Mechanical Engineering Department

Fluid Mechanics

(MEng 2113)

Chapter 3

Integral Relations For A Control Volume

Prepared by: Addisu Dagne May 2017

Chapter Contents

- Classification of Fluid Flow
- Fluid Flow Analysis Methods
- Basic Physical Laws of Fluid Mechanics
- The Continuity Equation
- The Bernoulli Equation and its application
- The Linear Momentum equation and its application
- The Angular Momentum equation and its application
- The Energy Equation

Classification of Fluid Flows

1) Uniform flow; steady flow

- If we look at a fluid flowing under normal circumstances - a river for example - the conditions (e.g. velocity, pressure) at one point will vary from those at another point, then we have **non-uniform flow**.
- If the conditions at one point vary as time passes, then we have **unsteady flow.**
- **Uniform flow:** If the flow velocity is the same magnitude and direction at every point in the flow it is said to be uniform. That is, the flow conditions **DO NOT change with position.**
- **Non-uniform:** If at a given instant, the velocity is not the same at every point the flow is non-uniform.

Classification of Fluid Flows

- **Steady**: A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with **time.**
- **Unsteady**: If at any point in the fluid, the conditions change with time, the flow is described as unsteady.
- Combining the above we can classify any flow in to one of four types:
- **Steady uniform flow**. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

Classification of Fluid Flows

- **Steady non-uniform flow.** Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.
- **Unsteady uniform flow.** At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.
- **Unsteady non-uniform flow.** Every condition of the flow may change from point to point and with time at every point. An example is surface waves in an open channel.

Classification of Fluid Flows

- **2) One-, two-, and three-dimensional flows**
  - A fluid flow is in general a three-dimensional, spatial and time dependent phenomenon:-

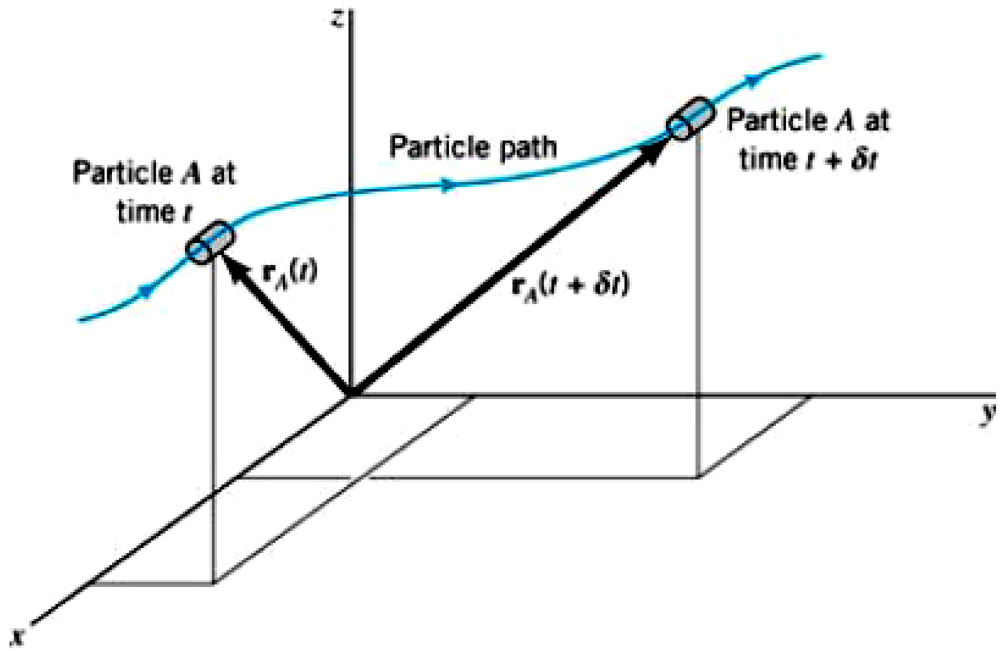
$$V = V(\vec{r},t) = u(\vec{r},t)\vec{i} + v(\vec{r},t)\vec{j} + w(\vec{r},t)\vec{k}$$

  - Where $\vec{r} = (x, y, z)$ is the position vector, $(\vec{i}, \vec{j}, \vec{k})$ are the unit vectors in the Cartesian coordinates, and (u, v, w) are the velocity components in these directions.
  - As defined above, the flow will be uniform if the velocity components are independent of spatial position (x, y, z) and will be steady if the velocity components are independent of time *t*.

Classification of Fluid Flows

2) One-, two-, and three-dimensional flows

- Accordingly, a fluid flow is called three-dimensional if all three velocity components are equally important.
- A three-dimensional flow problem will have the most complex characters and is the most difficult to solve.



Classification of Fluid Flows

- Fortunately, in many engineering applications, the flow can be considered as two-dimensional.
- In such a situation, one of the velocity components (say, w) is either identically zero or much smaller than the other two components, and the flow conditions vary essentially only in two directions (say, x and y).
- Hence, the velocity is reduced to $\boldsymbol{V} = u\vec{\boldsymbol{i}} + v\vec{\boldsymbol{j}}$ where (u, v) are functions of *(x, y)* (and possibly t).
- It is sometimes possible to further simplify a flow analysis by assuming that two of the velocity components are negligible, leaving the velocity field to be approximated as a **one-dimensional flow** field.
- That is, $\boldsymbol{V} = u\vec{\boldsymbol{i}}$ where the velocity **u** may vary across the section of flow.

Classification of Fluid Flows

- Typical examples are fully-developed flows in long uniform pipes and open-channels.
- One-dimensional flow problems will require only elementary analysis, and can be solved analytically in most cases.

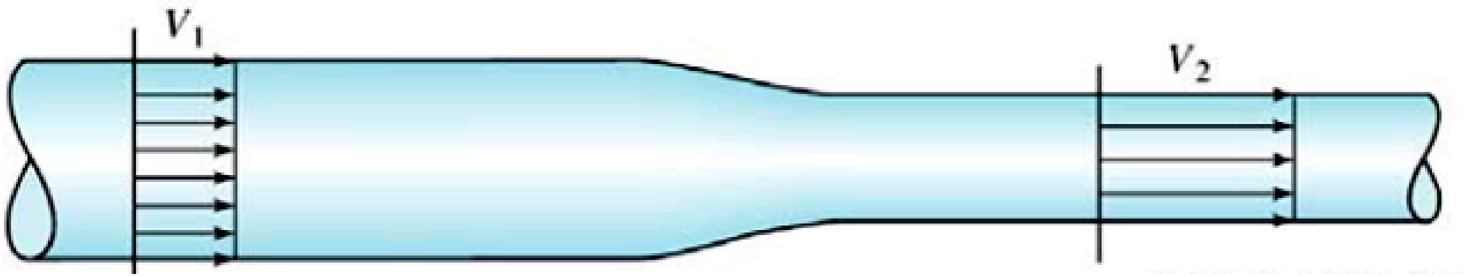


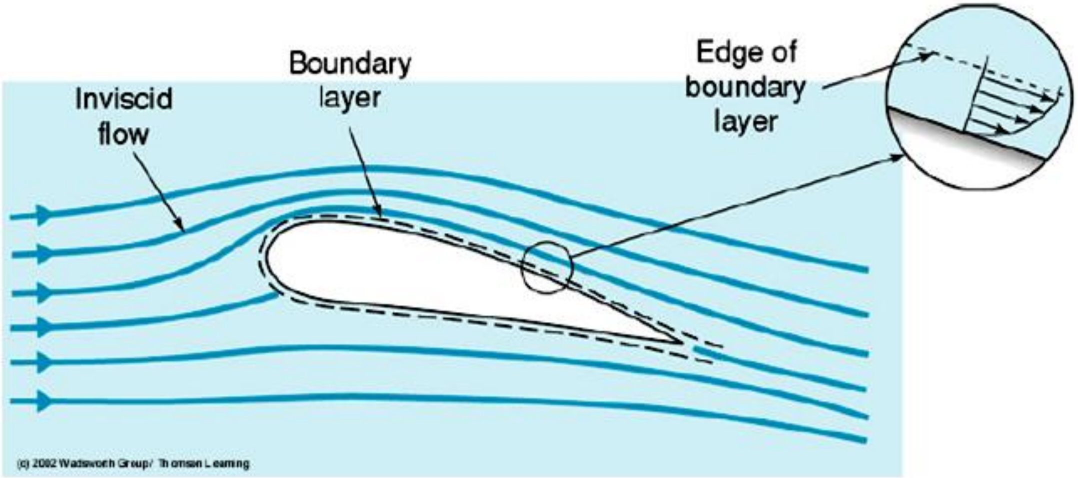
Fig. One-dimensional ideal flow along a pipe, where the velocity is uniform across the pipe section.

Classification of Fluid Flows

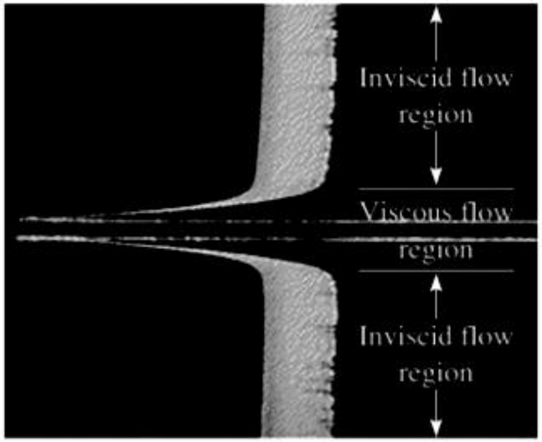
- **3) Viscous and inviscid flows**
  - An **inviscid flow** is one in which viscous effects do not significantly influence the flow and are thus neglected.
  - If the shear stresses in a flow are small and act over such small areas that they do not significantly affect the flow field the flow can be assumed as inviscid flow.
  - In a **viscous flow** the effects of viscosity are important and cannot be ignored.
  - Based on experience, it has been found that the primary class of flows, which can be modeled as inviscid flows, is **external flows,** that is, flows of an unbounded fluid which exist exterior to a body. Any viscous effects that may exist are confined to a thin layer, called a **boundary layer,** which is attached to the boundary**.**

Classification of Fluid Flows

- The velocity in a boundary layer is always zero at a fixed wall, a result of viscosity.
- For many flow situations, boundary layers are so thin that they can simply be ignored when studying the gross features of a flow around a streamlined body



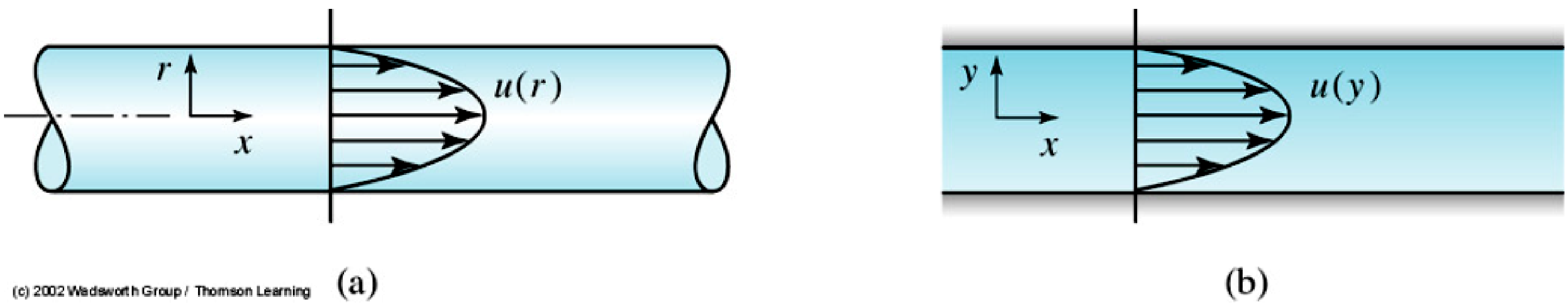
External flow around an airfoil.



Viscous flow in a boundary layer.

Classification of Fluid Flows

- Viscous flows include the broad class of **internal flows,** such as flows in pipes, hydraulic machines, and conduits and in open channels.
- In such flows viscous effects cause substantial "losses" and account for the huge amounts of energy that must be used to transport oil and gas in pipelines. The no-slip condition resulting in zero velocity at the wall, and the resulting shear stresses, lead directly to these losses.



Viscous internal flow: (a) in a pipe; (b) between two parallel plates.

Classification of Fluid Flows

4) Incompressible and compressible flows

- All fluids are compressible - even water - their density will change as pressure changes.
- Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density.
- As you will appreciate, liquids are quite difficult to compress – so under most steady conditions they are treated as incompressible. In some unsteady conditions very high pressure differences can occur and it is necessary to take these into account - even for liquids.
- Gases, on the contrary, are very easily compressed, it is essential in cases of high-speed flow to treat these as compressible, taking changes in pressure into account.

Classification of Fluid Flows

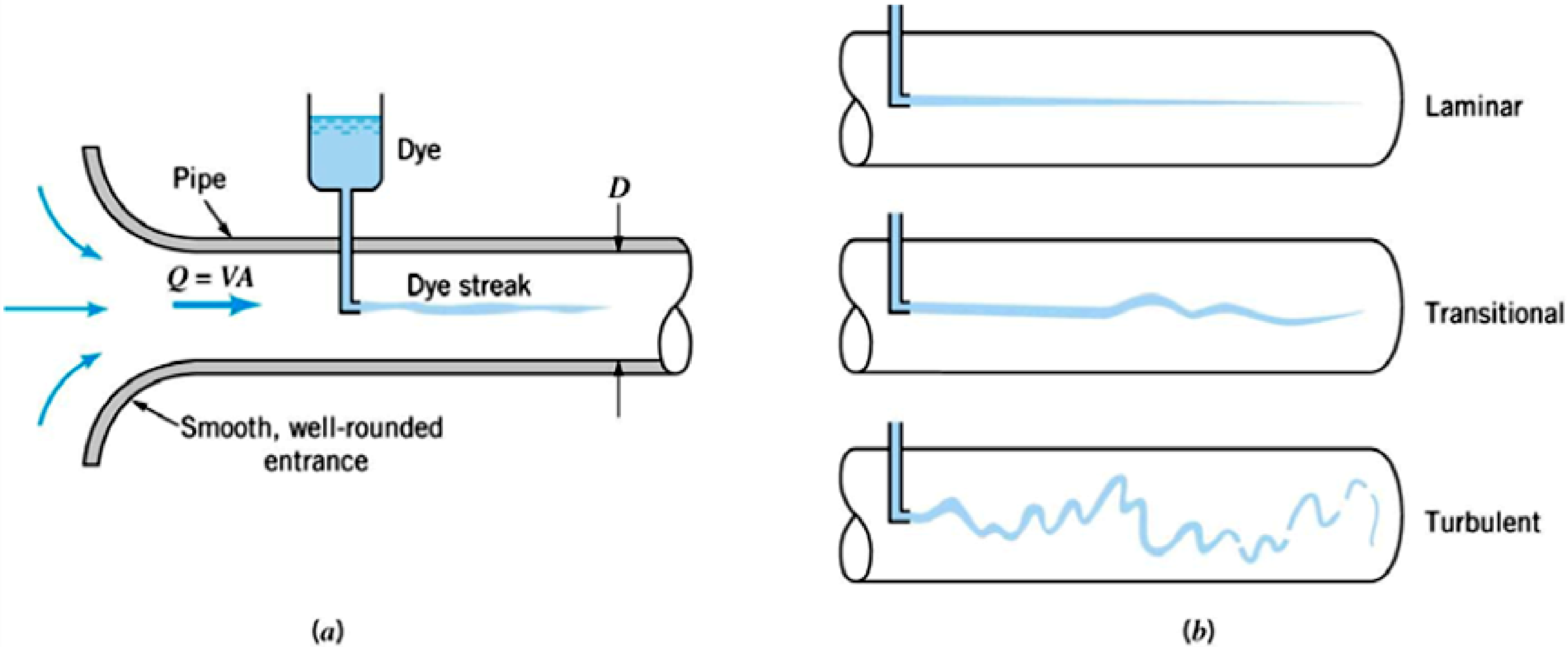
- Low-speed gas flows, such as the atmospheric flow referred to above, are also considered to be incompressible flows. The **Mach number is defined as**

$$M = \frac{V}{c}$$

- where V is the gas speed and c is the speed of sound.
- The Mach number is useful in deciding whether a particular gas flow can be studied as an incompressible flow.
- If $M < 0.3$, density variations are at most 3% and the flow is assumed to be incompressible; for standard air this corresponds to a velocity below about 100 m/s.
- If $M > 0.3$, the density variations influence the flow and compressibility effects should be accounted for.

Classification of Fluid Flows

5) Laminar and turbulent flows



- In the experiment shown above, a dye is injected into the middle of pipe flow of water. The dye streaks will vary, as shown in (b), depending on the flow rate in the pipe.

Classification of Fluid Flows

- The top situation is called **laminar flow,** and the lower is **turbulent flow,** occurring when the flow is sufficiently slow and fast, respectively.
- In laminar flow the motion of the fluid particles is very orderly with all particles moving in straight lines parallel to the pipe wall. There is essentially no mixing of neighboring fluid particles.
- In sharp contrast, mixing is very significant in turbulent flow, in which fluid particles move haphazardly in all directions.
- It is therefore impossible to trace motion of individual particles in turbulent flow.

Classification of Fluid Flows

- Whether the flow is laminar or not depends on the Reynolds number,

$$\mathrm{Re} \equiv \frac{\rho \bar{V} d}{\mu}$$

$\rho = density$, $\mu = \text{viscosity}$, $\bar{V} = \text{section-mean velocity}$, $d = \text{diameter of pipe}$

- and it has been demonstrated experimentally that

$$\mathrm{Re}\begin{cases} < 2,000 & \text{laminar flow} \\ \text{between } 2{,}000 \text{ and } 4{,}000 & \text{transitional flow} \\ > 4,000 & \text{turbulent flow} \end{cases}$$

Fluid Flow analysis methods

- In analyzing fluid motion, we might take one of two paths:
  1. Seeking to describe the detailed flow pattern at every point *(x, y, z) in the field or*
  2. *Working* with a finite region, making a balance of flow in versus flow out, and determining gross flow effects such as the force or torque on a body or the total energy exchange.
- The second is the "control volume" method and is the subject of this chapter.
- The first is the "differential" approach and is developed in Chap. 4.

Basic Physical Laws of Fluid Mechanics

- Statics problems basically require only the density of the fluid and knowledge of the position of the free surface, but most flow problems require the analysis of an arbitrary state of variable fluid motion defined by the geometry, the boundary conditions, and the laws of mechanics.
- This chapter and the next two chapters outline the three basic approaches to the analysis of arbitrary flow problems:
    1. Control volume, or large-scale, analysis (Chap. 3).
    2. Differential, or small-scale, analysis (Chap. 4).
    3. Experimental, or dimensional, analysis (Chap. 5).
- Control volume analysis is accurate for any flow distribution but is often based on average or "one dimensional" property values at the boundaries.

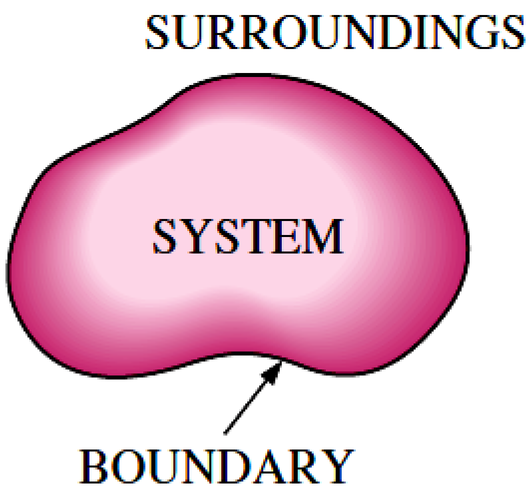
Basic Physical Laws of Fluid Mechanics

- The differential equation approach can be applied to any problem. Only a few problems, such as straight pipe flow, yield to exact analytical solutions.
- But the differential equations can be modeled numerically, and computational fluid dynamics (CFD) can be used to give good estimates for almost any geometry.
- The dimensional analysis applies to any problem, whether analytical, numerical, or experimental. It is particularly useful to reduce the cost of experimentation.

Basic Physical Laws of Fluid Mechanics

Systems and Control Volumes

- A system is defined *as a quantity of matter or a region in space chosen for study.*
- The mass or region outside the system is called the **surroundings.**
- The real or imaginary surface that separates the system from its surroundings is called the **boundary.**
- The boundary of a system can be fixed or movable.

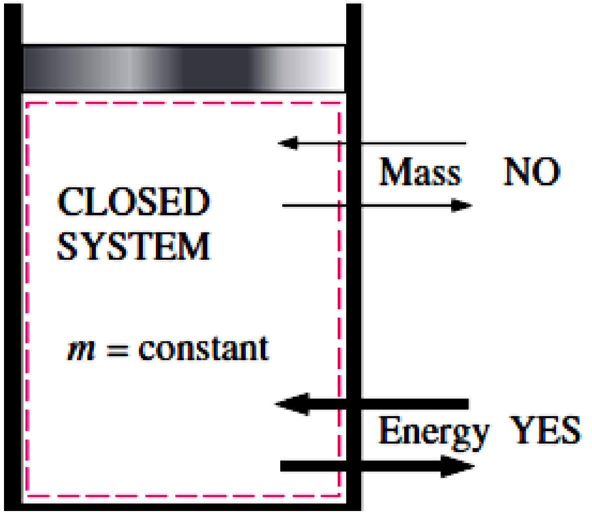


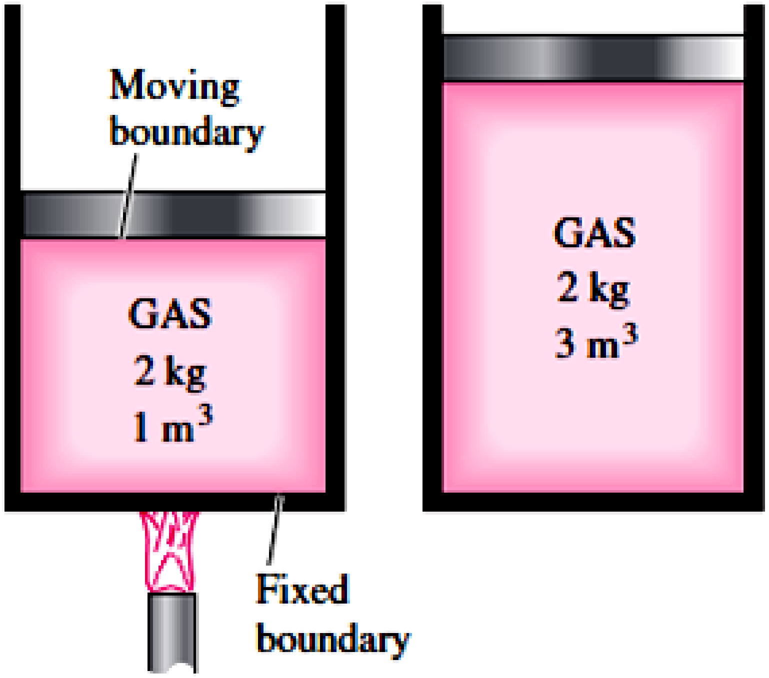
- Note that the boundary is the contact surface shared by both the system and the surroundings. Mathematically speaking, the boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.

Basic Physical Laws of Fluid Mechanics

Systems and Control Volumes

- Systems may be considered to be *closed or open*, depending on whether a fixed mass or a fixed volume in space is chosen for study.
- A **closed system** (also known as a **control mass)** consists of a fixed amount of mass, and no mass can cross its boundary. That is, no mass can enter or leave a closed system.
- But energy, in the form of heat or work, can cross the boundary; and the volume of a closed system does not have to be fixed.
- If, as a special case, even energy is not allowed to cross the boundary, that system is called an **isolated system.**





Basic Physical Laws of Fluid Mechanics

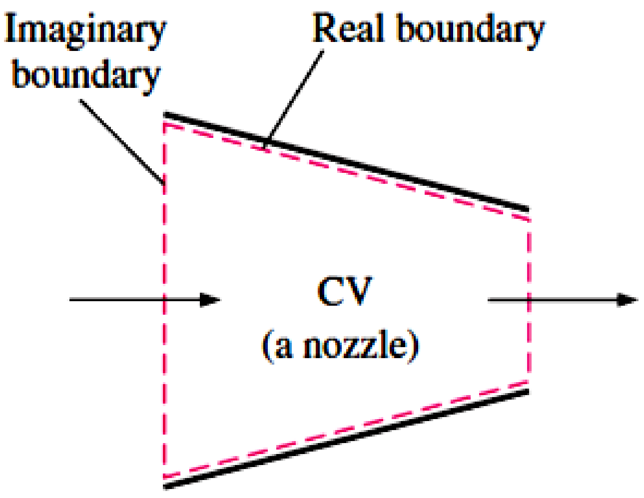
Systems and Control Volumes

- An **open system, or a control volume,** as it is often called, is a properly selected region in space.
- A *control volume* usually encloses a device that involves mass flow such as a compressor, turbine, or nozzle. Flow through these devices is best studied by selecting the region within the device as the control volume. Both mass and energy can cross the boundary of a control volume.
- A large number of engineering problems involve mass flow in and out of a system and, therefore, are modeled as *control volumes*.
- *A* water heater, *a* car radiator, a turbine, and a compressor all involve mass flow and should be analyzed as control volumes (open systems) instead of as control masses (closed systems).
- In general, *any arbitrary region in space can be* selected as a control volume. There are no concrete rules for the selection of control volumes, but the proper choice certainly makes the analysis much easier.

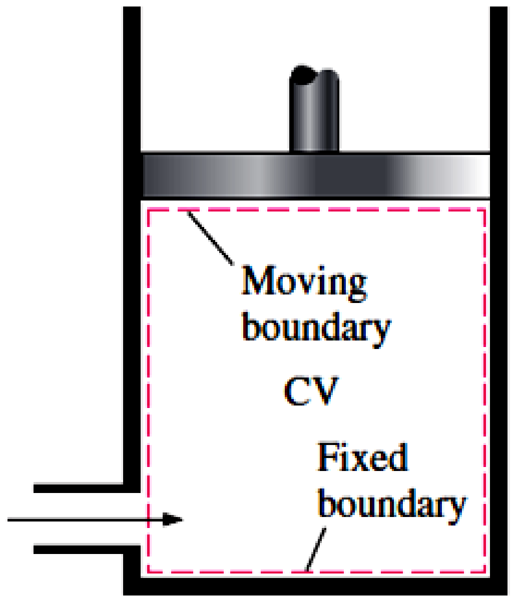
Basic Physical Laws of Fluid Mechanics

Systems and Control Volumes

- The boundaries of a control volume are called *a control surface*, and they can be *real* or *imaginary*. In the case of a nozzle, the inner surface of the nozzle forms the real part of the boundary, and the entrance and exit areas form the imaginary part, since there are no physical surfaces there.



(*a*) A control volume with real and imaginary boundaries



(*b*) A control volume with fixed and moving boundaries

Basic Physical Laws of Fluid Mechanics

- The laws of mechanics state what happens when there is an interaction between the system and its surroundings.
- First, the system is a fixed quantity of mass, denoted by *m*. Thus the mass of the system is conserved and does not change. This is a law of mechanics and has a very simple mathematical form, called *conservation of mass:*

$$m_{\text{syst}} = \text{const}$$

$$\frac{dm}{dt} = 0$$

- Second, if the surroundings exert a net force **F** on the system, **Newton's second** law states that the mass in the system will begin to accelerate

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{V}}{dt} = \frac{d}{dt}(m\mathbf{V})$$

Basic Physical Laws of Fluid Mechanics

- Newton's second law is called the linear momentum relation.
- Note that it is a vector law that implies the three scalar equations $F_x = ma_x$, $F_y = ma_y$, and $F_z = ma_z$.
- Third, if the surroundings exert a net moment **M** about the center of mass of the system, there will be a rotation effect

$$\mathbf{M} = \frac{d\mathbf{H}}{dt}$$

- where $\mathbf{H} = \sum(\mathbf{r} \times \mathbf{V})\, \delta m$ is the angular momentum of the system about its center of mass. This is called the angular momentum relation.

Basic Physical Laws of Fluid Mechanics

- Fourth, if heat δ*Q* is added to the system or work δW is done by the system, the system energy *dE must change* according to *the energy relation, or first law of* thermodynamics:

$$\delta Q - \delta W = dE$$

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

- Finally, the second law of thermodynamics relates entropy change *dS to heat added dQ and absolute temperature T:*

$$dS \geq \frac{\delta Q}{T}$$

- This is valid for a system and can be written in control volume form, but there are almost no practical applications in fluid mechanics

Basic Physical Laws of Fluid Mechanics

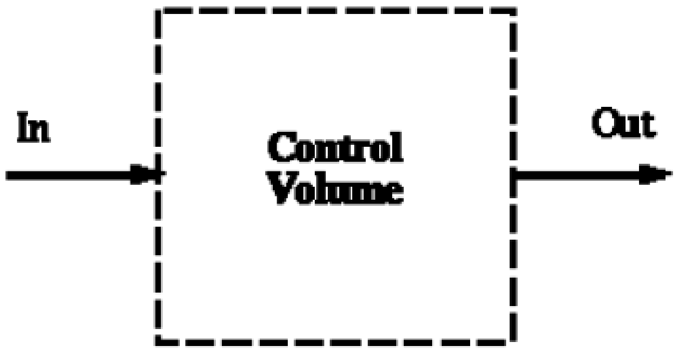
- The purpose of this chapter is to put the above four basic laws into the control volume form suitable for arbitrary regions in a flow: The four basic laws are:
    1. Conservation of mass
    2. The linear momentum relation
    3. The angular momentum relation
    4. The energy equation
- Wherever necessary to complete the analysis we also introduce a state relation such as the perfect-gas law.

Elementary Equations of Motion

- We shall derive the three basic control-volume relations in fluid mechanics:
  1. The principle of conservation of mass, from which the continuity equation is developed;
  2. The principle of conservation of energy, from which the energy equation is derived;
  3. The principle of conservation of linear momentum, from which equations evaluating dynamic forces exerted by flowing fluids may be established.

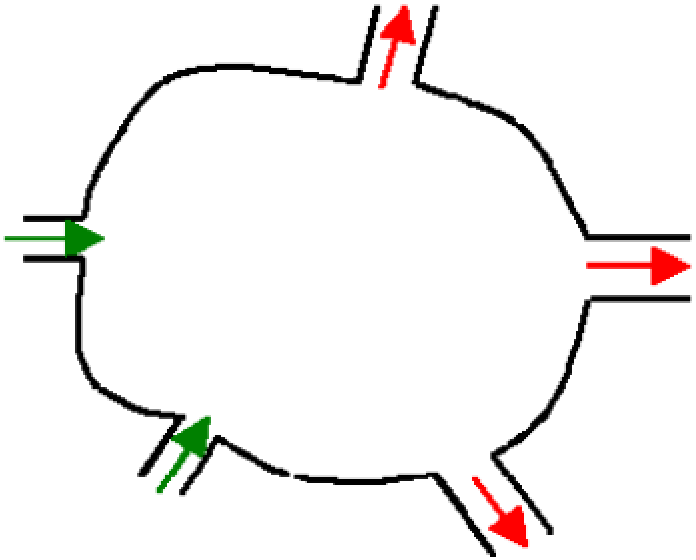
Control volume

- A control volume is a finite region, chosen carefully by the analyst for a particular problem, with open boundaries through which mass, momentum, and energy are allowed to cross.
- The analyst makes a budget, or balance, between the incoming and outgoing fluid and the resultant changes within the control volume. Therefore one can calculate the <u>gross properties (net force, total power output, total heat transfer, etc.) with this method.</u>
- With this method, however, we do not care about the details inside the control volume.



Control volume

- let us consider a control volume that can be a tank, reservoir or a compartment inside a system, and consists of some definite one-dimensional inlets and outlets, like the one shown.
- Let us denote for each of the inlets and outlets:-
  - V = velocity of fluid in a stream
  - A = sectional area of a stream
  - p = pressure of the fluid in a stream
  - ρ = density of the fluid



Control volume

- Then, the volume flow rate, or discharge (volume of flow crossing a section per unit time) is given by

$$Q = VA$$

- Similarly, the mass flow rate (mass of flow crossing a section per unit time) is given by

$$\dot{m} = \rho VA = \rho Q$$

- Then, the momentum flux, defined as the momentum of flow crossing a section per unit time, is given by $\dot{m}V$

Continuity equation

- By steadiness, the total mass of fluid contained in the control volume must be invariant with time.
- Therefore there must be an exact balance between the total rate of flow into the control volume and that out of the control volume:

Total Mass Outflow = Total Mass Inflow

- which translates into the following mathematical relation

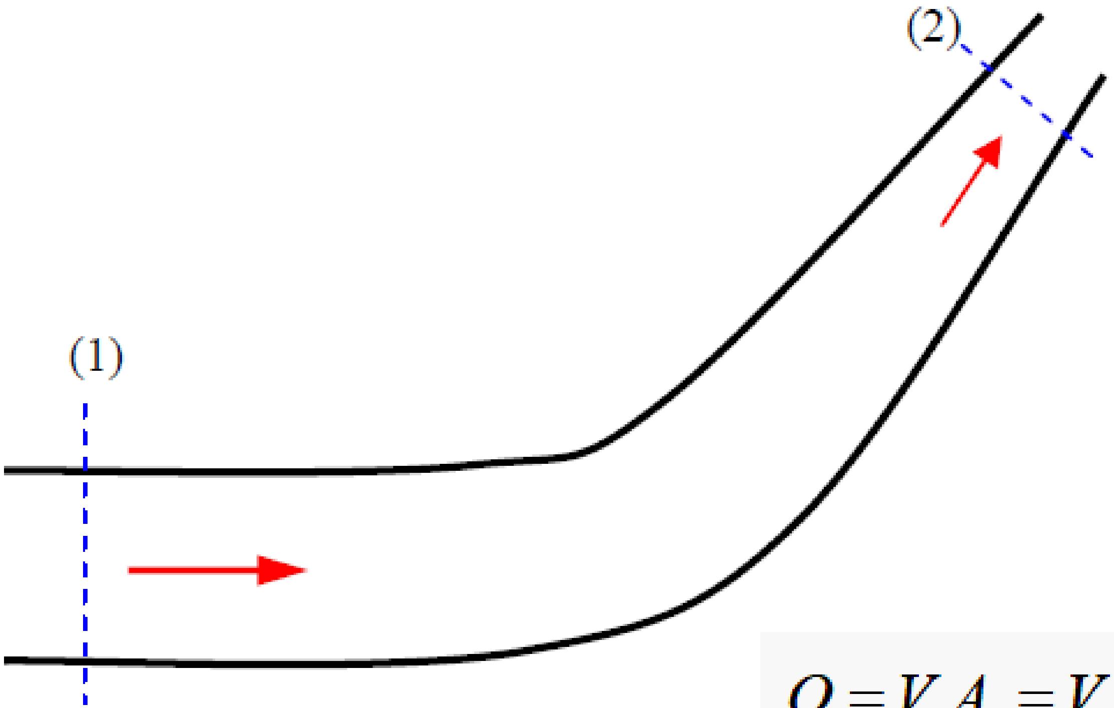
$$\sum_{i=1}^{M} \left(\rho_i V_i A_i \right)_{\text{in}} = \sum_{i=1}^{N} \left(\rho_i V_i A_i \right)_{\text{out}}$$

- Where M is the number of inlets, and N is the number of outlets.

Continuity equation

- If the fluid is incompressible, e.g. water, with ρ being effectively constant, then .

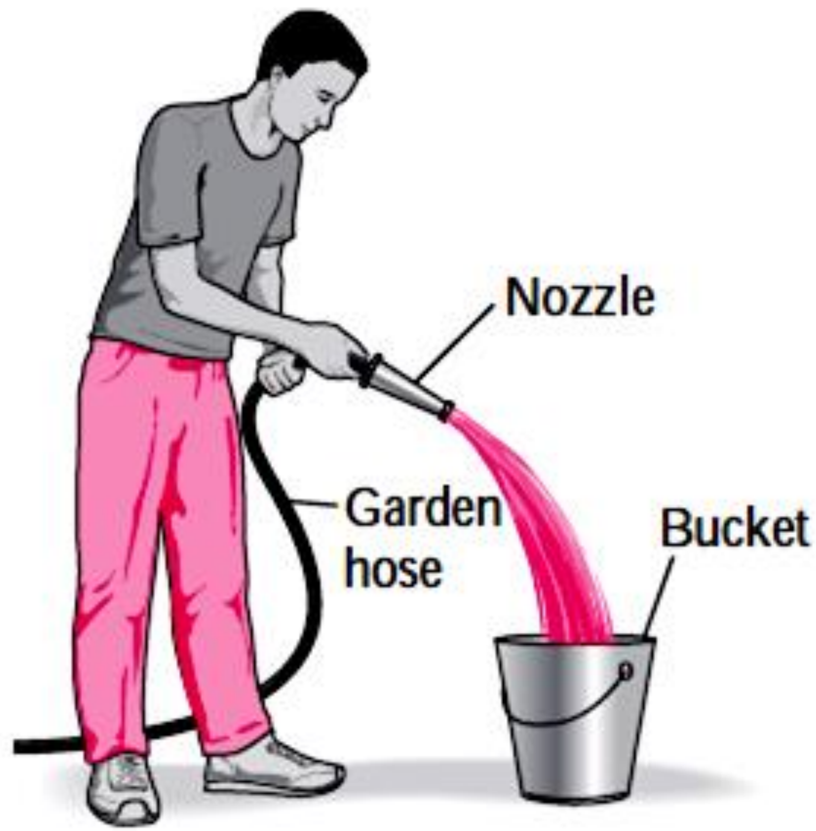
$$\sum_{i=1}^{M}(V_iA_i)_{\text{in}} = \sum_{i=1}^{N}(V_iA_i)_{\text{out}} \qquad \text{or} \qquad \sum_{i=1}^{M}(Q_i)_{\text{in}} = \sum_{i=1}^{N}(Q_i)_{\text{out}}$$



$$Q = V_1A_1 = V_2A_2 = \text{constant}$$

Example 1. Water Flow through a Garden Hose Nozzle

- A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket with water, determine
- (a) the volume and mass flow rates of water through the hose, and (b) the average velocity of water at the nozzle exit.



SOLUTION A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis (*a*) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}}\left(\frac{3.7854 \text{ L}}{1 \text{ gal}}\right) = \mathbf{0.757 \text{ L/s}}$$

$$\dot{m} = \rho\dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = \mathbf{0.757 \text{ kg/s}}$$

(*b*) The cross-sectional area of the nozzle exit is

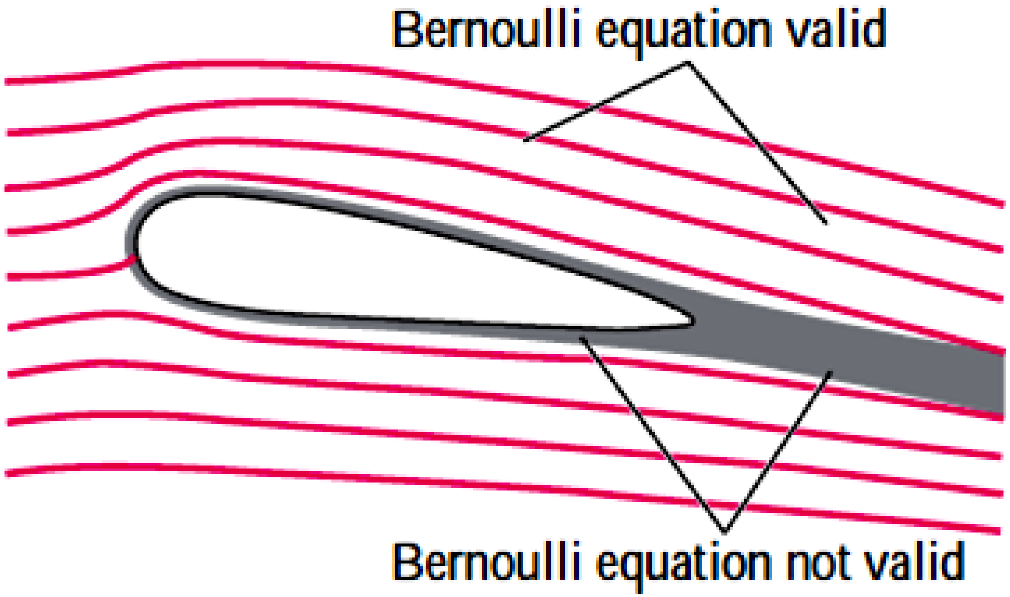
$$A_e = \pi r_e^2 = \pi(0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = \mathbf{15.1 \text{ m/s}}$$

The Bernoulli Equation

- The **Bernoulli** equation is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.
- Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.



Derivation of the Bernoulli Equation

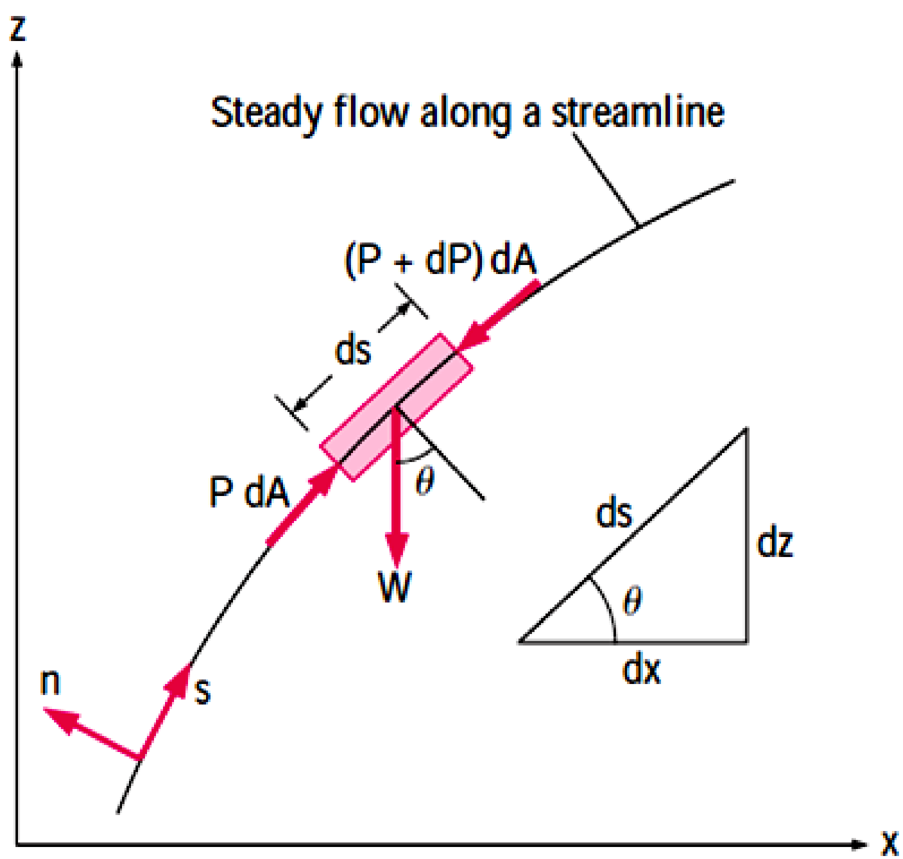
Assumptions

- Inviscid flow (ideal fluid, frictionless)
- Steady flow
- Along a streamline
- Constant density (incompressible flow)
- No shaft work or heat transfer

- Care must be exercised when applying the Bernoulli equation since it is an approximation that applies only to inviscid regions of flow.
- The Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.

Derivation of the Bernoulli Equation

- A streamline (a line which follows the direction of the fluid velocity) is chosen with the coordinates shown in Fig below.
- Around this line, a cylindrical element of fluid having the cross-sectional area dA and length ds is considered.



- Let p be the pressure acting on the lower face, and pressure p + dp acts on the upper face a distance ds away.
- The gravitational force acting on this element is its weight, ρgdAds.

Derivation of the Bernoulli Equation

- Applying Newton's second in the s-direction on a particle moving along a streamline gives

$$\sum F_s = ma_s$$

- The velocity may change with both position and time. In one-dimensional flow it therefore becomes a function of distance and time, *v = v(s, t). The* change in velocity dv over time dt may be written as

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \quad \text{and} \quad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

In steady flow $\partial V/\partial t = 0$ and thus $V = V(s)$, and the acceleration in the s-direction becomes

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V = V\frac{dV}{ds}$$

Derivation of the Bernoulli Equation

- Summing forces in the direction of motion, the s-direction results

$$pdA-(p+dp)dA-\rho g\ ds\ dA\sin\theta=\rho\ ds\ dA\ a_s$$

- Where $a_s=V\dfrac{dV}{ds}$ and $\sin\theta=dz/ds$
- On substituting and dividing the equation by ρgdA, *we can obtain* ***Euler's equation:***

$$\boxed{\frac{dp}{\rho g}+dz+\frac{V}{g}dV=0}$$

- Note that Euler's equation is valid also for compressible flow.

Derivation of the Bernoulli Equation

- Now if we further assume that the flow is incompressible so that the density is constant, we may integrate Euler's equation to get

$$\boxed{\frac{p}{\rho g} + z + \frac{V^2}{2g} = \text{constant}}$$

- The terms of in the equation represent energy per unit weight, and they have the units of length (m) so they are commonly termed heads.

$$\frac{v^2}{2g}: \quad \text{velocity head}$$

$$\frac{p}{\rho g}: \quad \text{pressure head}$$

$$z: \quad \text{potential head}$$

Derivation of the Bernoulli Equation

- A **head** corresponds to **energy per unit weight of flow** and has dimensions of length.
- **Piezometric head = pressure head + elevation head**, which is the level registered by a piezometer connected to that point in a pipeline.
- **Total head = piezometric head + velocity head.**
- It follows that for ideal steady flow the total energy head is constant along a streamline, but the constant may differ in different streamlines .
- Applying the Bernoulli equation to any two points on the same streamline, we have

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

Application of Bernoulli's equation

- Various problems on the one-dimensional flow of an ideal fluid can be solved by jointly using Bernoulli's theorem and the continuity equation.

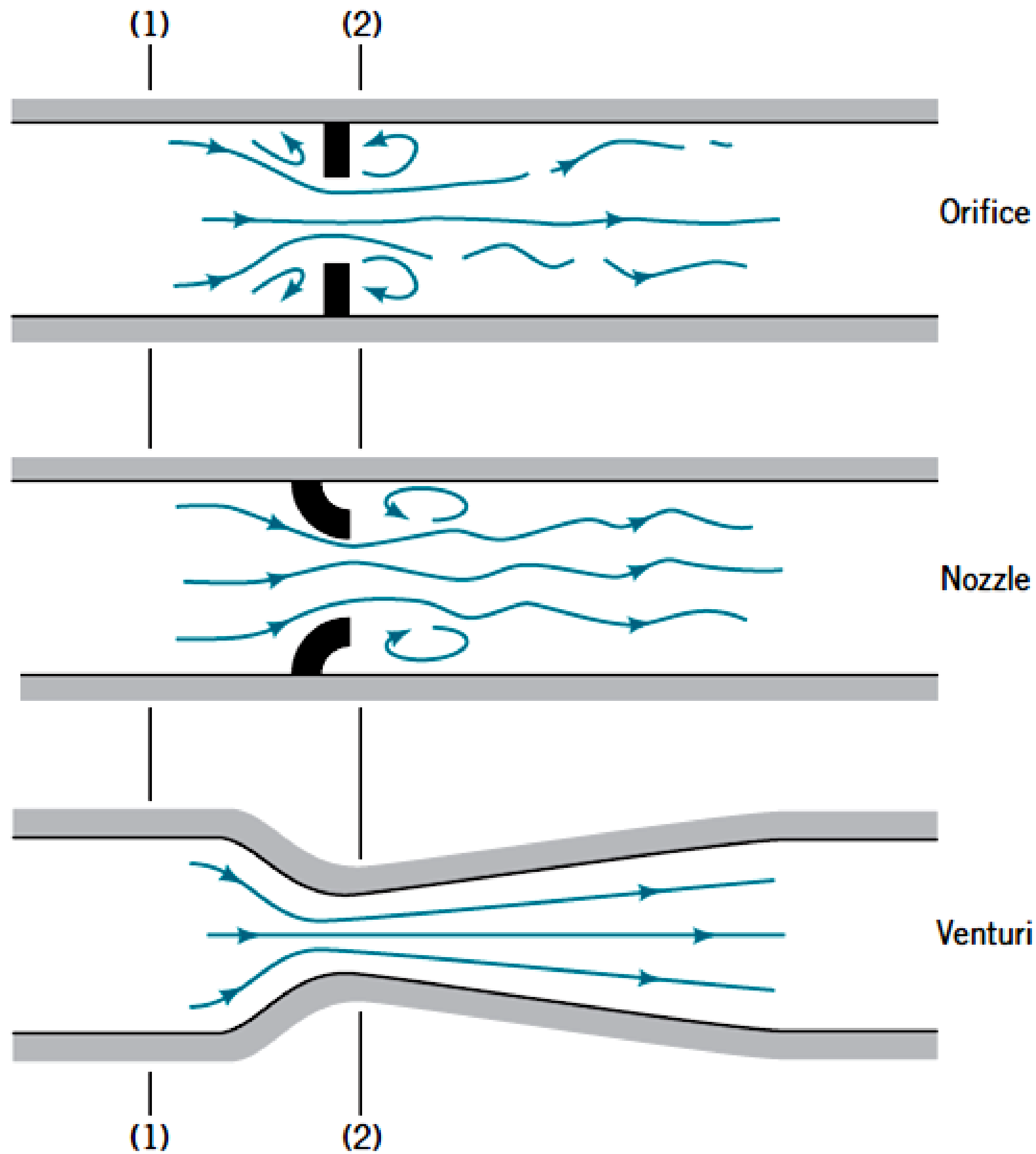
Venturi, nozzle and orifice meters

- The Venturi, nozzle, and orifice-meters are three similar types of devices for **measuring discharge** in a pipe.
- The Venturi meter consists of a rapidly converging section, which increases the velocity of flow and hence reduces the pressure.
- It then returns to the original dimensions of the pipe by a gently diverging 'diffuser' section.
- By measuring the pressure differences the discharge can be calculated.

Application of Bernoulli's equation

- We assume the flow is horizontal $(z_1 = z_2)$, steady, inviscid, and incompressible between points (1) and (2). The Bernoulli equation becomes

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$



Application of Bernoulli's equation

- If we assume the velocity profiles are uniform at sections (1) and (2), the continuity equation can be written as

$$Q = A_1V_1 = A_2V_2$$

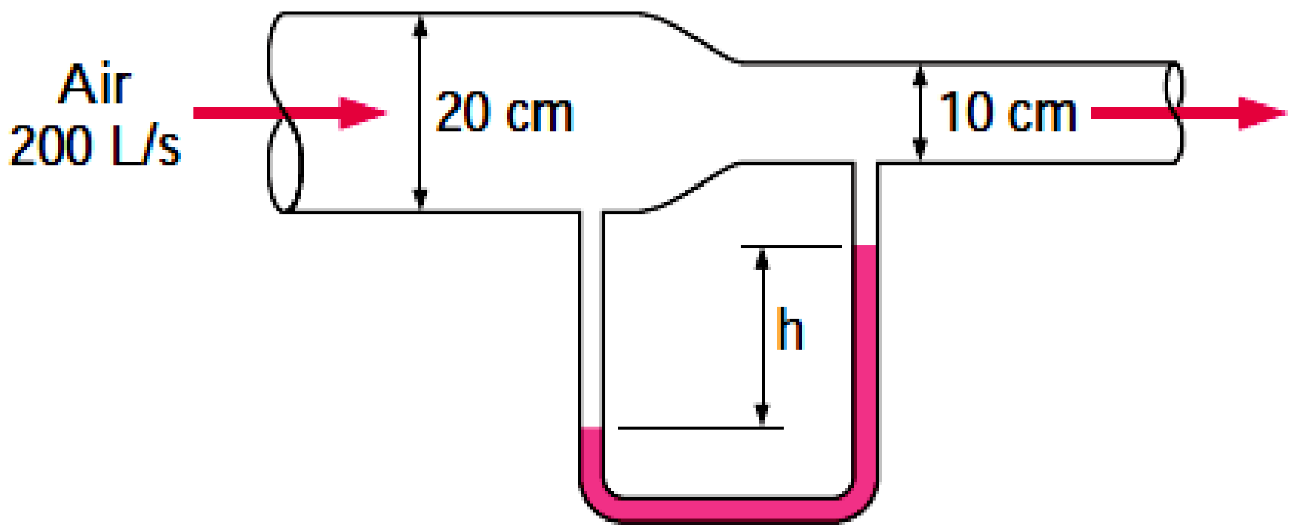
- Where A_2 is the small $(A_2 < A_1)$ flow area at section (2). Combination of these two equations results in the following theoretical flowrate

$$Q = A_2\sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

- Thus for a given flow geometry $(A_1 \text{ and } A_2)$ and the flowrate can be determined if the pressure difference, $p_1 - p_2$, is measured.

Example 2. Application of Bernoulli's equation

- Air flows through a pipe at a rate of 200 L/s. The pipe consists of two sections of diameters 20 cm and 10 cm with a smooth reducing section that connects them. The pressure difference between the two pipe sections is measured by a water manometer. Neglecting frictional effects, determine the differential height of water between the two pipe sections. Take the air density to be 1.20 kg/m^3.



Example 2. Application of Bernoulli's equation

- ***Assumptions*** **.**1The flow through the pipe is steady, incompressible, and with negligible friction (so that the Bernoulli equation is applicable). 2 The losses in the reducing section are negligible. 3 The pressure difference across an air column is negligible because of the low density of air, and thus the air column in the manometer can be ignored.
- **Analysis**. We take points 1 and 2 along the centerline of the pipe over the two tubes of the manometer. Noting that $z_1 = z_2$ (or, the elevation effects are negligible for gases), the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad P_1 - P_2 = \frac{\rho_{\text{air}}(V_2^2 - V_1^2)}{2} \qquad (1)$$

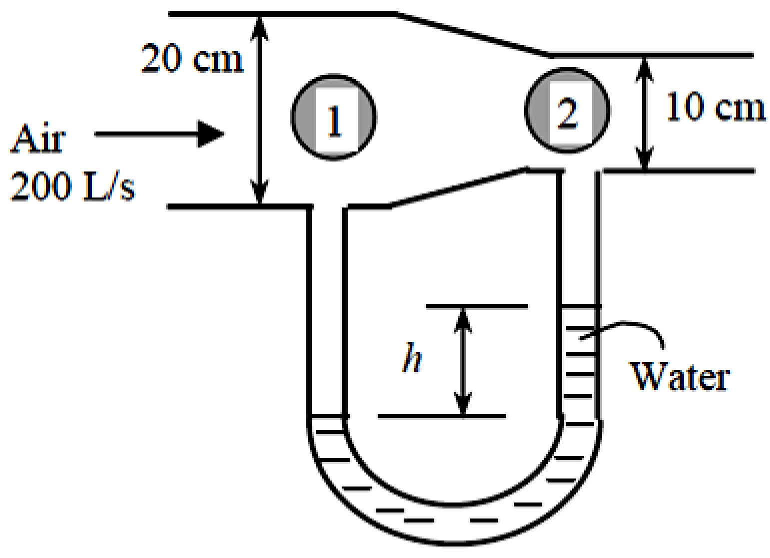
Example 2. Application of Bernoulli's equation

- We let the differential height of the water manometer be h. Then the pressure difference $P_2 - P_1$ can also be expressed as

$$P_1 - P_2 = \rho_w g h \qquad (2)$$

Combining Eqs. (1) and (2) and solving for *h*,

$$\frac{\rho_{\text{air}}(V_2^2 - V_1^2)}{2} = \rho_w g h \quad \rightarrow \quad h = \frac{\rho_{\text{air}}(V_2^2 - V_1^2)}{2g\rho_w} = \frac{V_2^2 - V_1^2}{2g\rho_w / \rho_{\text{air}}}$$



Example 2. Application of Bernoulli's equation

Calculating the velocities and substituting,

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.2\ \text{m}^3/\text{s}}{\pi(0.2\ \text{m})^2/4} = 6.37\ \text{m/s}$$

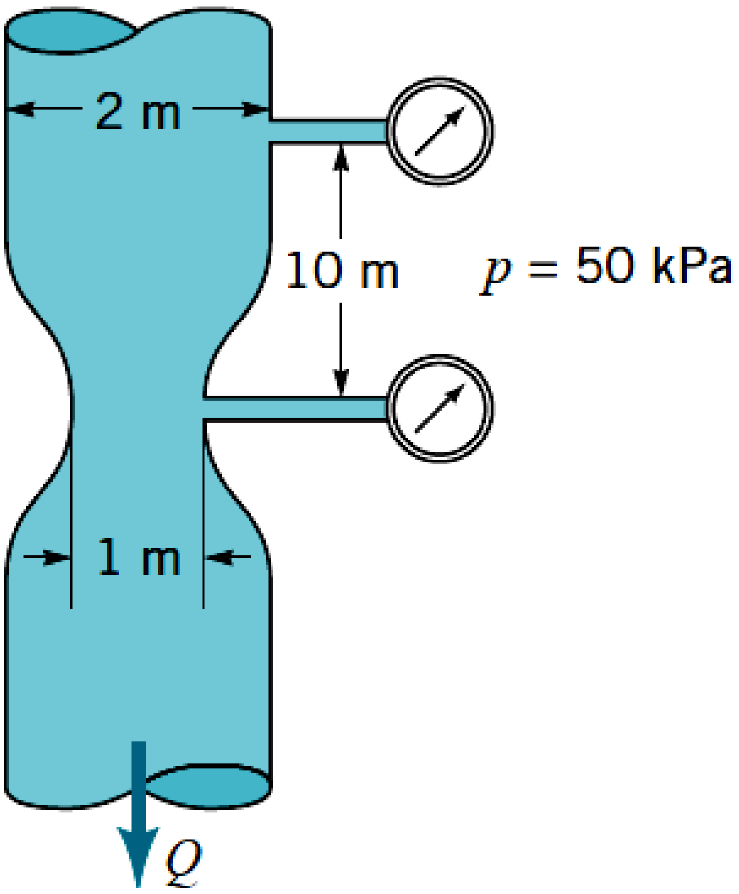
$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.2\ \text{m}^3/\text{s}}{\pi(0.1\ \text{m})^2/4} = 25.5\ \text{m/s}$$

$$h = \frac{(25.5\ \text{m/s})^2 - (6.37\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)(1000/1.20)} = 0.037\ \text{m} = \mathbf{3.7\ cm}$$

Therefore, the differential height of the water column will be 3.7 cm.

Exercise 1

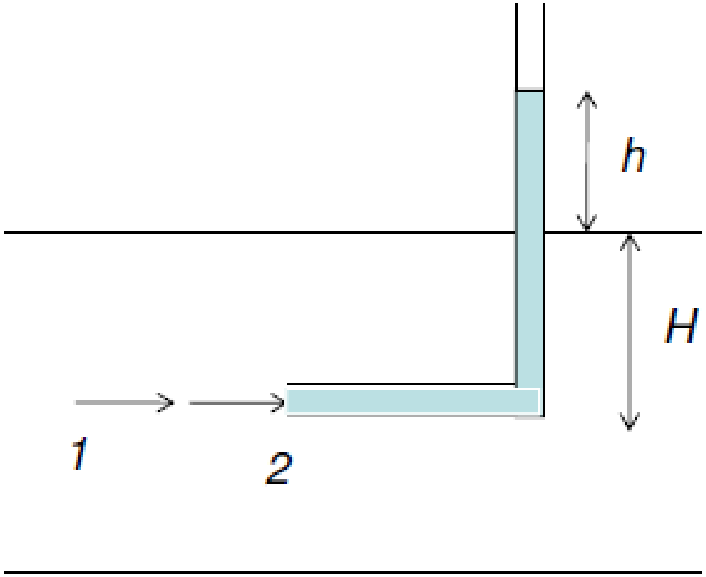
- Water (assumed inviscid and incompressible) flows steadily in the vertical variable-area pipe shown in the Fig. Determine the flowrate if the pressure in each of the gages reads 50 kPa.
- **Answer. Q= 11.4 m³/s**



Application of Bernoulli's equation- The Pitot Tube

- **The Pitot** Tube is a device used for measuring the velocity of flow at any point in a pipe or a channel.
- **Principle:** If the velocity at any point decreases, the pressure at that point increases due to the conservation of the kinetic energy into pressure energy. In simplest form, the Pitot tube consists of a glass tube, bent at right angles.

Let p_1 = pressure at section 1
p_2 = pressure at section 2
v_1 = velocity at section 1
v_2 = velocity at section 2 = 0
H = depth of tube in the liquid
h = rise of liquid in the tube above the free surface



Point 2 is just at the inlet of the Pitot-tube
Point 1 is far away from the tube

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{\ 2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{\ 2}}{2g}+z_2$$

But $z_1 = z_2$, and $v_2 = 0.$

$\frac{p_1}{\rho g} =$ Pressure head at 1=H

$\frac{p_2}{\rho g} =$ Pressure head at 2=h+H

Substituting these values, we get $H+\frac{v_1^{\ 2}}{2g}=h+H$

$$\Rightarrow v_1=\sqrt{2gh}$$

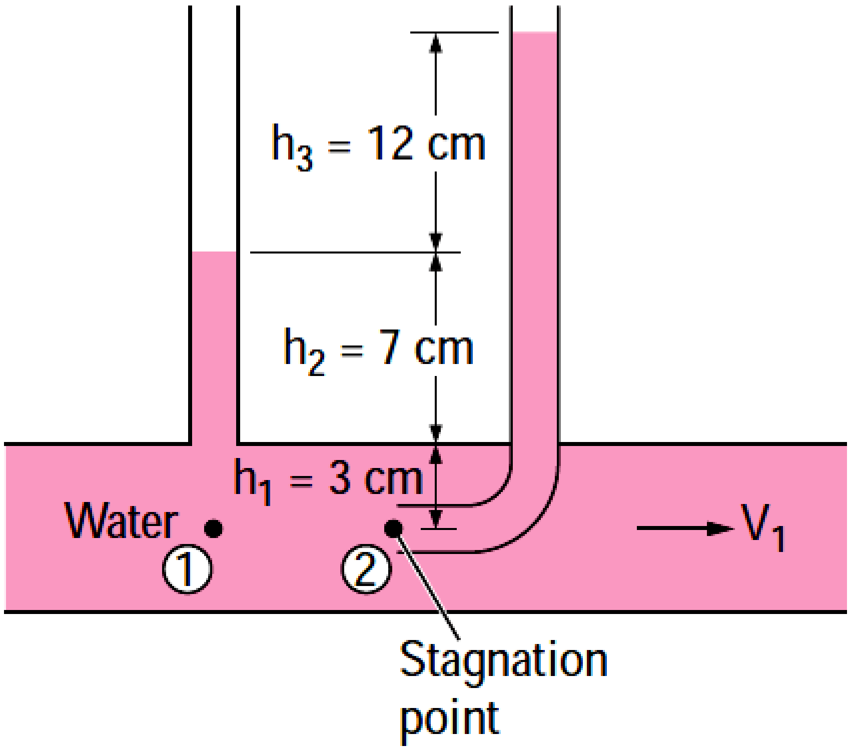
This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act}=C_v\sqrt{2gh}$$

$C_v \equiv$ coefficient of pitot-tube

Example 3. Velocity Measurement by a Pitot Tube

- A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in the Fig. , to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.



Analysis We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$$P_1 = \rho g(h_1 + h_2)$$

$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}^{\nearrow 0} + \cancel{z_2} \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

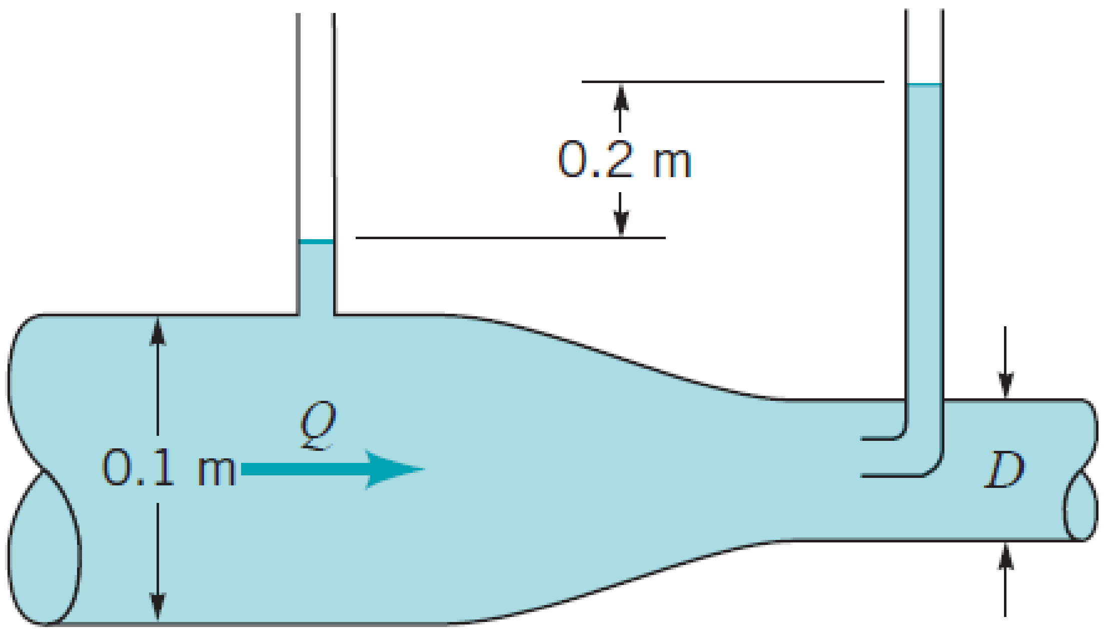
Solving for V_1 and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81\ \text{m/s}^2)(0.12\ \text{m})} = \mathbf{1.53\ m/s}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube.

Example 4

- Water flows through the pipe contraction shown in Fig. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, *D*.

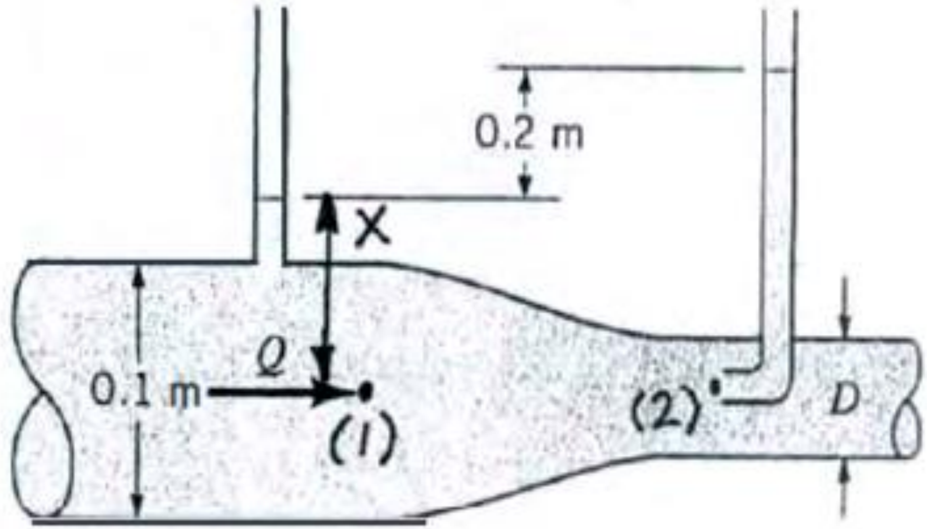


$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$ and $V_2 = 0$.

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma}$$



But

$$\frac{p_1}{\gamma} = x \quad \text{and} \quad \frac{p_2}{\gamma} = 0.2\,m + x \quad \text{so that}$$

$$x + \frac{V_1^2}{2g} = 0.2\,m + x \quad \text{or}$$

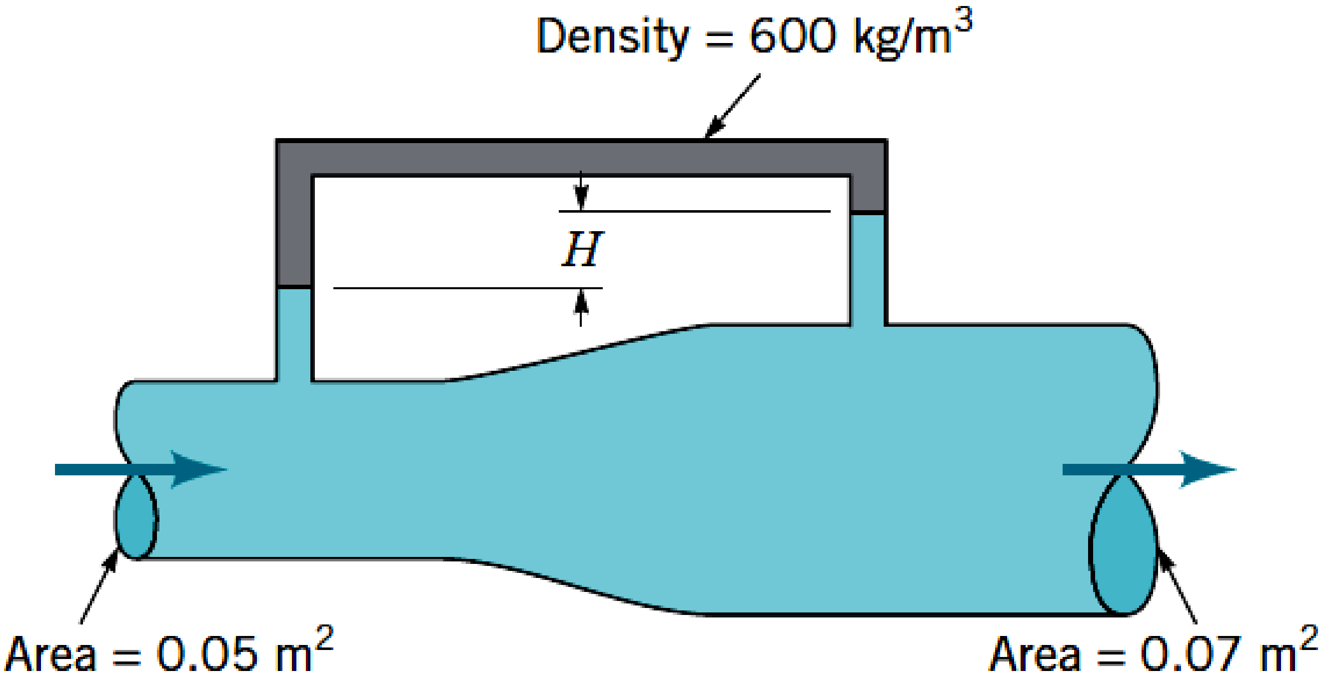
$$V_1 = \sqrt{2g\,(0.2\,m)} = \left(2\left(9.81\,\frac{m}{s^2}\right)(0.2\,m)\right)^{1/2} = 1.98\,\frac{m}{s}$$

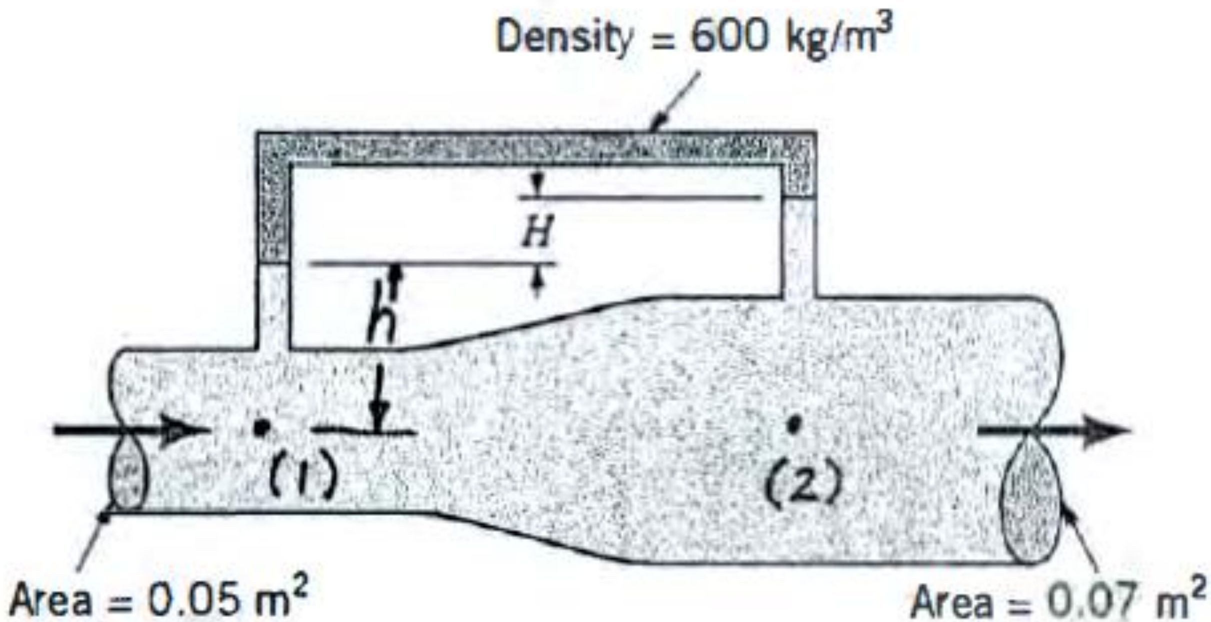
Thus,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.1\,m)^2\left(1.98\,\frac{m}{s}\right) = \underline{\underline{0.0156\,\frac{m^3}{s} \text{ for any } D}}$$

Example 5

- Water flows steadily through the variable area pipe shown in the Fig. with negligible viscous effects. Determine the manometer reading, H, if the flowrate is 0.5 m^3/s and the density of the manometer fluid is 600 kg/m^3.





From the Bernoulli equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2, \quad \text{where } z_1=z_2$$

Thus,

(1) $$p_2-p_1=\frac{\gamma}{2g}(V_1^2-V_2^2)=\frac{1}{2}\rho(V_1^2-V_2^2)$$

But, $Q=A_1V_1=A_2V_2$ so that

$$V_1 = \frac{Q}{A_1} = \frac{0.5\,\frac{m^3}{s}}{0.05\,m^2} = 10\,\frac{m}{s} \text{ and } V_2 = \frac{Q}{A_2} = \frac{0.5\,\frac{m^3}{s}}{0.07\,m^2} = 7.14\,\frac{m}{s}$$

Hence, from Eq. (1):

(2) $$p_2 - p_1 = \frac{1}{2}\left(999\,\frac{kg}{m^3}\right)\left[\left(10\,\frac{m}{s}\right)^2 - \left(7.14\,\frac{m}{s}\right)^2\right] = 24.5\times10^3\left(\frac{kg\cdot m}{s^2}\right)/m^2$$

$$= 24.5\times10^3\,\frac{N}{m^2}$$

For the manometer,

$$p_1 - \gamma_{H_2O}\,h - \gamma_{man}\,H = p_2 - \gamma_{H_2O}(h+H)$$

so that

(3) $$p_2 - p_1 = \gamma_{H_2O}(h+H) - \gamma_{H_2O}\,h - \gamma_{man}\,H = (\gamma_{H_2O} - \gamma_{man})H = g(\rho_{H_2O} - \rho_{man})H$$

Hence, from Eqs (2) and (3):

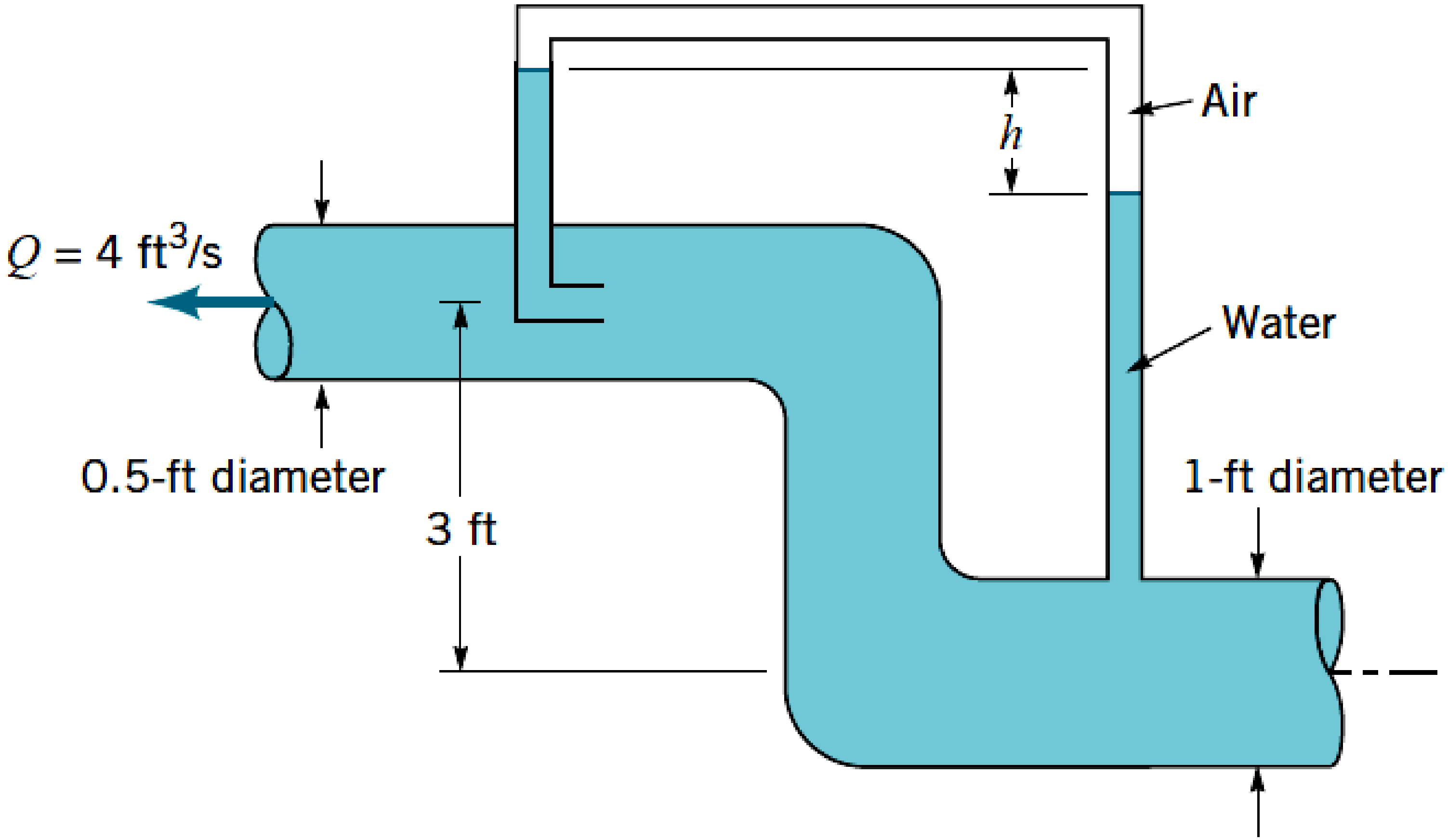
$$24.5\times10^3\,\frac{N}{m^2} = 9.81\,\frac{m}{s^2}\left(999\,\frac{kg}{m^3} - 600\,\frac{kg}{m^3}\right)H$$

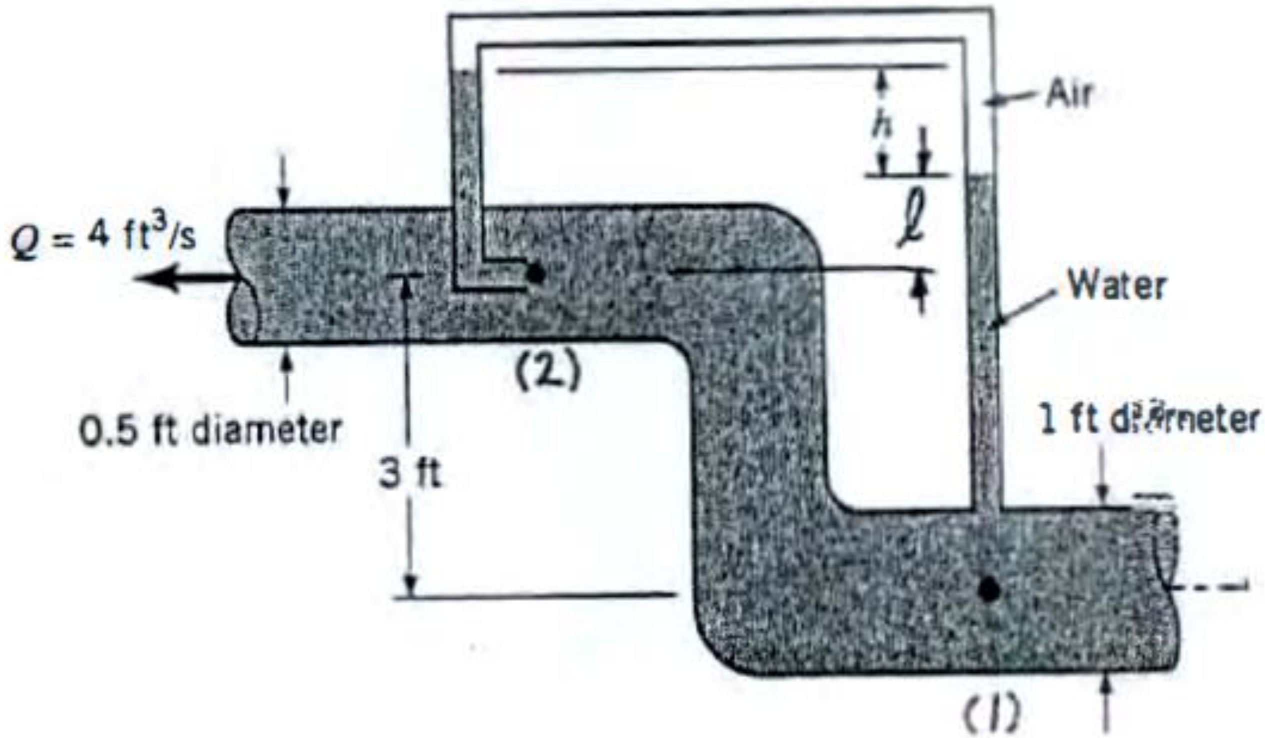
or

$$H = \underline{\underline{6.26\,m}}$$

Example 6

- Water, considered an inviscid, incompressible fluid, flows steadily as shown in Fig. below. Determine *h*.





$$p_1 + \gamma z_1 + \frac{1}{2}\rho V_1^2 = p_2 + \gamma z_2 + \frac{1}{2}\rho V_2^2$$

where $z_1 = 0$, $z_2 = 3\,ft$, $V_2 = 0$, and $V_1 = \frac{Q}{A_1} = \frac{4\,\frac{ft^3}{s}}{\frac{\pi}{4}(1\,ft)^2} = 5.09\,\frac{ft}{s}$

Thus,

$$p_1 + \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left(5.09\,\frac{ft}{s}\right)^2 = p_2 + 62.4\,\frac{lb}{ft^3}\,(3\,ft)$$

or

$$p_1 - p_2 = 162\,\frac{lb}{ft^2}$$

But from the manometer,

$$p_1 - \gamma(\ell + 3\,\text{ft}) + \gamma(h + \ell) = p_2$$

or

$$p_1 - 62.4\,\frac{\text{lb}}{\text{ft}^3}(3\,\text{ft}) + 62.4\,\frac{\text{lb}}{\text{ft}^3}\,h = p_2$$

Hence,

$p_1 = p_2 + 187 - 62.4h$ which when combined with Eq. (1) gives

$$p_2 + 187 - 62.4h - p_2 = 162$$

or

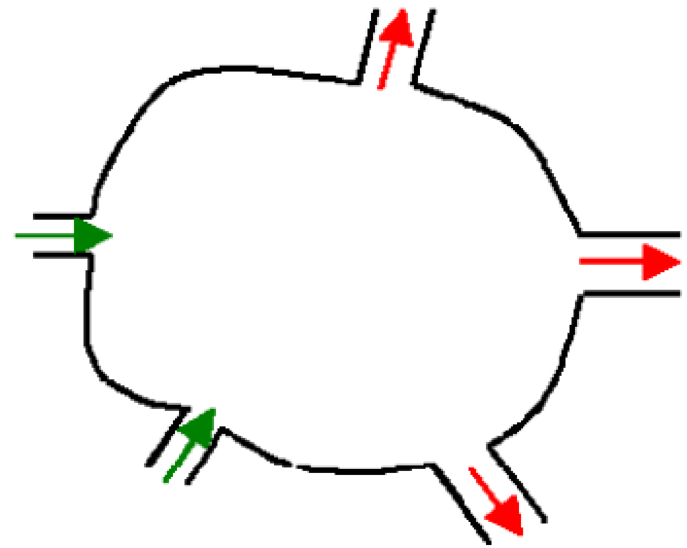
$$h = \underline{\underline{0.400\,\text{ft}}}$$

The Momentum Equation and its Application

- On applying Newton's second law of motion to the control volume

$$\sum \vec{F} = \sum_{i=1}^{M}\left(\rho_i V_i A_i \vec{V}_i\right)_{out} - \sum_{i=1}^{N}\left(\rho_i V_i A_i \vec{V}_i\right)_{in}$$

$$= \sum_{i=1}^{M}\left(\dot{m}_i \vec{V}_i\right)_{out} - \sum_{i=1}^{N}\left(\dot{m}_i \vec{V}_i\right)_{in}$$

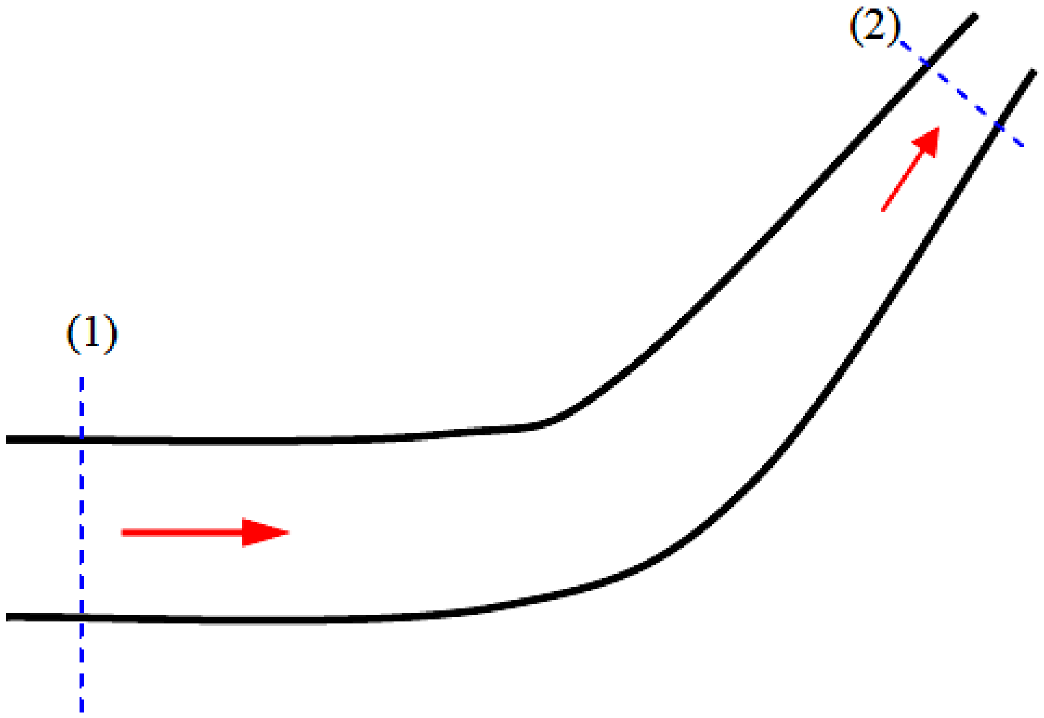


- Note that this equation
  - follows from the principle of conservation of linear momentum: resultant force on the control volume is balanced by the net rate of momentum flux (i.e., $\dot{m}\vec{V}$) out through the control surface.
  - is a vector equation. **Components of the forces and the velocities need to be considered.**

The Momentum Equation and its Application

- Further consider a steady-flow situation in which there is only one entrance (section 1) and one exit (section 2) across which uniform profiles can be assumed. By continuity

$$\dot{m}_1 = \dot{m}_2 = \rho Q = \text{mass flow rate}$$



The Momentum Equation and its Application

The momentum equation now reduces to $\sum \vec{F} = \rho Q\left(\vec{V}_2 - \vec{V}_1\right)$

or in terms of their components in (x, y, z) coordinates

$$\sum F_x = \rho Q\left[\left(V_x\right)_2 - \left(V_x\right)_1\right]$$
$$\sum F_y = \rho Q\left[\left(V_y\right)_2 - \left(V_y\right)_1\right]$$
$$\sum F_z = \rho Q\left[\left(V_z\right)_2 - \left(V_z\right)_1\right]$$

where $\left(V_x\right)_1$ is the x-component of the velocity at section 1, and so on.

The Momentum Equation and its Application

- On applying the momentum equation, one needs to pay attention to the following two aspects

Forces

- $\sum \vec{F}$ represents **all forces acting on the control volume,** including
- Surface forces resulting from the surrounding acting on the control volume:
  - Impact force, which is usually the unknown to be found, on the control surface in contact with a solid boundary
  - Pressure force on the control surface which cuts a flow inlet or exit. Remember that the pressure force is always a compressive force.
- Body force that results from gravity.

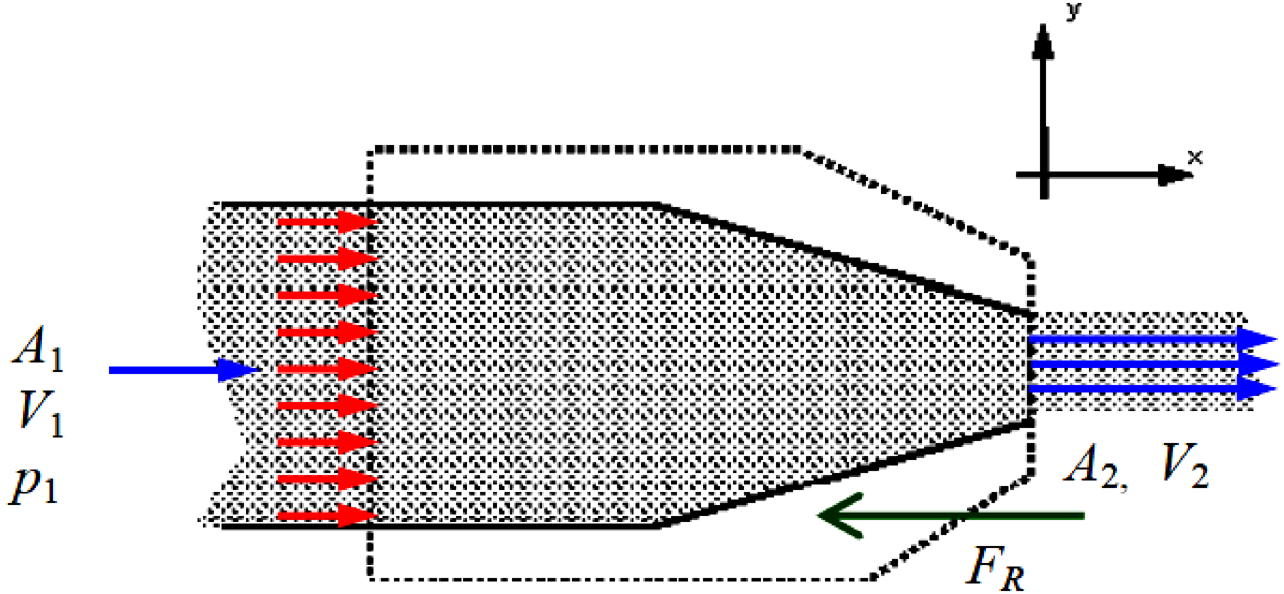
Application of the momentum Equation: Force on a pipe nozzle

- A simple application of the momentum equation is to find the force on the nozzle at the outlet of a pipe. Because the fluid is contracted at the nozzle forces are induced in the nozzle. Anything holding the nozzle (e.g. a fireman) must be strong enough to withstand these forces.

Steps in analysis:

- Draw a control volume
- Decide on a coordinate-axis system
- Calculate the total force, given by the rate of change of momentum across the control volume
- Calculate the pressure force F_p
- Calculate the body force F_B
- Calculate the resultant reaction force F_R

1 & 2. Control volume and co-ordinate axis are shown in the figure below.



Notice how this is a one-dimensional system which greatly simplifies matters.

3. Calculate the total force

$$\sum F = \rho Q \left(V_2 - V_1\right)$$

By continuity, $Q = A_1 V_1 = A_2 V_2$, so

$$\sum F = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

4. Calculate the pressure force (red arrows)

$$F_p = \text{ pressure force at 1 } - \text{ pressure force at 2} = p_1 A_1 - p_2 A_2$$

We use the Bernoulli equation to calculate the pressure

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^{\,2}}{2g}$$

Since the nozzle is horizontal, $z_1 = z_2$, and the pressure outside is atmospheric, $p_2 = 0$, and with continuity the Bernoulli equation gives

$$p_1 = \frac{\rho Q^2}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

$$\Rightarrow F_p = \frac{\rho Q^2 A_1}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

5. Calculate the **body** force

The only body force is the weight due to gravity in the *y*-direction - but we need not consider this as the only forces we are considering are in the *x*-direction.

6. Calculate the **reaction** force that the nozzle acts on the fluid (green arrow)

Since the indicated direction of the reaction force is opposite to *x*-axis, a negative sign is included

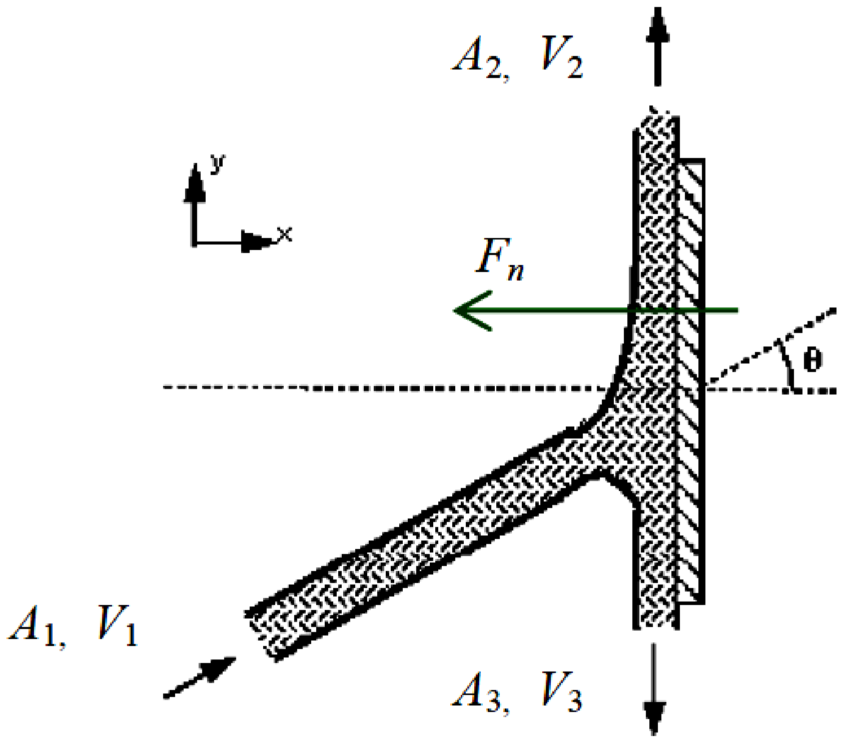
$$\sum F = -F_R + F_p + \cancel{F_B} = \rho Q^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

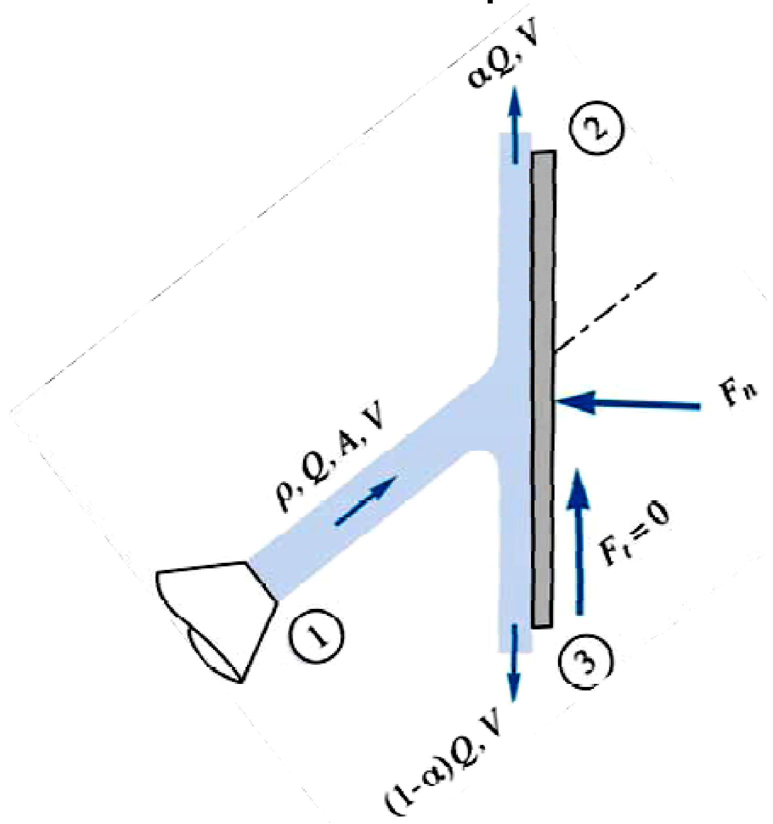
$$\Rightarrow F_R = \frac{\rho Q^2 A_1}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) - \rho Q^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right) = \frac{\rho Q^2}{2A_1}\left(\frac{A_1}{A_2} - 1\right)^2$$

So the fireman must be able to resist the force of F_R.

Force due to a two-dimensional jet hitting an inclined plane

- Consider a two-dimensional jet hitting a flat plate at an angle θ. For simplicity gravity and friction are neglected from this analysis.
- We want to find the reaction force normal to the plate so we choose the axis system such that it is normal to the plane.





Force due to a two-dimensional jet hitting an inclined plane

We do not know the velocities of flow in each direction. To find these we can apply the Bernoulli equation

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g} = \frac{p_3}{\rho g} + z_3 + \frac{V_3^2}{2g}$$

The height differences are negligible i.e., $z_1 = z_2 = z_3$, and the pressures are all atmospheric = 0. So

$$V_1 = V_2 = V_3 = V$$

By continuity

$$Q_1 = Q_2 + Q_3 \quad \Rightarrow \quad V_1 A_1 = V_2 A_2 + V_3 A_3$$

$$\Rightarrow \quad A_1 = A_2 + A_3$$

Using this we can calculate the forces in the same way as before.

2. Calculate the pressure force

All zero as the pressure is everywhere atmospheric.

3. Calculate the body force

As the control volume is small, hence the weight of fluid is small, we can ignore the body forces.

4. Calculate the resultant reaction force

$$\sum F_x = -F_n + \cancel{F_p} + \cancel{F_B} = -\rho Q_1 V \cos\theta \qquad \Rightarrow \qquad F_n = \rho Q_1 V \cos\theta$$

which is the force exerted **on the fluid by the plate**.

We can further find out how much discharge goes along in each direction on the plate. Along the plate, in the y-direction, the total force must be zero, $\sum F_y = 0$, since friction is ignored.

Also in the y-direction: $V_{1y} = V\sin\theta$, $V_{2y} = V$, $V_{3y} = -V$, so

$$\sum F_y = \rho\left[\left(Q_2V_{2y} + Q_3V_{3y}\right) - Q_1V_{1y}\right] = \rho V\left[Q_2 - Q_3 - Q_1\sin\theta\right] = \rho V^2\left[A_2 - A_3 - A_1\sin\theta\right]$$

Setting this to zero, we get

$$0 = A_2 - A_3 - A_1 \sin\theta$$

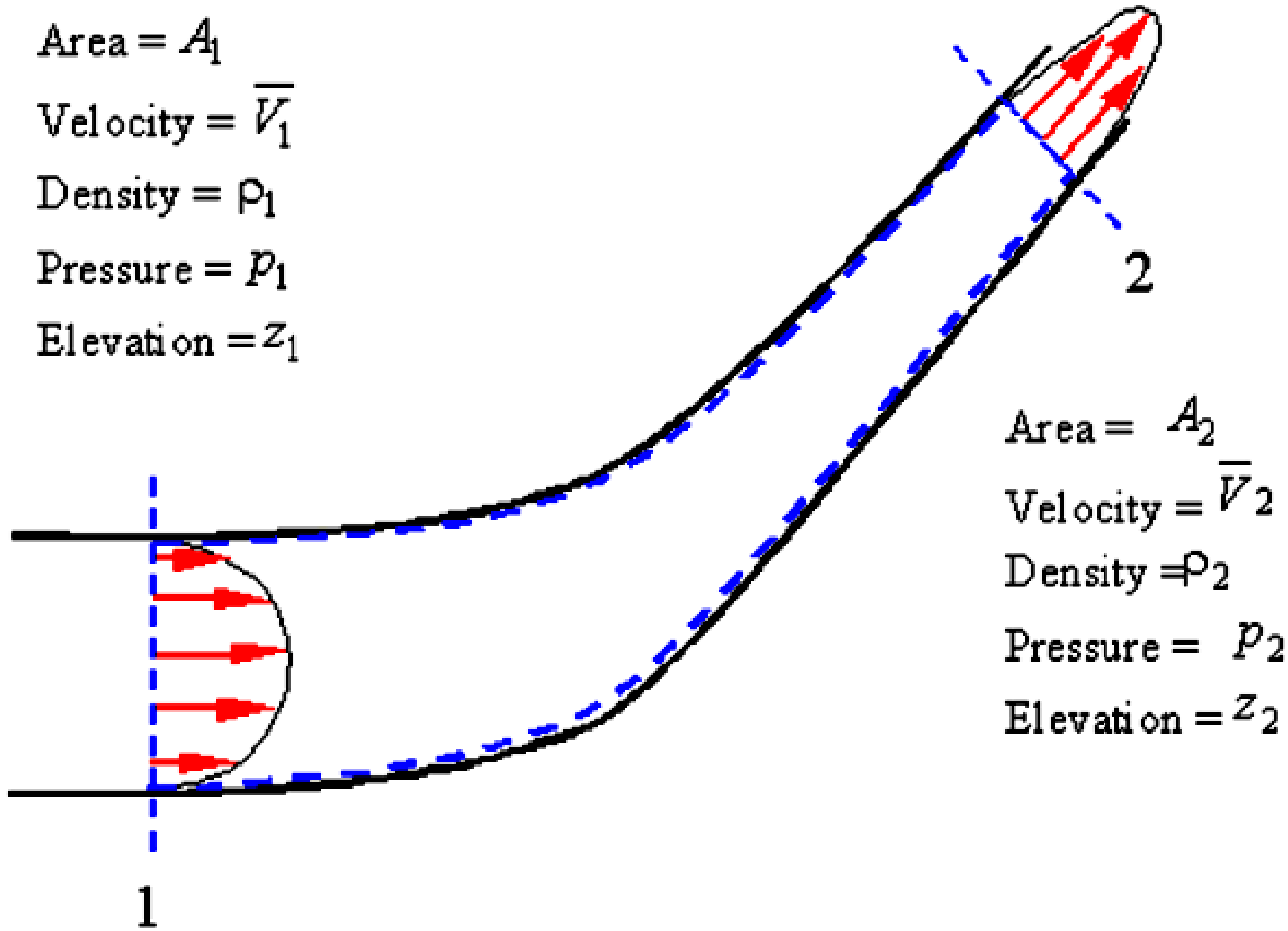
and as found earlier we have $A_1 = A_2 + A_3$, so on solving

$$A_2 = A_3\left(\frac{1+\sin\theta}{1-\sin\theta}\right)$$

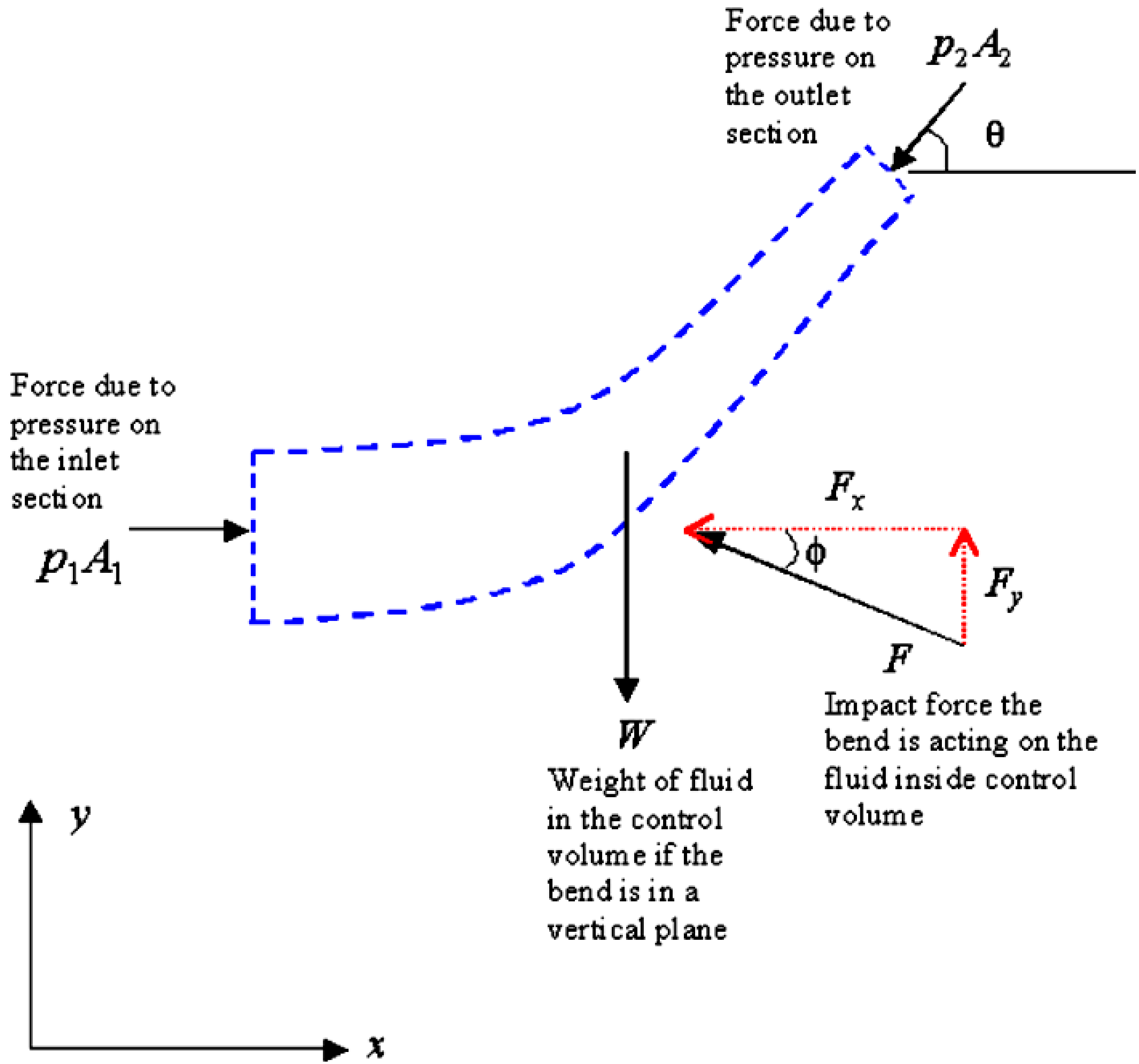
by which we readily obtain that $\dfrac{Q_2}{Q_1} = \alpha = \dfrac{1}{2}(1+\sin\theta)$, $\quad \dfrac{Q_3}{Q_1} = 1-\alpha = \dfrac{1}{2}(1-\sin\theta)$

So we know how the discharge is divided between the two jets leaving the plate.

Flow past a pipe bend



- Consider the pipe bend shown above. We may first draw a free body diagram for the control volume with the forces:


Force due to pressure on the outlet section
p_2A_2
θ
Force due to pressure on the inlet section
p_1A_1
F_x
ϕ
F_y
F
Impact force the bend is acting on the fluid inside control volume
W
Weight of fluid in the control volume if the bend is in a vertical plane
y
x

Paying due regard to the positive x and y directions, we may write the summation of forces in these two directions:

$$\sum F_x = p_1 A_1 - p_2 A_2 \cos\theta - F_x$$
$$\sum F_y = F_y - p_2 A_2 \sin\theta - W$$

Relating these components to the net change of momentum flux through the inlet and exit surfaces

x-Direction

$$p_1 A_1 - p_2 A_2 \cos\theta - F_x = \rho Q\left(\bar{V}_2 \cos\theta - \bar{V}_1\right)$$

y-Direction

$$F_y - p_2 A_2 \sin\theta - W = \rho Q\left(\bar{V}_2 \sin\theta - 0\right)$$

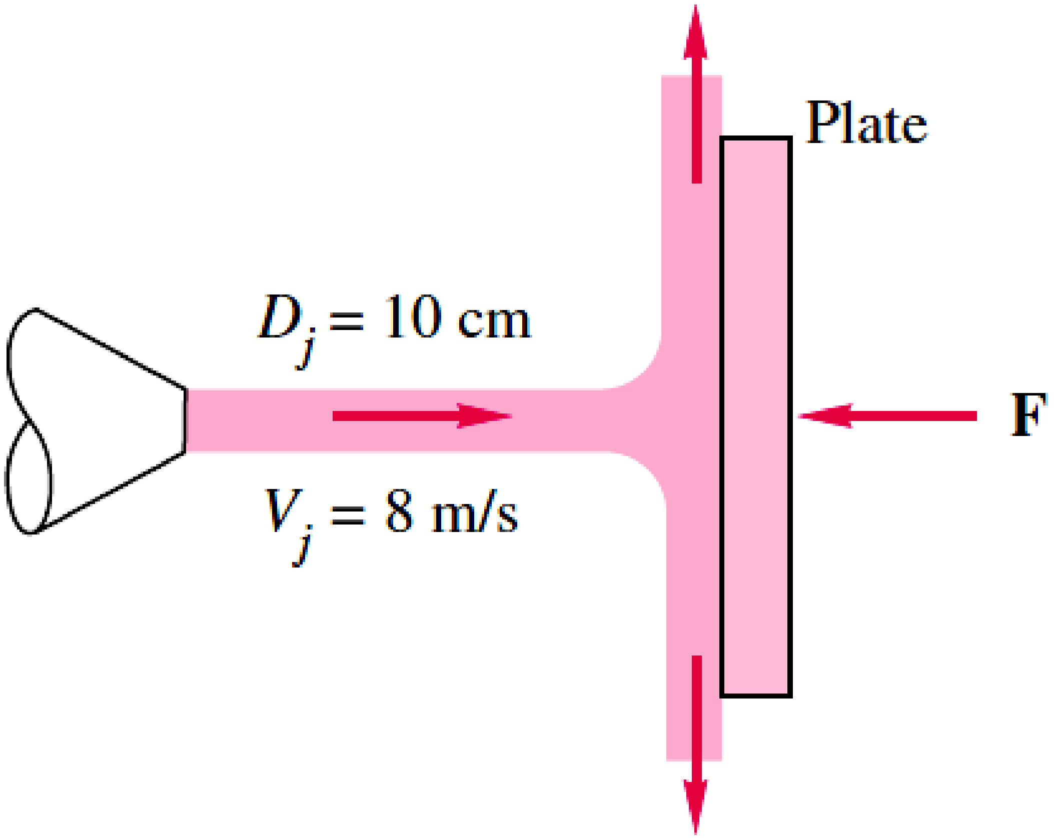
From these two equations and using the continuity equation and the Bernoulli equation, we may calculate the two force components. The magnitude and direction of the resultant force from the bend on the fluid are

$$F = \sqrt{F_x^2 + F_y^2}$$
$$\phi = \tan^{-1}\left(F_y / F_x\right)$$

As a reaction, the impact force on the pipe bend is equal in magnitude, but opposite in direction to the one on the fluid.

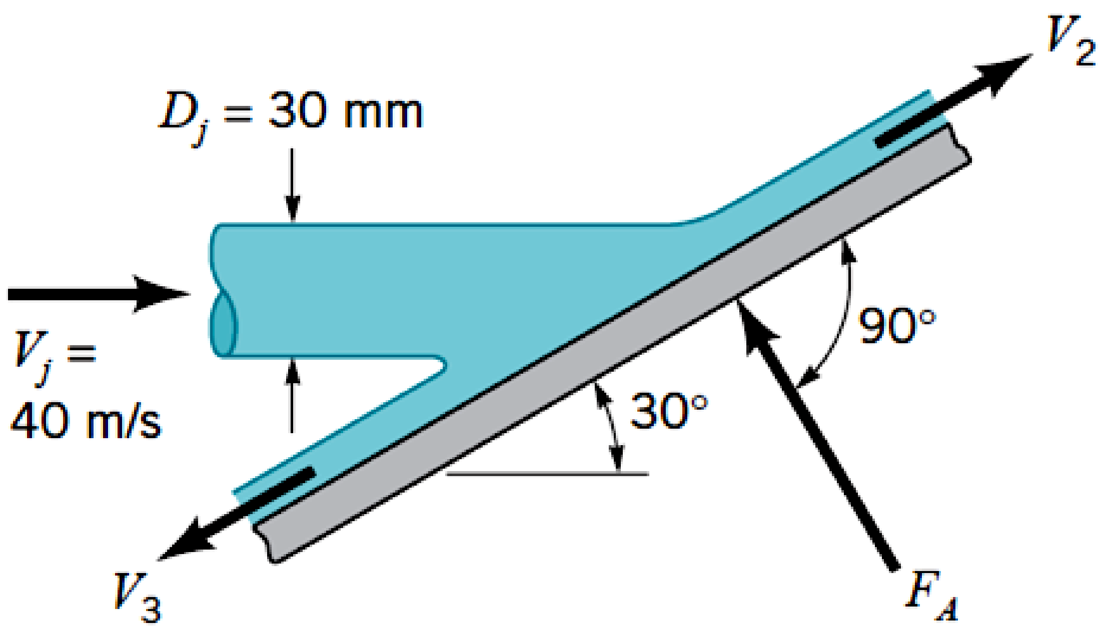
Example 7

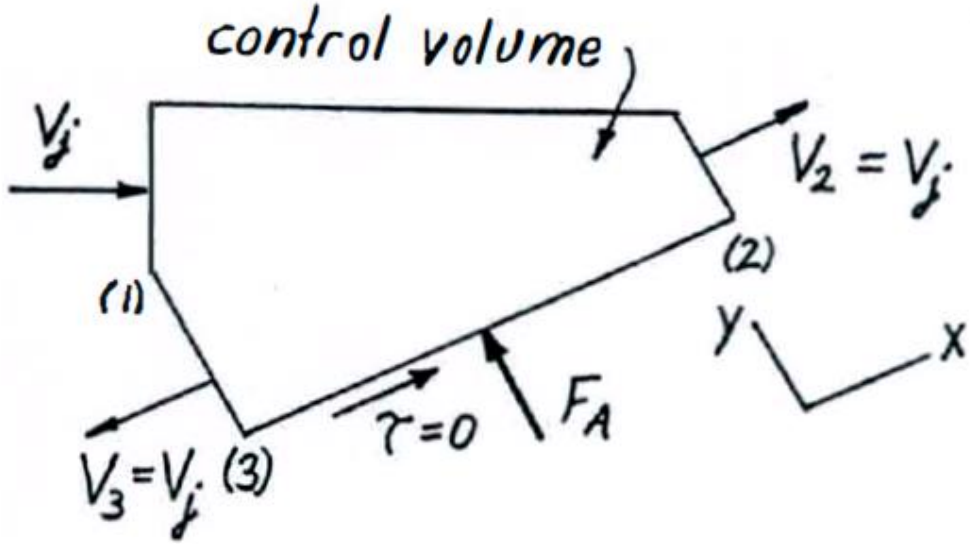
- The water jet in Fig. shown strikes normal to a fixed plate. Neglect gravity and friction, and compute the force F in newtons required to hold the plate fixed. **Ans. 503 N**



Example 8

- A horizontal circular jet of air strikes a stationary flat plate as indicated in the Fig. The jet velocity is 40 m/s and the jet diameter is 30 mm. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine: **(a)** the magnitude of FA, the anchoring force required to hold the plate stationary; **(b)** the fraction of mass flow along the plate surface in each of the two directions shown.



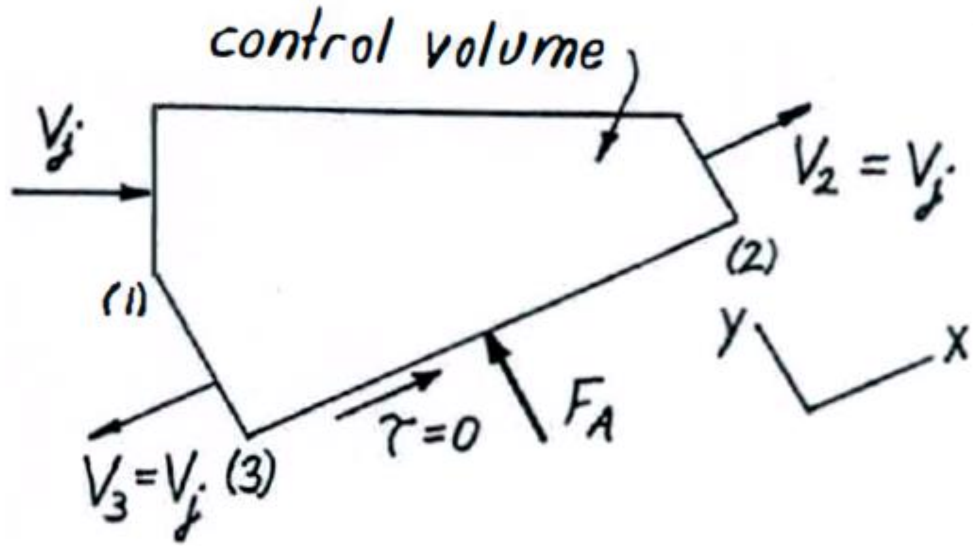


- To determine the magnitude of F_A we apply the component of the linear momentum equation along the direction of F_A

$$F_A = \dot{m} V_j \sin 30^\circ = \rho A_j V_j V_j \sin 30^\circ = \rho \frac{\pi D_j^2}{4} V_j^2 \sin 30^\circ$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030 m)^2}{(4)} \left(40 \frac{m}{s}\right)^2 (\sin 30^\circ) \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right) = \underline{\underline{0.696 N}}$$



(b) To determine the fraction of mass flow along the plate surface in each of the 2 directions shown in the sketch above, we apply the component of the linear momentum equation parallel to the surface of the plate, $\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \Sigma F_x$, to obtain

$$R_{\text{along plate surface}} = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_j V_j \cos 30° \qquad (1)$$

Since the air velocity magnitude remains constant, the value of $R_{\text{along plate surface}}$ is zero.* Thus from Eq. 1 we obtain

$$\dot{m}_3 V_3 = \dot{m}_2 V_2 - \dot{m}_j V_j \cos 30^\circ \qquad (2)$$

Since $V_3 = V_2 = V_j$, Eq. 2 becomes

$$\dot{m}_3 = \dot{m}_2 - \dot{m}_j \cos 30^\circ \qquad (3)$$

From conservation of mass we conclude that

$$\dot{m}_j = \dot{m}_2 + \dot{m}_3 \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$\dot{m}_3 = \dot{m}_j - \dot{m}_3 - \dot{m}_j \cos 30^\circ$$

or

$$\dot{m}_3 = \dot{m}_j \frac{(1 - \cos 30^\circ)}{2} = \dot{m}_j (0.0670)$$

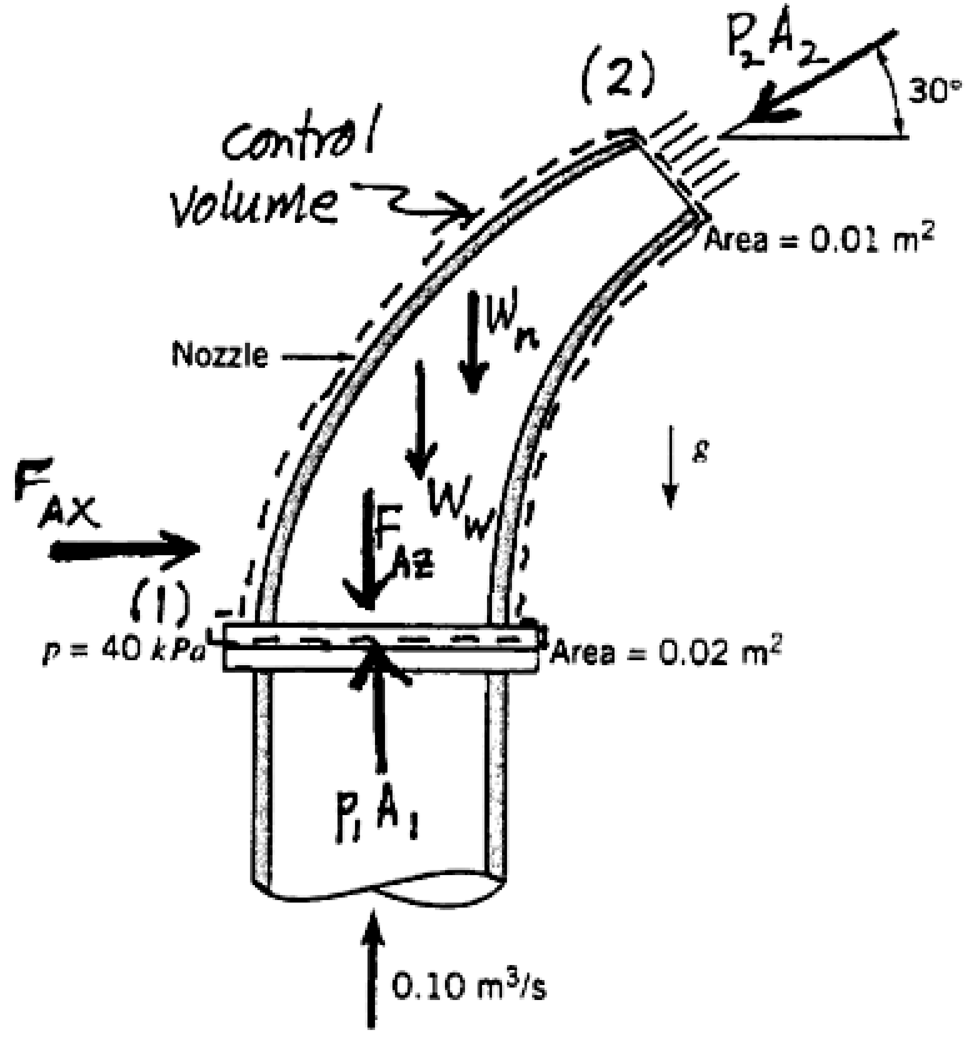
and

$$\dot{m}_2 = \dot{m}_j (1 - 0.067) = \dot{m}_j (0.933)$$

Thus, $\underline{\underline{\dot{m}_2 \text{ involves } 93.3\% \text{ of } \dot{m}_j}}$ and $\underline{\underline{\dot{m}_3 \text{ involves } 6.7\% \text{ of } \dot{m}_j}}$.

Example 9

- A nozzle is attached to a vertical pipe and discharges water into the atmosphere as shown in the figure. When the discharge is 0.1 m^3/s, the gage pressure at the flange is 40 KPa. Determine the vertical component of the anchoring force required to hold the nozzle in place. The nozzle has a weight of 200 N, and the volume of water in the nozzle is 0.012 m^3.



- Application of the vertical or z- direction component of the linear momentum to the flow through the control volume leads to

$$\dot{m}(V_2 \sin 30^\circ - V_1) = p_1 A_1 - F_{Az} - W_n - W_w - \overset{0\ \text{gage}}{\cancel{p_2}} A_2 \sin 30^\circ \qquad (1)$$

Solving Eq. 1 for F_{Az} yields

$$F_{Az} = p_1 A_1 - W_n - W_w - \dot{m}(V_2 \sin 30^\circ - V_1) \qquad (2)$$

For $\dot{m}$ we use $\dot{m} = \rho Q$

For W_w we use $W_w = \forall_w \gamma_w$

From conservation of mass we obtain

$$Q_2 = Q_1$$

or

$$V_2 = \frac{Q_1}{A_2}$$

Also, we not that $V_1 = \dfrac{Q_1}{A_1}$

Thus, Eq. 2 becomes

$$F_{Az} = p_1 A_1 - W_n - \forall_w \gamma_w - \rho Q \left(\frac{Q}{A_2} \sin 30^\circ - \frac{Q}{A_1} \right)$$

or

$$F_{Az} = (40\ kPa)\left(1 \frac{N}{m^2 \cdot Pa}\right)\left(1000 \frac{Pa}{kPa}\right)\left(0.02\ m^2\right) - 200\ N$$

$$- (0.012\ m^3)\left(9.8 \frac{kN}{m^3}\right)\left(1000 \frac{N}{kN}\right)$$

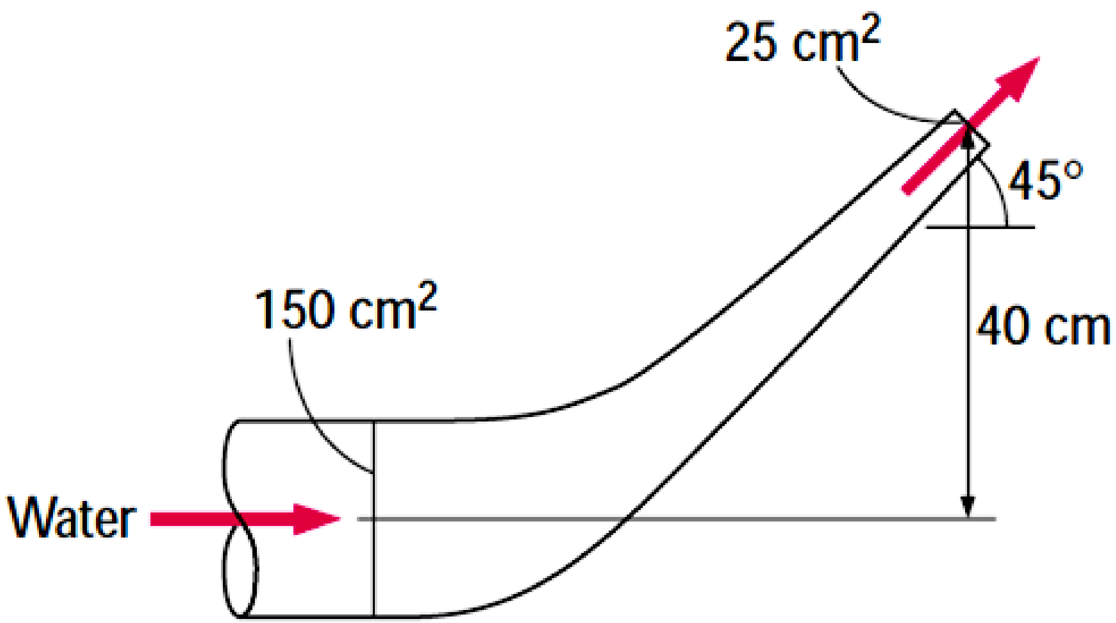
$$- \left(999 \frac{kg}{m^3}\right)\left(0.01 \frac{m^3}{s}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right)\left[\left(\frac{0.01 \frac{m^3}{s}}{0.01\ m^2}\right)\sin 30^\circ - \left(\frac{0.01 \frac{m^3}{s}}{0.02\ m^2}\right)\right]$$

and

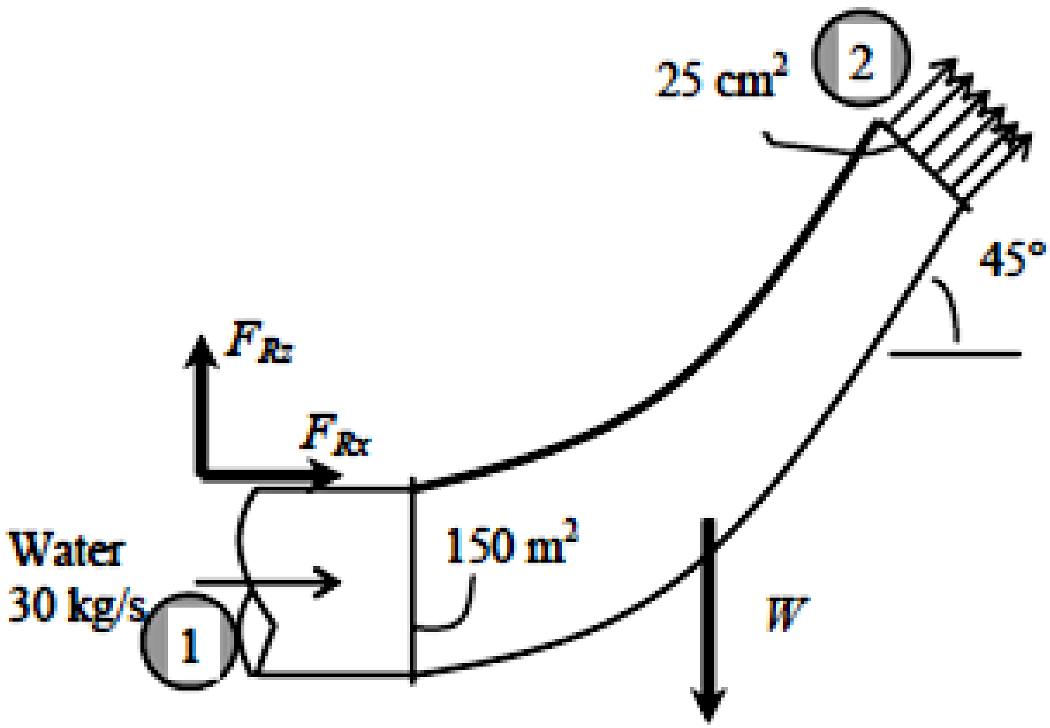
$$F_{Az} = 800N - 200N - 117.6N - 0N = \underline{\underline{482}}\ N \text{ downward}$$

Example 10

- A reducing elbow is used to deflect water flow at a rate of 30 kg/s in a horizontal pipe upward by an angle $\theta = 45^{\circ}$ from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 150 cm^2 at the inlet and 25 cm^2 at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. The mass of the elbow and the water in it is 50 kg. Determine the anchoring force needed to hold the elbow in place.



- **Assumptions** 1 The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). 2 The weight of the elbow and the water in it is considered. 3 The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. The momentum-flux correction factor for each inlet and outlet is given to be β = 1.03.



Analysis The weight of the elbow and the water in it is

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30\text{ kg/s}$. Noting that $\dot{m} = \rho A V$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

- Taking the center of the inlet cross section as the reference level ($z_1 = 0$) *and noting that* $P_2 = P_{atm}$, *the Bernoulli equation* for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \;\rightarrow\; P_1-P_2=\rho g\left(\frac{V_2^2-V_1^2}{2g}+z_2-z_1\right)$$

$$\rightarrow\; P_{1,\text{ gage}}=\rho g\left(\frac{V_2^2-V_1^2}{2g}+z_2\right)$$

Substituting,

$$P_{1,\text{ gage}}=(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2-(2\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)=73.9\text{ kN/m}^2=73.9\text{ kPa}$$

- The momentum equation for steady one-dimensional flow is

$$\sum\vec{F}=\sum_{\text{out}}\beta\dot{m}\vec{V}-\sum_{\text{in}}\beta\dot{m}\vec{V}$$

- We let the x- and z- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and z axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1 \quad \text{and} \quad F_{Rz} - W = \beta \dot{m} V_2 \sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot{m}(V_2\cos\theta - V)_1 - P_{1,\text{gage}}A_1 = 1.03(30\text{ kg/s})[(12\cos 45°\text{-}2)\text{ m/s}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) - (73.9\text{ kN/m}^2)(0.0150\text{ m}^2)$$

$$= -0.908\text{ kN}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta + W = 1.03(30\text{ kg/s})(12\sin 45°\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + 0.4905\text{ kN} = 0.753\text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18\text{ kN}}, \quad \theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.753}{-0.908} = \mathbf{-39.7°}$$

- **Discussion .** Note that the magnitude of the anchoring force is 1.18 kN, and its line of action makes –39.7◦ from +x direction. Negative value for F_{Rx} indicates the assumed direction is wrong.

The Angular Momentum Equation and its Application

- Many engineering problems involve the moment of the linear momentum of flow streams, and the rotational effects caused by them.
- Such problems are best analyzed by the angular momentum equation, also called the moment of momentum equation.
- An important class of fluid devices, called **turbomachines**, which include centrifugal pumps, turbines, and fans, is analyzed by the angular momentum equation.
- For steady two dimensional flow the angular momentum equation is given by

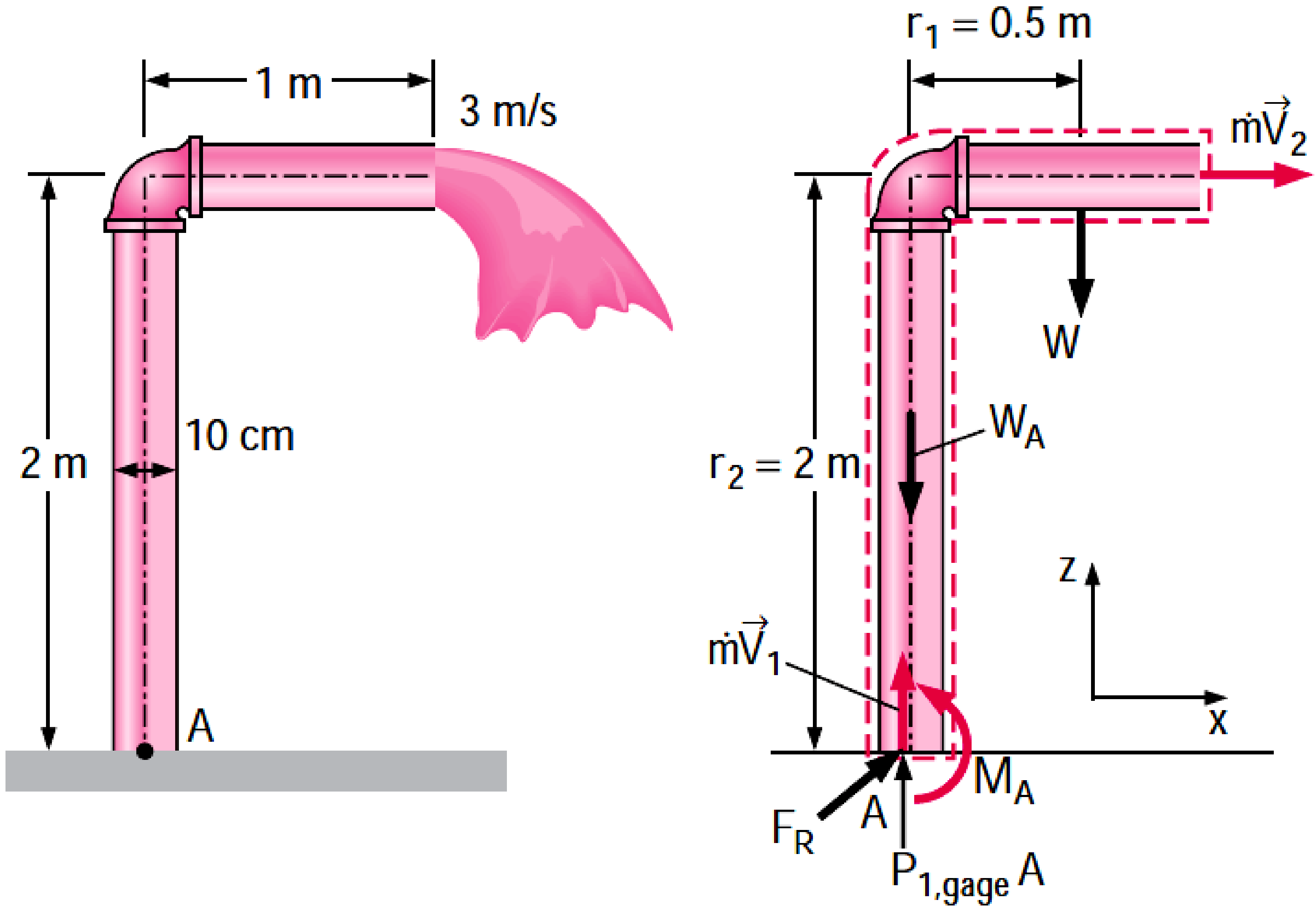
$$\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$$

The Angular Momentum Equation and its Application

- It states that the net torque acting on the control volume during steady flow is equal to the difference between the outgoing and incoming angular momentum flow rates.
- where r represents the average normal distance between the point about which moments are taken and the line of action of the force or velocity, provided that the sign convention for the moments is observed.
- That is, all moments in the counterclockwise direction are positive, and all moments in the clockwise direction are negative.

Example. Bending Moment Acting at the Base of a Water Pipe

- Underground water is pumped to a sufficient height through a 10-cmdiameter pipe that consists of a 2-m-long vertical and 1-m-long horizontal section, as shown in the Fig. below. Water discharges to atmospheric air at an average velocity of 3 m/s, and the mass of the horizontal pipe section when filled with water is 12 kg per meter length. The pipe is anchored on the ground by a concrete base. Determine the bending moment acting at the base of the pipe (point A) and the required length of the horizontal section that would make the moment at point A zero.


1 m
3 m/s
2 m
10 cm
A
$r_1 = 0.5$ m
$\dot{m}\vec{V}_2$
W
W_A
$r_2 = 2$ m
z
x
$\dot{m}\vec{V}_1$
M_A
F_R
A
$P_{1,gage}$ A

Analysis We take the entire L-shaped pipe as the control volume, and designate the inlet by 1 and the outlet by 2. We also take the *x*- and *z*-coordinates as shown. The control volume and the reference frame are fixed.

The conservation of mass equation for this one-inlet, one-outlet, steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and $V_1 = V_2 = V$ since A_c = constant. The mass flow rate and the weight of the horizontal section of the pipe are

$$\dot{m} = \rho A_c V = (1000\ \text{kg/m}^3)[\pi(0.10\ \text{m})^2/4](3\ \text{m/s}) = 23.56\ \text{kg/s}$$

$$W = mg = (12\ \text{kg/m})(1\ \text{m})(9.81\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 118\ \text{N}$$

To determine the moment acting on the pipe at point *A*, we need to take the moment of all forces and momentum flows about that point. This is a steady-flow problem, and all forces and momentum flows are in the same plane. Therefore, the angular momentum equation in this case can be expressed as

$$\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$$

where *r* is the average moment arm, V is the average speed, all moments in the counterclockwise direction are positive, and all moments in the clockwise direction are negative.

- The free-body diagram of the L-shaped pipe is given in the Fig. Noting that the moments of all forces and momentum flows passing through point *A* are zero, the only force that yields a moment about point *A is the weight W* of the horizontal pipe section, and the only momentum flow that yields a moment is the outlet stream (both are negative since both moments are in the clockwise direction).
- Then the angular momentum equation about point *A becomes*

$$M_A - r_1 W = -r_2 \dot{m} V_2$$

Solving for M_A and substituting give

$$\begin{aligned} M_A &= r_1 W - r_2 \dot{m} V_2 \\ &= (0.5\ \text{m})(118\ \text{N}) - (2\ \text{m})(23.56\ \text{kg/s})(3\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) \\ &= -82.5\ \text{N}\cdot\text{m} \end{aligned}$$

The negative sign indicates that the assumed direction for M_A is wrong and should be reversed. Therefore, a moment of 82.5 N · m acts at the stem of the pipe in the clockwise direction. That is, the concrete base must apply a 82.5 N · m moment on the pipe stem in the clockwise direction to counteract the excess moment caused by the exit stream.

The weight of the horizontal pipe is $w = W/L = 118$ N per m length. Therefore, the weight for a length of L m is Lw with a moment arm of $r_1 = L/2$. Setting $M_A = 0$ and substituting, the length L of the horizontal pipe that will cause the moment at the pipe stem to vanish is determined to be

$$0 = r_1 W - r_2 \dot{m} V_2 \quad \rightarrow \quad 0 = (L/2)Lw - r_2 \dot{m} V_2$$

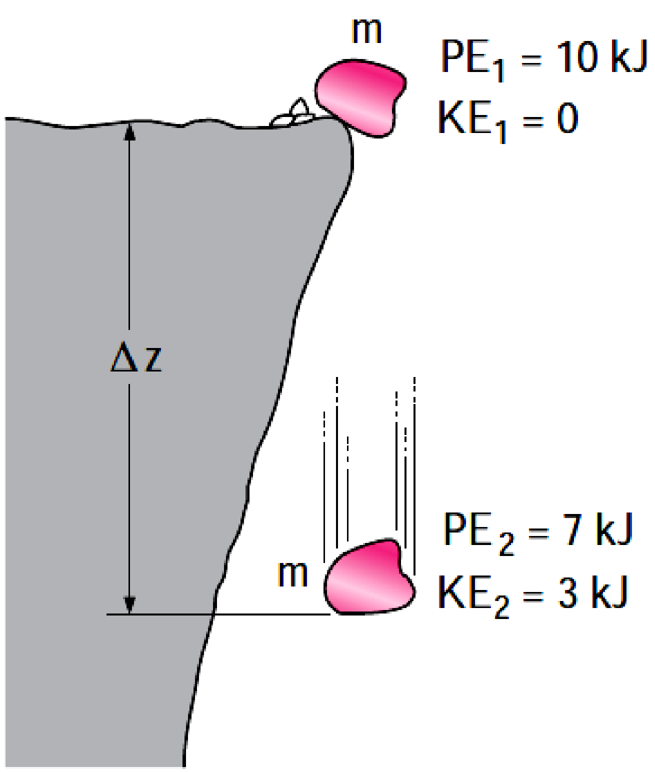
or

$$L = \sqrt{\frac{2 r_2 \dot{m} V_2}{w}} = \sqrt{\frac{2 \times 141.4 \text{ N} \cdot \text{m}}{118 \text{ N/m}}} = \mathbf{2.40\ m}$$

Discussion Note that the pipe weight and the momentum of the exit stream cause opposing moments at point A. This example shows the importance of accounting for the moments of momentums of flow streams when performing a dynamic analysis and evaluating the stresses in pipe materials at critical cross sections.

THE ENERGY EQUATION

- **The first law of thermodynamics,** also known as the **conservation of energy principle,** states that *energy can be neither created nor destroyed during a process; it can only change forms.*



Energy cannot be created or destroyed during a process; it can only change forms.

The Energy Equation

- The change in the energy content of a system is equal to the difference between the energy input and the energy output, and the conservation of energy principle for any system can be expressed simply as

$$E_{in} - E_{out} = \Delta E$$

- The energy content of a fixed quantity of mass (a closed system) can be changed by two mechanisms: *heat transfer Q* and *work transfer W*. Then the conservation of energy for a fixed quantity of mass can be expressed in rate form as

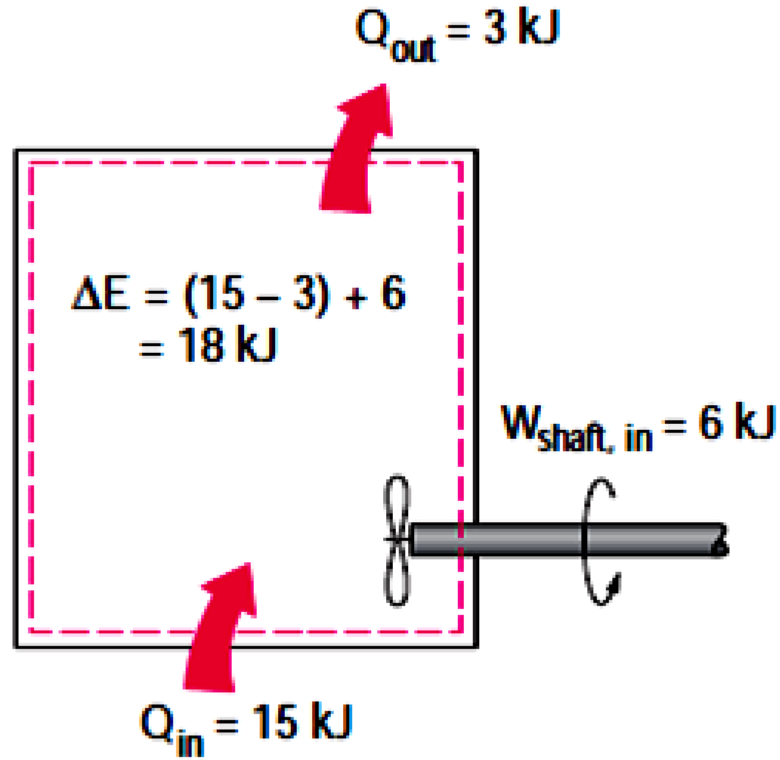
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{dE_{sys}}{dt} \quad \text{or} \quad \dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{sys} \rho e \, dV$$

- Where $\dot{Q}_{\text{net in}} = \dot{Q}_{in} - \dot{Q}_{out}$ is the net rate of heat transfer to the system (negative, if from the system)

The Energy Equation

- $\dot{W}_{net\ in} = \dot{W}_{in} - \dot{W}_{out}$ is the net power input to the system in all forms (negative, if power output)
- $d\dot{E}_{sys}/dt$ is the rate of change of the total energy content of the system.

 (N.B: The overdot stands for time rate)



For simple compressible systems, total energy consists of internal, kinetic, and potential energies, and it is expressed on a unit-mass basis as

$$e = u + ke + pe = u + \frac{V^2}{2} + gz$$

Energy Transfer by Heat, *Q*

- The transfer of thermal energy from one system to another as a result of a temperature difference is called **heat transfer.**
- A process during which there is no heat transfer is called an **adiabatic process.**
- There are two ways a process can be adiabatic: Either the system is well insulated so that only a negligible amount of heat can pass through the system boundary, or both the system and the surroundings are at the same temperature and therefore there is no driving force (temperature difference) for heat transfer.
- An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work transfer.

Energy Transfer by Work, *W*

- An energy interaction is **work** if it is associated with a force acting through a distance.
- A rising piston, a rotating shaft, and an electric wire crossing the system boundary are all associated with work interactions.
- The time rate of doing work is called **power** and is denoted by $\dot{W}$.
- Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, fans, and mixers consume work.
- Work-consuming devices transfer energy to the fluid, and thus increase the energy of the fluid. A fan in a room, for example, mobilizes the air and increases its kinetic energy.

Energy Transfer by Work, *W*

- A system may involve numerous forms of work, and the total work can be expressed as

$$W_{total} = W_{shaft} + W_{pressure} + W_{viscous} + W_{other}$$

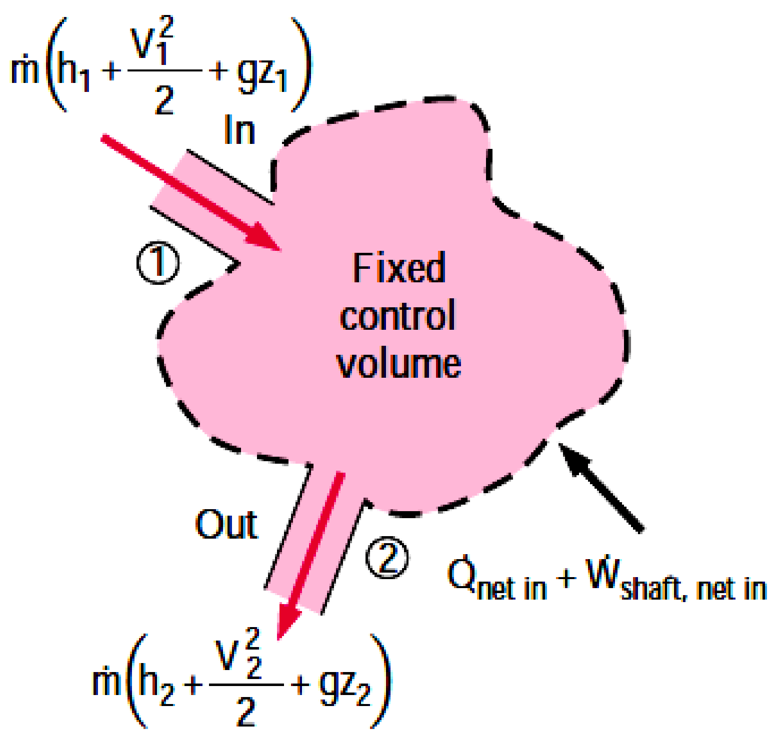
- W_{shaft} is the work transmitted by a rotating shaft
- $W_{pressure}$ is the work done by the pressure forces on the control surface,
- $W_{viscous}$ is the work done by the normal and shear components of viscous forces on the control surface,
- W_{other} is the work done by other forces such as electric, magnetic, and surface tension, which are insignificant for simple compressible systems
- $W_{viscous}$ is usually very small relative to other terms. So it is not considered in control volume analysis.

Energy Analysis of Steady Flows

- For steady flows, the energy equation is given by

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

- It states that the net rate of energy transfer to a control volume by heat and work transfers during steady flow is equal to the difference between the rates of outgoing and incoming energy flows with mass.



Energy Analysis of Steady Flows

- Many practical problems involve just one inlet and one outlet. The mass flow rate for such **single-stream devices** remains constant, and the energy equation reduces to

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

- where subscripts 1 and 2 stand for inlet and outlet, respectively.
- on a unit-mass basis

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

- Using the definition of enthalpy $h = u + P/\rho$ and rearranging, the steady-flow energy equation can also be expressed as

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$

Energy Analysis of Steady Flows

- where u is the internal energy, P/ρ is the flow energy, $V^2/2$ is the kinetic energy, and gz is the potential energy of the fluid, all per unit mass. These relations are valid for both compressible and incompressible flows

$$\underbrace{w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1}_{\text{Mechanical Energy Input}} = \underbrace{\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2}_{\text{Mechanical Energy Output}} + (u_2 - u_1 - q_{\text{net in}})$$

- If the flow is ideal with no irreversibilities such as friction, the total mechanical energy must be conserved. Thus u_2 - u_1- $q_{\text{net in}}$ must be equal to zero.

Ideal flow (no mechanical energy loss): $q_{\text{net in}} = u_2 - u_1$

Energy Analysis of Steady Flows

- u_2 - u_1- $q_{net\ in}$ represents the mechanical energy loss

Mechanical energy loss: $$e_{mech,\ loss} = u_2 - u_1 - q_{net\ in}$$

- For single-phase fluids (a gas or a liquid), we have

$$u_2 - u_1 = c_v(T_2 - T_1)$$

where c_v is the constant-volume specific heat.

- The steady-flow energy equation on a unit-mass basis can be written conveniently as a **mechanical energy balance as**

$$e_{mech,\ in} = e_{mech,\ out} + e_{mech,\ loss}$$

$$w_{shaft,\ net\ in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{mech,\ loss}$$

Energy Analysis of Steady Flows

- Noting that

$$w_{\text{shaft, net in}} = w_{\text{shaft, in}} - w_{\text{shaft, out}} = w_{\text{pump}} - w_{\text{turbine}}$$

- the mechanical energy balance can be written more explicitly as

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

- where w_{pump} is the mechanical work input (due to the presence of a pump, fan, compressor, etc.) and $w_{turbine}$ is the mechanical work output.
- Multiplying the above energy equation by the mass flow rate $\dot{m}$ *gives*

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

Energy Analysis of Steady Flows

Where

- $\dot{W}_{pump}$ is the shaft power input through the pump's shaft,
- $\dot{W}_{turbine}$ turbine is the shaft power output through the turbine's shaft, and
- $\dot{E}_{mech,\ loss}$, loss is the *total* mechanical power loss, which consists of pump and turbine losses as well as the frictional losses in the piping network.

$$\dot{E}_{mech,\ loss} = \dot{E}_{mech\ loss,\ pump} + \dot{E}_{mech\ loss,\ turbine} + \dot{E}_{mech\ loss,\ piping}$$

- The energy equation can be expressed in its most common form in terms of heads as

$$\boxed{\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump,\ u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine,\ e} + h_L}$$

Energy Analysis of Steady Flows

- Where

$$h_{\text{pump, u}} = \frac{w_{\text{pump, u}}}{g} = \frac{\dot{W}_{\text{pump, u}}}{\dot{m}g} = \frac{\eta_{\text{pump}}\dot{W}_{\text{pump}}}{\dot{m}g}$$

is the useful head delivered to the fluid by the pump

$$h_{\text{turbine, e}} = \frac{w_{\text{turbine, e}}}{g} = \frac{\dot{W}_{\text{turbine, e}}}{\dot{m}g} = \frac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$$

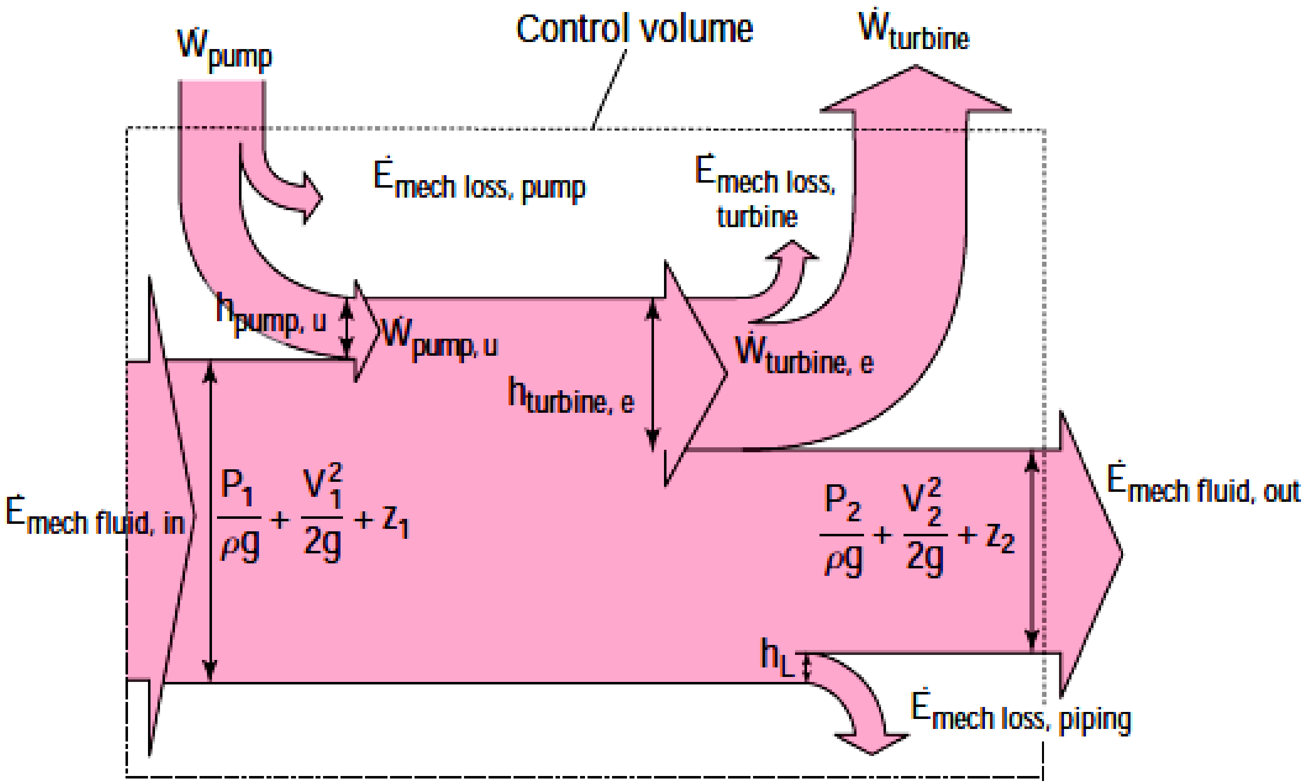
is the extracted head removed from the fluid by the turbine.

$$h_L = \frac{\dot{e}_{\text{mech loss, piping}}}{g} = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$$

is the irreversible head loss between 1 and 2 due to all components of the piping system other than the pump or turbine.

Energy Analysis of Steady Flows

- Note that the head loss h_L represents the frictional losses associated with fluid flow in piping, and it does not include the losses that occur within the pump or turbine due to the inefficiencies of these devices—these losses are taken into account by η_{pump} and $\eta_{turbine}$



Energy Analysis of Steady Flows

- The pump head is zero if the piping system does not involve a pump, a fan, or a compressor, and the turbine head is zero if the system does not involve a turbine.
- Also, the head loss h_L can sometimes be ignored when the frictional losses in the piping system are negligibly small compared to the other terms

Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

- When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus $h_L = e_{\text{mech loss, piping}}/g \cong 0$, and $h_{\text{pump, u}} = h_{\text{turbine, e}} = 0$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \qquad \text{or} \qquad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

- which is the **Bernoulli equation**

Energy Analysis of Steady Flows

Kinetic Energy Correction Factor, α

- the kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components
- This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation by $\alpha V_{avg}^2/2$, where α is the **kinetic energy correction factor.**
- The correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.
- The kinetic energy correction factors are often ignored (i.e., a is set equal to 1) in an elementary analysis since (1) most flows encountered in practice are turbulent, for which the correction factor is near unity, and

Energy Analysis of Steady Flows

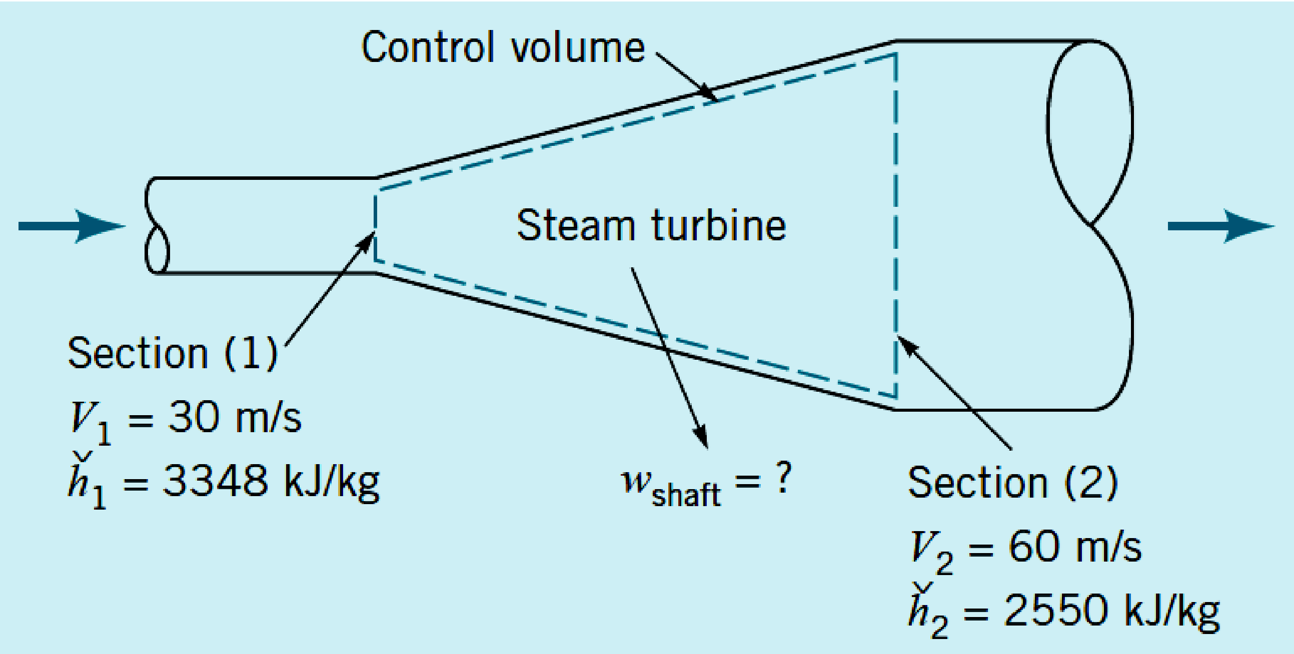
- (2) the kinetic energy terms are often small relative to the other terms in the energy equation, and multiplying them by a factor less than 2.0 does not make much difference.
- When the kinetic energy correction factors are included, the energy equations for *steady incompressible flow* become

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

Example 1

- A steam turbine generator unit used to produce electricity. Assume the steam enters a turbine with a velocity of 30 m/s and enthalpy, h_1, of 3348 kJ/kg .The steam leaves the turbine as a mixture of vapor and liquid having a velocity of 60 m/s and an enthalpy of 2550 kJ/kg. The flow through the turbine is adiabatic, and changes in elevation are negligible. Determine the work output involved per unit mass of steam through-flow.



Solution

0 (elevation change is negligible)

0 (adiabatic flow)

$$\dot{m}\left[\breve{h}_2 - \breve{h}_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} \quad \textbf{(1)}$$

The work output per unit mass of steam through-flow, $w_{\text{shaft net in}}$, can be obtained by dividing Eq. 1 by the mass flow rate, $\dot{m}$, to obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{\dot{m}} = \breve{h}_2 - \breve{h}_1 + \frac{V_2^2 - V_1^2}{2} \quad \textbf{(2)}$$

Since $w_{\text{shaft net out}} = -w_{\text{shaft net in}}$, we obtain

$$w_{\substack{\text{shaft}\\ \text{net out}}} = \breve{h}_1 - \breve{h}_2 + \frac{V_1^2 - V_2^2}{2}$$

Solution

$$w_{\substack{\text{shaft}\\ \text{net out}}} = 3348\ \text{kJ/kg} - 2550\ \text{kJ/kg} + \frac{[(30\ \text{m/s})^2 - (60\ \text{m/s})^2][1\ \text{J/(N·m)}]}{2[1\ (\text{kg·m})/(\text{N·s}^2)](1000\ \text{J/kJ})}$$

Thus,

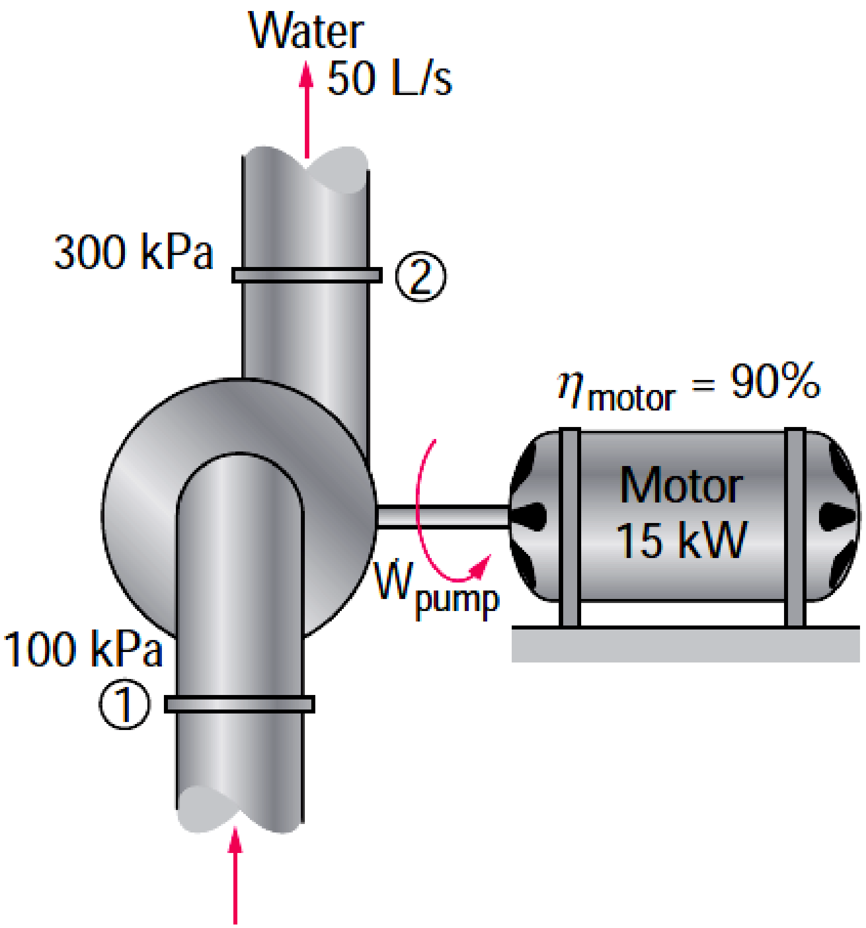
$$w_{\substack{\text{shaft}\\ \text{net out}}} = 3348\ \text{kJ/kg} - 2550\ \text{kJ/kg} - 1.35\ \text{kJ/kg}$$

$$= 797\ \text{kJ/kg} \qquad \textbf{(Ans)}$$

Example 2. Pumping Power and Frictional Heating in a Pump

- The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent. The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine
  - *a)* *the mechanical efficiency* of the pump and
  - b) *the temperature rise of water as it flows through* the pump due to the mechanical inefficiency.

Example 2. Pumping Power and Frictional Heating in a Pump



- Schematic for Example 2

Example 2. Pumping Power and Frictional Heating in a Pump

Assumptions **1** The flow is steady and incompressible. **2** The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. **3** The elevation difference between the inlet and outlet of the pump is negligible, $z_1 \cong z_2$. **4** The inlet and outlet diameters are the same and thus the inlet and outlet velocities and kinetic energy correction factors are equal, $V_1 = V_2$ and $\alpha_1 = \alpha_2$.

Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 and its specific heat to be 4.18 kJ/kg · °C.

Analysis (*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(50 \text{ L/s}) = 50 \text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}} \dot{W}_{\text{electric}} = (0.90)(15 \text{ kW}) = 13.5 \text{ kW}$$

Example 2. Pumping Power and Frictional Heating in a Pump

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50 \text{ kg/s})\left(\frac{(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3}\right)\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) = 10 \text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10 \text{ kW}}{13.5 \text{ kW}} = 0.741 \quad \text{or} \quad 74.1\%$$

Example 2. Pumping Power and Frictional Heating in a Pump

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta\dot{E}_{\text{mech, fluid}} = 13.5 - 10 = 3.5\ \text{kW}$$

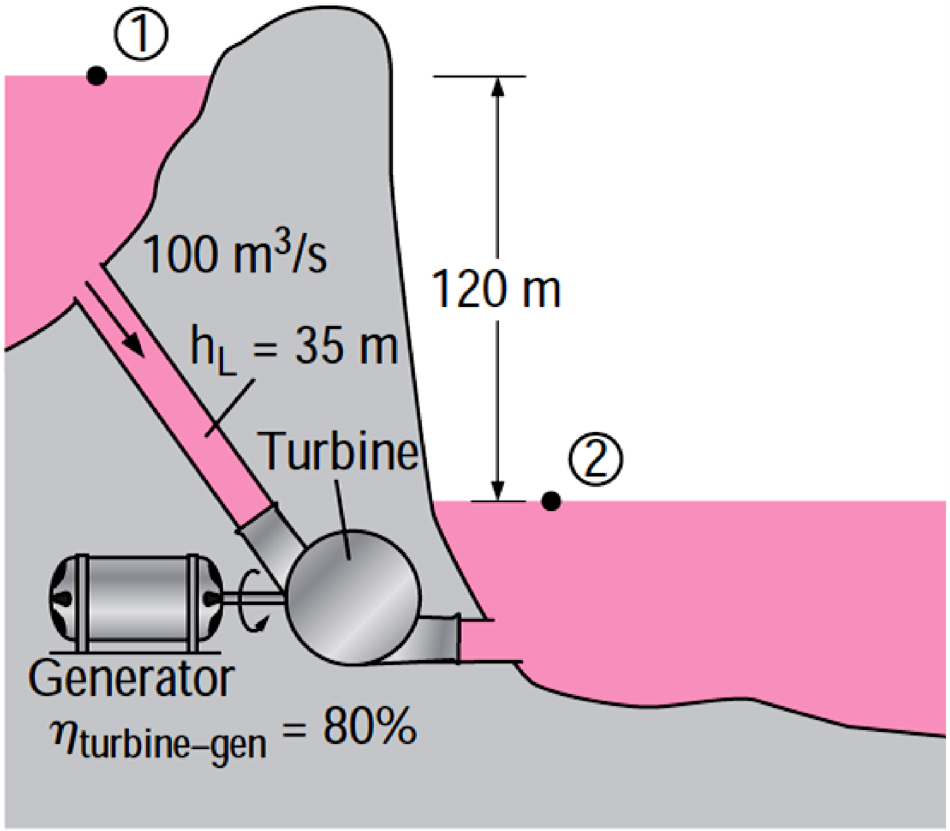
The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance, $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$. Solving for ΔT,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5\ \text{kW}}{(50\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C})} = 0.017^\circ\text{C}$$

Therefore, the water will experience a temperature rise of 0.017°C due to mechanical inefficiency, which is very small, as it flows through the pump.

Example 3. Hydroelectric Power Generation from a Dam

- In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output.



Example 3. Hydroelectric Power Generation from a Dam

SOLUTION The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(100 \text{ m}^3/\text{s}) = 10^5 \text{ kg/s}$$

We take point 2 as the reference level, and thus $z_2 = 0$. Also, both points 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the flow velocities are negligible at both points ($V_1 = V_2 = 0$). Then the energy equation for steady, incompressible flow reduces to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + h_{pump, u}^{\nearrow 0} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2^{\nearrow 0} + h_{turbine, e} + h_L \rightarrow$$

$$h_{turbine, e} = z_1 - h_L$$

Example 3. Hydroelectric Power Generation from a Dam

Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{\text{turbine, e}} = z_1 - h_L = 120 - 35 = 85 \text{ m}$$

$$\dot{W}_{\text{turbine, e}} = \dot{m}gh_{\text{turbine, e}} = (10^5 \text{ kg/s})(9.81 \text{ m/s}^2)(85 \text{ m})\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 83{,}400 \text{ kW}$$

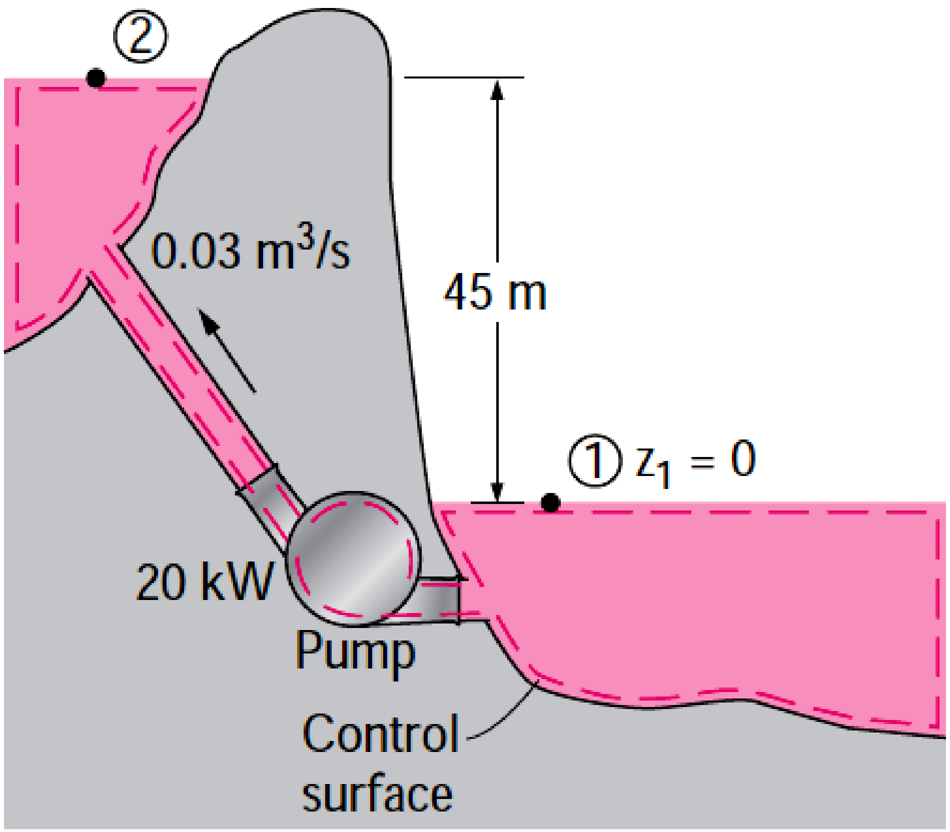
Therefore, a perfect turbine–generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine–gen}}\dot{W}_{\text{turbine, e}} = (0.80)(83.4 \text{ MW}) = \mathbf{66.7 \text{ MW}}$$

Discussion Note that the power generation would increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine–generator unit.

Example 4. Head and Power Loss During Water Pumping

- Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water. The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m^3/s, determine the irreversible head loss of the system and the lost mechanical power during this process.



Properties We take the density of water to be 1000 kg/m^3.

Analysis The mass flow rate of water through the system is

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.03\ \text{m}^3/\text{s}) = 30\ \text{kg/s}$$

We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($V_1 = V_2 = 0$). Then the energy equation for steady incompressible flow for a control volume between 1 and 2 reduces to

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \frac{V_1^2}{2}\xrightarrow{}0 + gz_1\xrightarrow{}0\right) + \dot{W}_{pump}$$

$$= \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{V_2^2}{2}\xrightarrow{}0 + gz_2\right) + \dot{W}_{turbine}\xrightarrow{}0 + \dot{E}_{mech,\ loss}$$

$$\dot{W}_{pump} = \dot{m}gz_2 + \dot{E}_{mech,\ loss} \quad \rightarrow \quad \dot{E}_{mech,\ loss} = \dot{W}_{pump} - \dot{m}gz_2$$

Substituting, the lost mechanical power and head loss are determined to be

$$\dot{E}_{mech,\ loss} = 20\ \text{kW} - (30\ \text{kg/s})(9.81\ \text{m/s}^2)(45\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)$$

$$= \mathbf{6.76\ kW}$$

Example 4. Solution…..

Noting that the entire mechanical losses are due to frictional losses in piping and thus $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech, loss, piping}}$, the irreversible head loss is determined to be

$$h_L = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g} = \frac{6.76 \text{ kW}}{(30 \text{ kg/s})(9.81 \text{ m/s}^2)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)\left(\frac{1000 \text{ N}\cdot\text{m/s}}{1 \text{ kW}}\right) = \mathbf{23.0 \text{ m}}$$

Discussion The 6.76 kW of power is used to overcome the friction in the piping system. Note that the pump could raise the water an additional 23 m if there were no irreversible head losses in the system. In this ideal case, the pump would function as a turbine when the water is allowed to flow from the upper reservoir to the lower reservoir and extract 20 kW of power from the water.

End of Chapter 3

Next Lecture

Chapter 4: Differential Relations For A Fluid Flow