 kPa, and 100 kPa, assuming isentropic flow.



The stagnation temperature and stagnation pressure are

$$T_o = T + \frac{\vec{V}^2}{2C_P}$$

$$T_o = 400\,K + \frac{(180\,m/s)^2}{2\left(1.005\frac{kJ}{kg\cdot K}\right)}\frac{\frac{kJ}{kg}}{1000\frac{m^2}{s^2}}$$

$$= (400 + 16.1)\,K = 416.1\,K$$

$$P_o = P\left(\frac{T_o}{T}\right)^{\frac{k}{k-1}} = 871\,kPa\left(\frac{416.1\,K}{400\,K}\right)^{\frac{1.4}{1.4-1}}$$

$$= 1000\,kPa$$

For air k = 1.4 and from Table or using the equation below the critical pressure ratio is $P^*/P_o = 0.528$. The critical pressure for this nozzle is

$$\left(\frac{2}{k+1}\right)^{k/(k-1)} = \frac{P^*}{P_o}$$

$$P^* = 0.528\,P_o$$

$$= 0.528(1000\,kPa) = 528\,kPa$$

Therefore, for a back pressure of 528 kPa, $M = 1$ at the nozzle exit and the flow is choked. For a back pressure of 700 kPa, the nozzle is not choked. The flow rate will not increase for back pressures below 528 kPa.

For the back pressure of 700 kPa,

$$\frac{P_B}{P_o} = \frac{700\,kPa}{1000\,kPa} = 0.700 > \frac{P^*}{P_o}$$

Thus, $P_E = P_B = 700$ kPa. For this pressure ratio Table 1 gives

$$M_E = 0.7324$$

$$\frac{T_E}{T_o} = 0.9031$$

$$T_E = 0.9031\,T_o = 0.9031(416.1\,K) = 375.8\,K$$

$$C_E = \sqrt{kRT_E}$$

$$= \sqrt{1.4(0.287\frac{kJ}{kg\cdot K})(375.8K)\frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}}}$$

$$= 388.6\frac{m}{s}$$

$$\vec{V}_E = M_E C_E = (0.7324)(388.6\frac{m}{s})$$

$$= 284.6\frac{m}{s}$$

$$\rho_E = \frac{P_E}{RT_E} = \frac{(700kPa)}{(0.287\frac{kJ}{kg\cdot K})(375.8K)}\frac{kJ}{m^3 kPa}$$

$$= 6.4902\frac{kg}{m^3}$$

Then

$$\dot{m} = \rho_E A_E \vec{V}_E$$

$$= 6.4902\frac{kg}{m^3}(730\,cm^2)(284.6\frac{m}{s})\frac{m^2}{(100\,cm)^2}$$

$$= 134.8\frac{kg}{s}$$

For the back pressure of 528 kPa,

$$\frac{P_E}{P_o} = \frac{528\,kPa}{1000\,kPa} = 0.528 = \frac{P^*}{P_o}$$

This is the critical pressure ratio and $M_E = 1$ and $P_E = P_B = P^* = 528$ kPa.

$$\frac{T_E}{T_o} = \frac{T^*}{T_o} = 0.8333$$

$$T_E = 0.8333\,T_o = 0.8333(416.1\,K) = 346.7\,K$$

And since $M_E = 1$,

$$\vec{V}_E = C_E = \sqrt{kRT_E}$$

$$= \sqrt{1.4(0.287\frac{kJ}{kg \cdot K})(346.7\,K)\frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}}}$$

$$= 373.2\frac{m}{s}$$

$$\rho_E = \rho^* = \frac{P^*}{RT^*} = \frac{(528\,kPa)}{(0.287\frac{kJ}{kg \cdot K})(346.7\,K)}\frac{kJ}{m^3 kPa}$$

$$= 5.3064\frac{kg}{m^3}$$

$$\dot{m} = \rho_E A_E \vec{V}_E$$

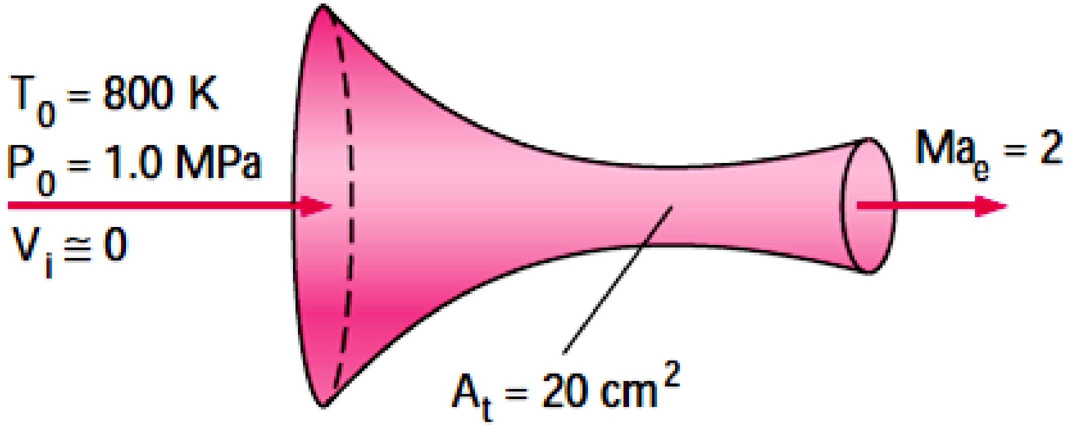
$$= 5.3064 \frac{kg}{m^3}(730\, cm^2)(373.2 \frac{m}{s}) \frac{m^2}{(100\, cm)^2}$$

$$= 144.6 \frac{kg}{s}$$

For a back pressure less than the critical pressure, 528 kPa in this case, the nozzle is choked and the mass flow rate will be the same as that for the critical pressure. Therefore, at a back pressure of 100 kPa the mass flow rate will be 144.6 kg/s.

Example -8

Air enters a converging–diverging nozzle, shown in Fig., at 1.0 MPa and 800 K with a negligible velocity. The flow is steady, one-dimensional, and isentropic with k = 1.4. For an exit Mach number of Ma = 2 and a throat area of 20 cm^2, determine (a) the throat conditions, (b) the exit plane conditions, including the exit area, and (c) the mass flow rate through the nozzle.



SOLUTION Air flows through a converging–diverging nozzle. The throat and the exit conditions and the mass flow rate are to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air is given to be $k = 1.4$. The gas constant of air is 0.287 kJ/kg · K.

Analysis The exit Mach number is given to be 2. Therefore, the flow must be sonic at the throat and supersonic in the diverging section of the nozzle. Since the inlet velocity is negligible, the stagnation pressure and stagnation temperature are the same as the inlet temperature and pressure, $P_0 = 1.0$ MPa and $T_0 = 800$ K. Assuming ideal-gas behavior, the stagnation density is

$$\rho_0 = \frac{P_0}{RT_0} = \frac{1000 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(800 \text{ K})} = 4.355 \text{ kg/m}^3$$

(*a*) At the throat of the nozzle Ma = 1, and from Table A–13 we read

$$\frac{P^*}{P_0} = 0.5283 \qquad \frac{T^*}{T_0} = 0.8333 \qquad \frac{\rho^*}{\rho_0} = 0.6339$$

Thus,

$$P^* = 0.5283P_0 = (0.5283)(1.0 \text{ MPa}) = \mathbf{0.5283\ MPa}$$

$$T^* = 0.8333T_0 = (0.8333)(800 \text{ K}) = \mathbf{666.6\ K}$$

$$\rho^* = 0.6339\rho_0 = (0.6339)(4.355 \text{ kg/m}^3) = \mathbf{2.761\ kg/m^3}$$

Also,

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(666.6 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)}$$

$$= \mathbf{517.5\ m/s}$$

(*b*) Since the flow is isentropic, the properties at the exit plane can also be calculated by using data from Table A–13. For Ma = 2 we read

$$\frac{P_e}{P_0} = 0.1278 \quad \frac{T_e}{T_0} = 0.5556 \quad \frac{\rho_e}{\rho_0} = 0.2300 \quad Ma_e^* = 1.6330 \quad \frac{A_e}{A^*} = 1.6875$$

Thus,

$$P_e = 0.1278P_0 = (0.1278)(1.0\ \text{MPa}) = \mathbf{0.1278\ MPa}$$

$$T_e = 0.5556T_0 = (0.5556)(800\ \text{K}) = \mathbf{444.5\ K}$$

$$\rho_e = 0.2300\rho_0 = (0.2300)(4.355\ \text{kg/m}^3) = \mathbf{1.002\ kg/m^3}$$

$$A_e = 1.6875A^* = (1.6875)(20\ \text{cm}^2) = \mathbf{33.75\ cm^2}$$

and

$$V_e = Ma_e^* c^* = (1.6330)(517.5\ \text{m/s}) = \mathbf{845.1\ m/s}$$

The nozzle exit velocity could also be determined from $V_e = Ma_e c_e$, where c_e is the speed of sound at the exit conditions:

$$V_e = Ma_e c_e = Ma_e\sqrt{kRT_e} = 2\sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(444.5\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)}$$

$$= 845.2\ \text{m/s}$$

(c) Since the flow is steady, the mass flow rate of the fluid is the same at all sections of the nozzle. Thus it may be calculated by using properties at any cross section of the nozzle. Using the properties at the throat, we find that the mass flow rate is

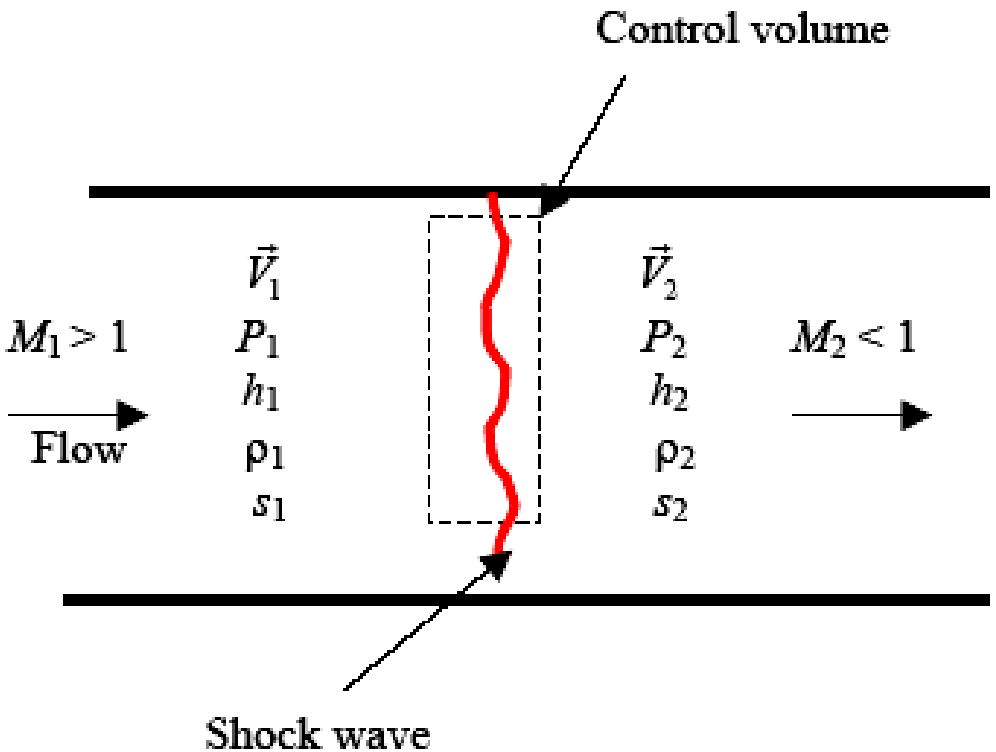
$$\dot{m} = \rho^* A^* V^* = (2.761\ \text{kg/m}^3)(20 \times 10^{-4}\ \text{m}^2)(517.5\ \text{m/s}) = \mathbf{2.86\ kg/s}$$

Discussion Note that this is the highest possible mass flow rate that can flow through this nozzle for the specified inlet conditions.

Normal Shocks

In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging-diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a ***normal shock*.**

The normal shock causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript 1) and after (subscript 2*)* a shock are related by



We assume steady-flow with no heat and work interactions and no potential energy changes. We have the following

Conservation of mass

$$\rho_1 A\vec{V}_1 = \rho_2 A\vec{V}_2$$

$$\rho_2\vec{V}_2 = \rho_2\vec{V}_2$$

Conservation of energy

$$h_1 + \frac{\vec{V}_1^{\,2}}{2} = h_2 + \frac{\vec{V}_2^{\,2}}{2}$$

$$h_{o1} = h_{o2}$$

$$for\ \ ideal\ \ gases: T_{o1} = T_{o2}$$

Conservation of momentum

Rearranging $\frac{dP}{\rho} + V\,dV = 0$ and integrating yield

$$A(P_1 - P_2) = \dot{m}(\vec{V}_2 - \vec{V}_1)$$

Increase of entropy

$$s_2 - s_1 \geq 0$$

Thus, we see that from the conservation of energy, the stagnation temperature is constant across the shock. However, the stagnation pressure decreases across the shock because of irreversibilities. The ordinary (static) temperature rises drastically because of the conversion of kinetic energy into enthalpy due to a large drop in fluid velocity.

We can show that the following relations apply across the shock.

$$\frac{T_2}{T_1} = \frac{1 + M_1^2 (k-1)/2}{1 + M_2^2 (k-1)/2}$$

$$\frac{P_2}{P_1} = \frac{M_1 \sqrt{1 + M_1^2 (k-1)/2}}{M_2 \sqrt{1 + M_2^2 (k-1)/2}}$$

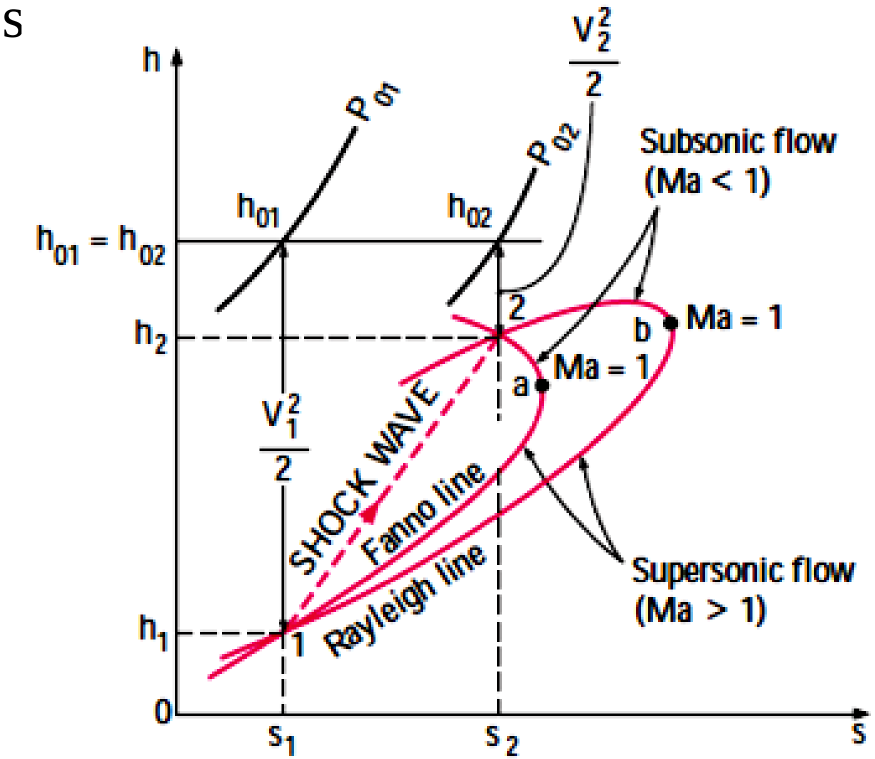
$$M_2^2 = \frac{M_1^2 + 2/(k-1)}{2M_1^2 k/(k-1) - 1}$$

The entropy change across the shock is obtained by applying the entropy-change equation for an ideal gas, constant properties, across the shock:

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

We can combine the conservation of mass and energy relations into a single equation and plot it on an h-s diagram, using property relations. The resultant curve is called the **Fanno line,** and it is the locus of states that have the same value of stagnation enthalpy and mass flux (mass flow per unit flow area).

Likewise, combining the conservation of mass and momentum equations into a single equation and plotting it on the h-s diagram yield a curve called the **Rayleigh line.**



The points of maximum entropy on these lines (points a and b) correspond to Ma = 1. The state on the upper part of each curve is subsonic and on the lower part supersonic.

The Fanno and Rayleigh lines intersect at two points (points 1 and 2), which represent the two states at which all three conservation equations are satisfied. One of these (state 1) corresponds to the state before the shock, and the other (state 2) corresponds to the state after the shock.

Note that the flow is supersonic before the shock and subsonic afterward. Therefore the flow must change from supersonic to subsonic if a shock is to occur. The larger the Mach number before the shock, the stronger the shock will be. In the limiting case of Ma = 1, the shock wave simply becomes a sound wave. Notice from Fig. that entropy increases, $s_2 > s_1$. This is expected since the flow through the shock is adiabatic but irreversible.

Table 2

$$T_{01} = T_{02}$$

$$Ma_2 = \sqrt{\frac{(k-1)Ma_1^2 + 2}{2kMa_1^2 - k + 1}}$$

$$\frac{P_2}{P_1} = \frac{1 + kMa_1^2}{1 + kMa_2^2} = \frac{2kMa_1^2 - k + 1}{k + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)Ma_1^2}{2 + (k-1)Ma_1^2} = \frac{V_1}{V_2}$$

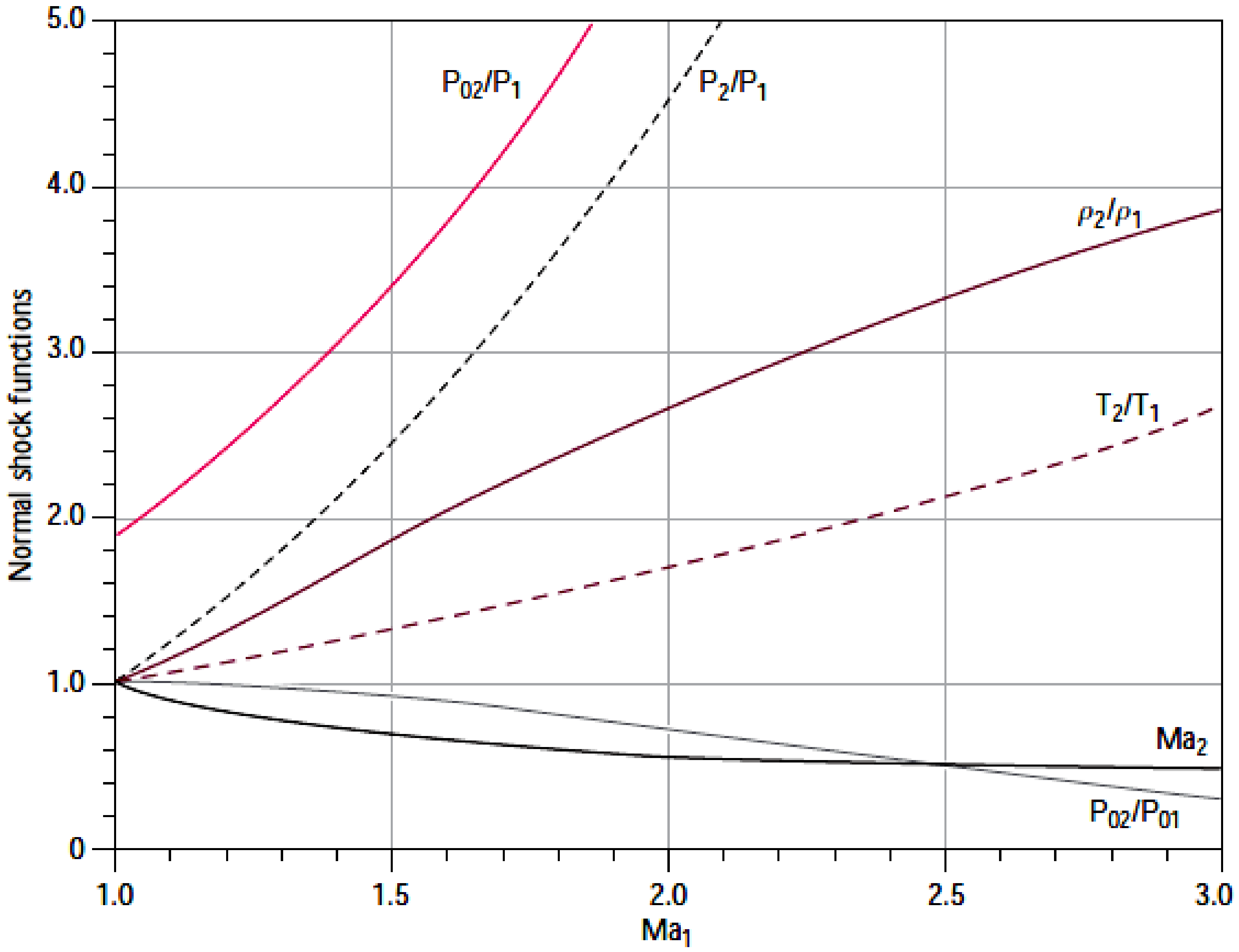
$$\frac{T_2}{T_1} = \frac{2 + Ma_1^2(k-1)}{2 + Ma_2^2(k-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{Ma_1}{Ma_2}\left[\frac{1 + Ma_2^2(k-1)/2}{1 + Ma_1^2(k-1)/2}\right]^{(k+1)/[2(k-1)]}$$

$$\frac{P_{02}}{P_1} = \frac{(1 + kMa_1^2)[1 + Ma_2^2(k-1)/2]^{k/(k-1)}}{1 + kMa_2^2}$$

One-dimensional normal shock functions for an ideal gas with $k = 1.4$

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.8929
1.1	0.9118	1.2450	1.1691	1.0649	0.9989	2.1328
1.2	0.8422	1.5133	1.3416	1.1280	0.9928	2.4075
1.3	0.7860	1.8050	1.5157	1.1909	0.9794	2.7136
1.4	0.7397	2.1200	1.6897	1.2547	0.9582	3.0492
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
1.6	0.6684	2.8200	2.0317	1.3880	0.8952	3.8050
1.7	0.6405	3.2050	2.1977	1.4583	0.8557	4.2238
1.8	0.6165	3.6133	2.3592	1.5316	0.8127	4.6695
1.9	0.5956	4.0450	2.5157	1.6079	0.7674	5.1418
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.1	0.5613	4.9783	2.8119	1.7705	0.6742	6.1654
2.2	0.5471	5.4800	2.9512	1.8569	0.6281	6.7165
2.3	0.5344	6.0050	3.0845	1.9468	0.5833	7.2937
2.4	0.5231	6.5533	3.2119	2.0403	0.5401	7.8969
2.5	0.5130	7.1250	3.3333	2.1375	0.4990	8.5261
2.6	0.5039	7.7200	3.4490	2.2383	0.4601	9.1813
2.7	0.4956	8.3383	3.5590	2.3429	0.4236	9.8624
2.8	0.4882	8.9800	3.6636	2.4512	0.3895	10.5694
2.9	0.4814	9.6450	3.7629	2.5632	0.3577	11.3022
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
5.0	0.4152	29.000	5.0000	5.8000	0.0617	32.6335
∞	0.3780	∞	6.0000	∞	0	∞



P02/P1
P2/P1
ρ2/ρ1
T2/T1
Ma2
P02/P01
5.0
4.0
3.0
2.0
1.0
0
1.0
1.5
2.0
2.5
3.0
Ma1
Normal shock functions

Example -9

Air flowing with a velocity of 600 m/s, a pressure of 60 kPa, and a temperature of 260 K undergoes a normal shock. Determine the velocity and static and stagnation conditions after the shock and the entropy change across the shock.

The Mach number before the shock is

$$M_1 = \frac{\vec{V}_1}{C_1} = \frac{\vec{V}_1}{\sqrt{kRT_1}}$$

$$= \frac{600\,\dfrac{m}{s}}{\sqrt{1.4(0.287\,\dfrac{kJ}{kg\cdot K})(260K)\dfrac{1000\,\dfrac{m^2}{s^2}}{\dfrac{kJ}{kg}}}}$$

$$= 1.856$$

For M_1 = 1.856, Table 1 gives

$$\frac{P_1}{P_{o1}} = 0.1597, \quad \frac{T_1}{T_{o1}} = 0.5921$$

For M_x = 1.856, Table 2 gives the following results.

$$M_2 = 0.6045, \ \frac{P_2}{P_1} = 3.852, \ \frac{\rho_2}{\rho_1} = 2.4473$$

$$\frac{T_2}{T_1} = 1.574, \ \frac{P_{o2}}{P_{o1}} = 0.7875, \ \frac{P_{o2}}{P_1} = 4.931$$

From the conservation of mass with $A_2 = A_1$.

$$\vec{V}_2 \rho_2 = \vec{V}_1 \rho_1$$

$$\vec{V}_2 = \frac{\vec{V}_1}{\dfrac{\rho_2}{\rho_1}} = \frac{600\,\dfrac{m}{s}}{2.4473} = 245.2\,\frac{m}{s}$$

$$P_2 = P_1 \frac{P_2}{P_1} = 60\,kPa(3.852) = 231.1\,kPa$$

$$T_2 = T_1 \frac{T_2}{T_1} = 260\,K(1.574) = 409.2\,K$$

$$T_{o1} = \frac{T_1}{\left(\dfrac{T_1}{T_{o1}}\right)} = \frac{260\,K}{0.5921} = 439.1\,K = T_{o2}$$

$$P_{o1} = \frac{P_1}{\left(\dfrac{P_1}{P_{o1}}\right)} = \frac{60\,kPa}{0.1597} = 375.6\,kPa$$

$$P_{o2} = P_{o1}\frac{P_{o2}}{P_{o1}} = 375.6\,kPa(0.7875) = 295.8\,kPa$$

The entropy change across the shock is

$$s_2 - s_1 = C_P \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

$$s_2 - s_1 = 1.005\frac{kJ}{kg\cdot K}\ln(1.574) - 0.287\frac{kJ}{kg\cdot K}\ln(3.852)$$

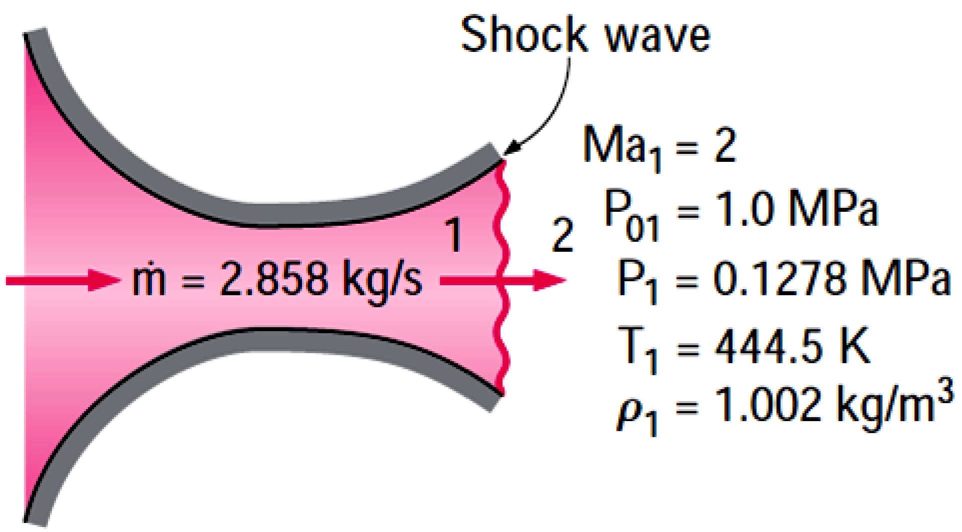
$$= 0.0688\frac{kJ}{kg\cdot K}$$

You are encouraged to read about the following topics in the text:

•Oblique shocks

Example 10. Shock Wave in a Converging–Diverging Nozzle

- If the air flowing through the converging–diverging nozzle of Example 8 experiences a normal shock wave at the nozzle exit plane (see fig below), determine the following after the shock: (a) the stagnation pressure, static pressure, static temperature, and static density; (b) the entropy change across the shock; (c) the exit velocity; and (d) the mass flow rate through the nozzle. Assume steady, one-dimensional, and isentropic flow with k = 1.4 from the nozzle inlet to the shock location.



SOLUTION Air flowing through a converging–diverging nozzle experiences a normal shock at the exit. The effect of the shock wave on various properties is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. **3** The shock wave occurs at the exit plane.

Properties The constant-pressure specific heat and the specific heat ratio of air are $c_p = 1.005$ kJ/kg · K and $k = 1.4$. The gas constant of air is 0.287 kJ/kg · K.

Analysis (a) The fluid properties at the exit of the nozzle just before the shock (denoted by subscript 1) are those evaluated in Example 8 at the nozzle exit to be

$$P_{01} = 1.0 \text{ MPa} \qquad P_1 = 0.1278 \text{ MPa} \qquad T_1 = 444.5 \text{ K} \qquad \rho_1 = 1.002 \text{ kg/m}^3$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table 2. For Ma1= 2.0, we read

$$\text{Ma}_2 = 0.5774 \qquad \frac{P_{02}}{P_{01}} = 0.7209 \qquad \frac{P_2}{P_1} = 4.5000 \qquad \frac{T_2}{T_1} = 1.6875 \qquad \frac{\rho_2}{\rho_1} = 2.6667$$

Then the stagnation pressure P_{02}, static pressure P_2, static temperature T_2, and static density ρ_2 after the shock are

$$P_{02} = 0.7209P_{01} = (0.7209)(1.0\text{ MPa}) = \mathbf{0.721\text{ MPa}}$$

$$P_2 = 4.5000P_1 = (4.5000)(0.1278\text{ MPa}) = \mathbf{0.575\text{ MPa}}$$

$$T_2 = 1.6875T_1 = (1.6875)(444.5\text{ K}) = \mathbf{750\text{ K}}$$

$$\rho_2 = 2.6667\rho_1 = (2.6667)(1.002\text{ kg/m}^3) = \mathbf{2.67\text{ kg/m}^3}$$

(*b*) The entropy change across the shock is

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= (1.005\text{ kJ/kg}\cdot\text{K})\ln(1.6875) - (0.287\text{ kJ/kg}\cdot\text{K})\ln(4.5000)$$

$$= \mathbf{0.0942\text{ kJ/kg}\cdot\text{K}}$$

Thus, the entropy of the air increases as it experiences a normal shock, which is highly irreversible.

(*c*) The air velocity after the shock can be determined from $V_2 = Ma_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock:

$$V_2 = Ma_2 c_2 = Ma_2 \sqrt{kRT_2}$$

$$= (0.5774)\sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(750.1\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)}$$

$$= \mathbf{317\ m/s}$$

(*d*) The mass flow rate through a converging–diverging nozzle with sonic conditions at the throat is not affected by the presence of shock waves in the nozzle. Therefore, the mass flow rate in this case is the same as that determined in Example 12–7:

$$\dot{m} = \mathbf{2.86\ kg/s}$$

Discussion This result can easily be verified by using property values at the nozzle exit after the shock at all Mach numbers significantly greater than unity.

Oblique Shocks

- Not all shock waves are normal shocks (perpendicular to the flow direction). For example, when the space shuttle travels at supersonic speeds through the atmosphere, it produces a complicated shock pattern consisting of inclined shock waves called **oblique shocks.**

Reading Assignment

- Read about oblique shock waves: characteristic features, governing equations, calculation o properties;
- Textbook to Read
  - **[YUNUS A. CENGEL] Fluid Mechanics. Fundamentals and Applications**

End of Course