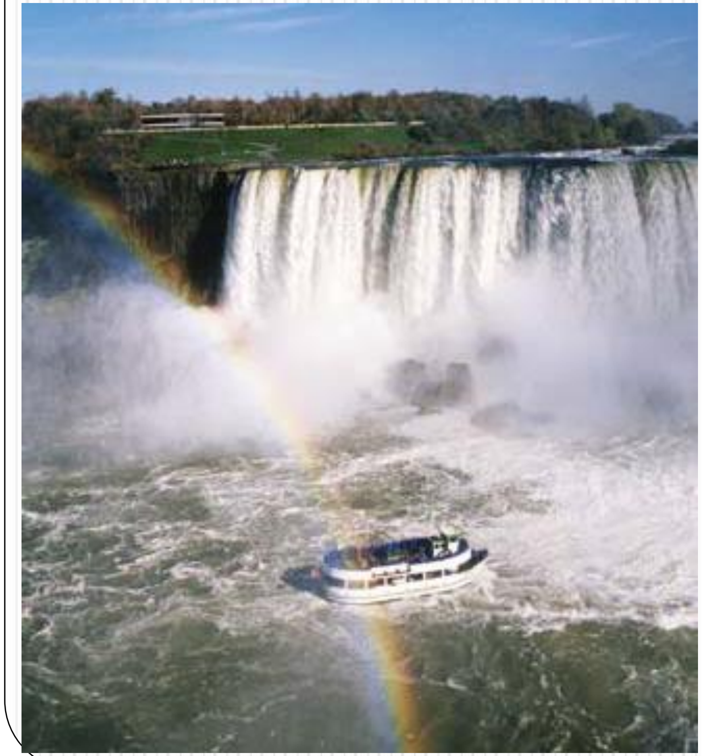
DMiT

School of Mechanical and Industrial Engineering

Fluid Mechanics

(MEng 2113)



Chapter 1

Introduction to Fluid Mechanics

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Definition

- **Mechanics** is the oldest physical science that deals with both stationary and moving bodies under the influence of forces.
- The branch of mechanics that deals with bodies at rest is called **statics**, while the branch that deals with bodies in motion is called **dynamics**.
- The subcategory **fluid mechanics** is defined as the science that deals with the behavior of fluids at rest (*fluid statics)* or in motion (*fluid dynamics), and the interaction of fluids with solids* or other fluids at the boundaries.
- The study of fluids at rest is called **fluid statics**.
- The study of fluids in motion, where pressure forces are not considered, is called **fluid kinematics** and if the pressure forces are also considered for the fluids in motion. that branch of science is called **fluid dynamics**.

Definition

- Fluid mechanics itself is also divided into several categories.
- The study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **hydrodynamics.**
- A subcategory of hydrodynamics is **hydraulics,** which deals with liquid flows in pipes and open channels.
- **Gas dynamics** deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds.
- The category **aerodynamics** deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds.
- Some other specialized categories such as **meteorology, oceanography, and hydrology** deal with naturally occurring flows.

What is a Fluid?

- A substance exists in three primary phases: solid, liquid, and gas. A substance in the liquid or gas phase is referred to as a **fluid.**
- Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.
- In solids stress is proportional to *strain*, *but in fluids stress is proportional* to *strain rate*.
- *When a constant shear force is applied*, *a solid eventually* stops deforming, at some fixed strain angle, whereas a fluid never stops deforming and approaches a certain rate of strain.

What is a Fluid?

- Consider a rectangular rubber block tightly placed between two plates. As the upper plate is pulled with a force *F* while the lower plate is held fixed, the rubber block deforms, as shown in the Fig. below. The angle of deformation a (called the *shear strain* or *angular displacement*) increases in proportion to the applied force *F*.

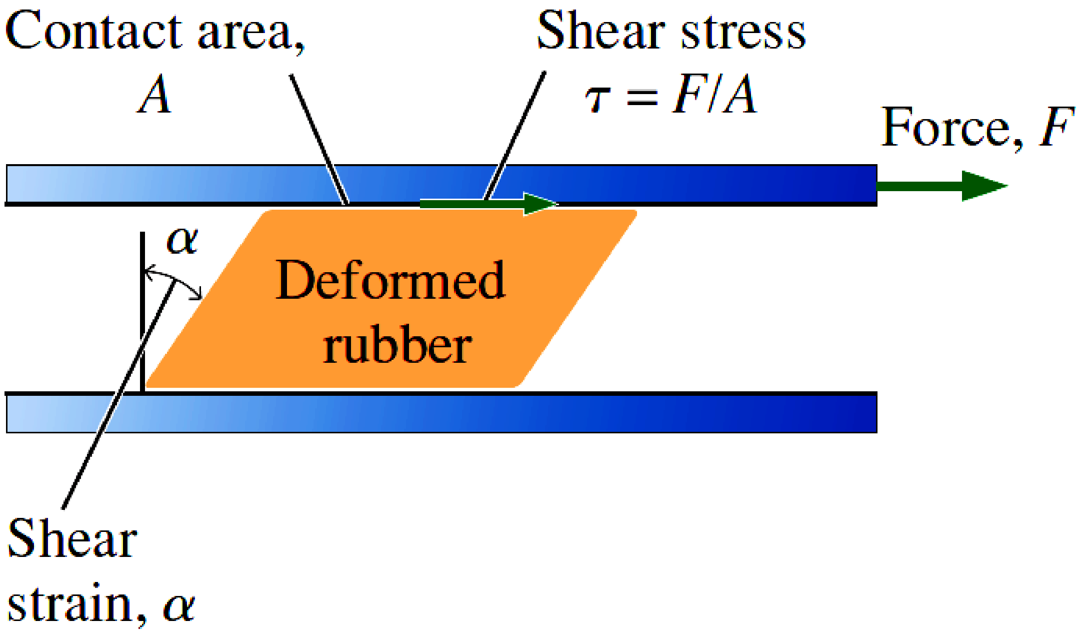
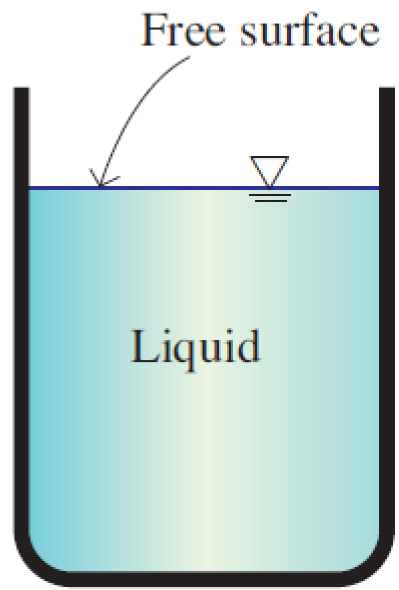
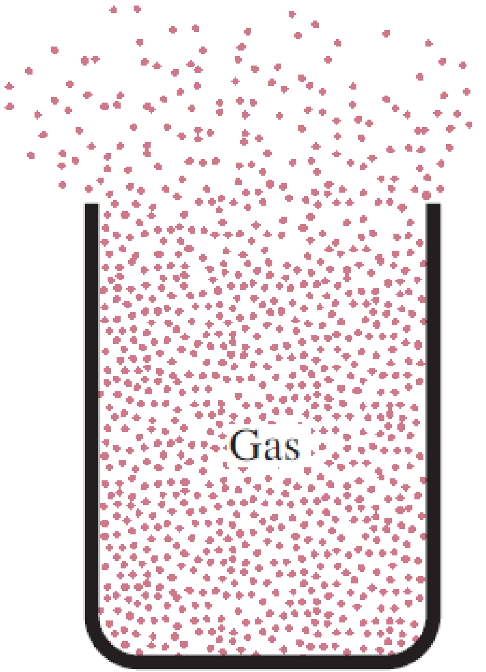


Figure. 1.1 Deformation of a rubber eraser placed between two parallel plates under the influence of a shear force.

What is a Fluid?

- In a liquid, molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules.
- As a result, a liquid takes the shape of the container it is in, and it forms a free surface in a larger container in a gravitational field.
- A gas, on the other hand, expands until it encounters the walls of the container and fills the entire available space.
- This is because the gas molecules are widely spaced, and the cohesive forces between them are very small.
- Unlike liquids, gases cannot form a free surface





What is a Fluid?

- Differences between liquid and gases

Liquid	Gases
Difficult to compress and often regarded as incompressible	Easily to compress – changes of volume is large, cannot normally be neglected and are related to temperature
Occupies a fixed volume and will take the shape of the container	No fixed volume, it changes volume to expand to fill the containing vessels
A free surface is formed if the volume of container is greater than the liquid.	Completely fill the vessel so that no free surface is formed.

Application areas of Fluid Mechanics

- Mechanics of fluids is extremely important in many areas of engineering and science. Examples are:
- **Biomechanics**
  - Blood flow through arteries and veins
  - Airflow in the lungs
  - Flow of cerebral fluid
- **Households**
  - Piping systems for cold water, natural gas, and sewage
  - Piping and ducting network of heating and air-conditioning systems
  - refrigerator, vacuum cleaner, dish washer, washing machine, water meter, natural gas meter, air conditioner, radiator, etc.
- **Meteorology and Ocean Engineering**
  - Movements of air currents and water currents

Application areas of Fluid Mechanics

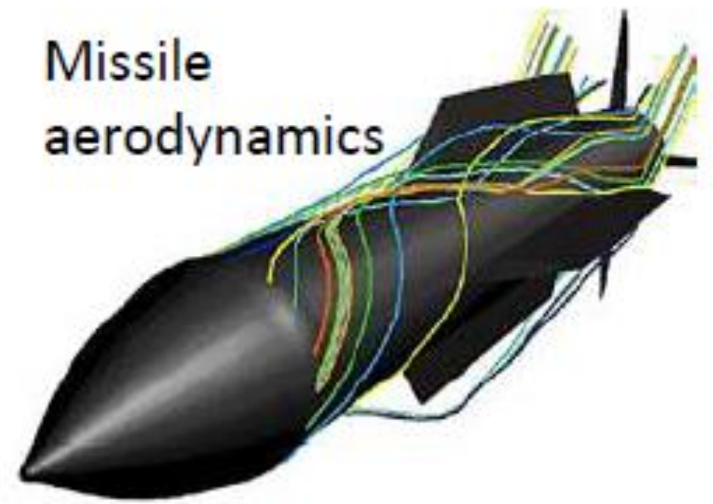
- **Mechanical Engineering**
  - Design of pumps, turbines, air-conditioning equipment, pollution-control equipment, etc.
  - Design and analysis of aircraft, boats, submarines, rockets, jet engines, wind turbines, biomedical devices, the cooling of electronic components, and the transportation of water, crude oil, and natural gas.
- **Civil Engineering**
  - Transport of river sediments
  - Pollution of air and water
  - Design of piping systems
  - Flood control systems
- **Chemical Engineering**
  - Design of chemical processing equipment

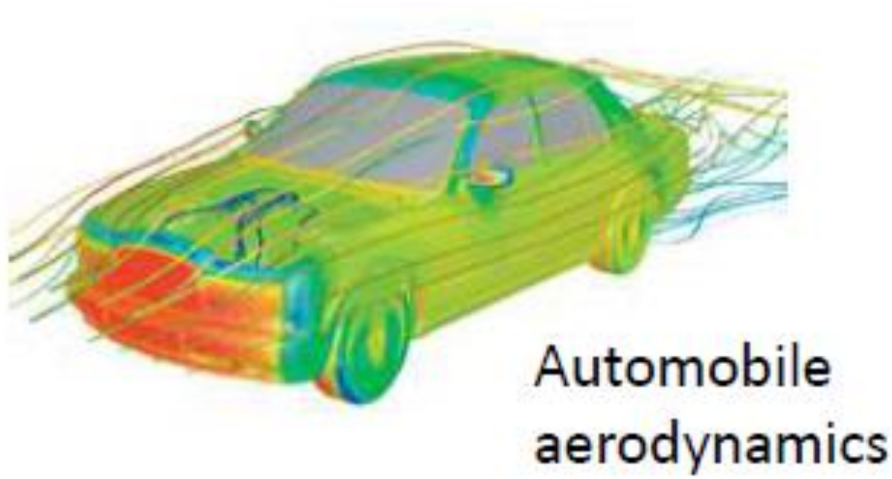
Application areas of Fluid Mechanics

- **Turbomachines:** pump, turbine, fan, blower, propeller, etc.
- **Military:** Missile, aircraft, ship, underwater vehicle, dispersion of chemical agents, etc.
- **Automobile:** IC engine, air conditioning, fuel flow, external aerodynamics, etc.
- **Medicine:** Heart assist device, artificial heart valve, Lab-on-a-Chip device, glucose monitor, controlled drug delivery, etc.
- **Electronics:** Convective cooling of generated heat.
- **Energy:** Combuster, burner, boiler, gas, hydro and wind turbine, etc.
- **Oil and Gas:** Pipeline, pump, valve, offshore rig, oil spill cleanup, etc.
- Almost everything in our world is either in contact with a fluid or is itself a fluid.

Application areas of Fluid Mechanics

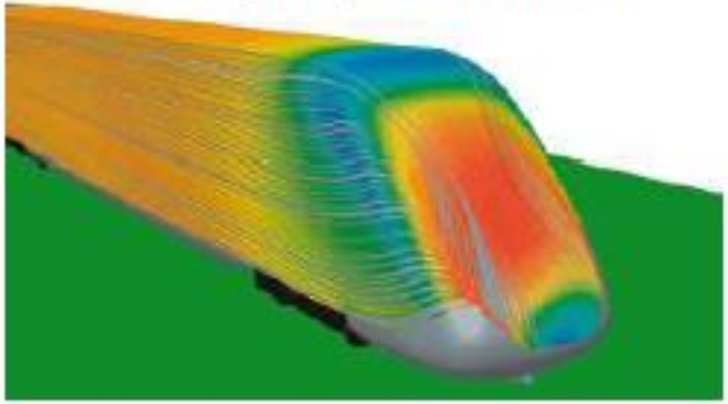
- The number of fluid engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few.
- When you think about it, almost everything on this planet either is a fluid or moves within or near a fluid.



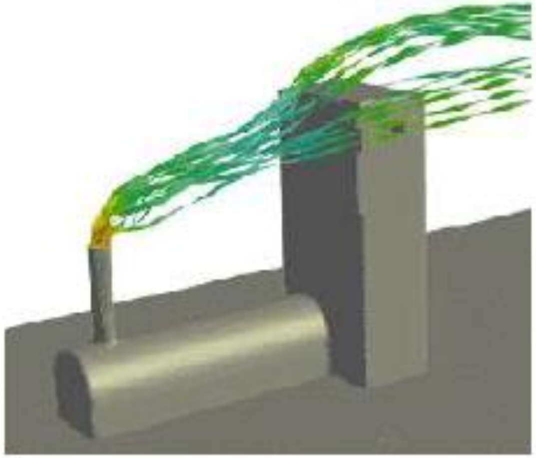


Application areas of Fluid Mechanics

High speed train

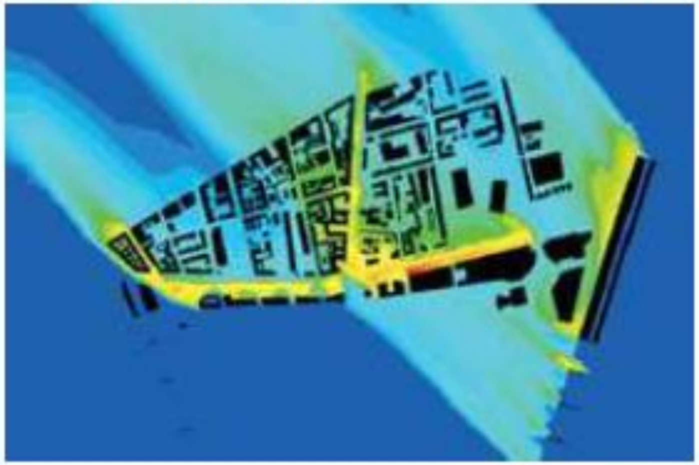


Wind turbines

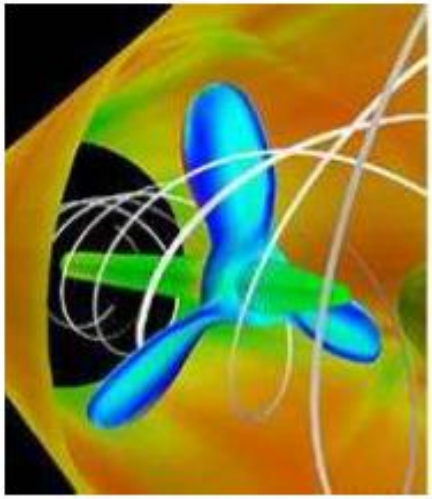


Smoke from a stack

Pollutant dispersion over a city



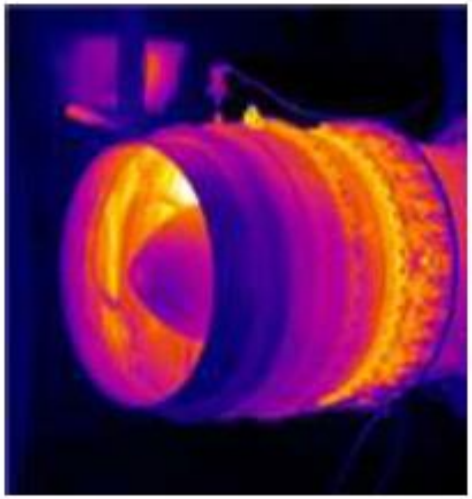
Application areas of Fluid Mechanics



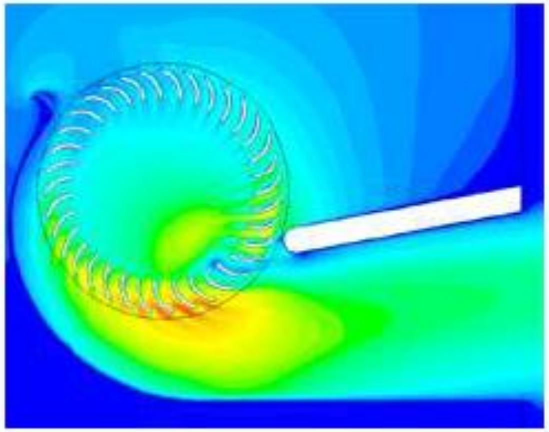
Propeller

Fan

Centrifugal Pump

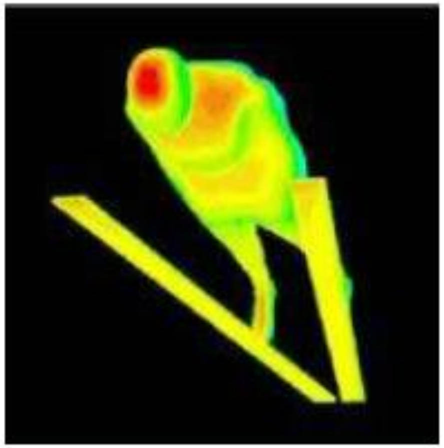


Jet Engine
Propulsion

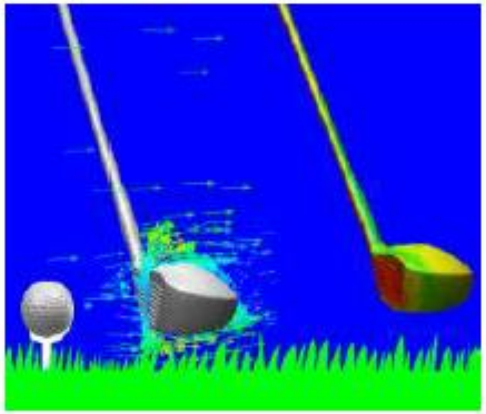


Crossflow fan

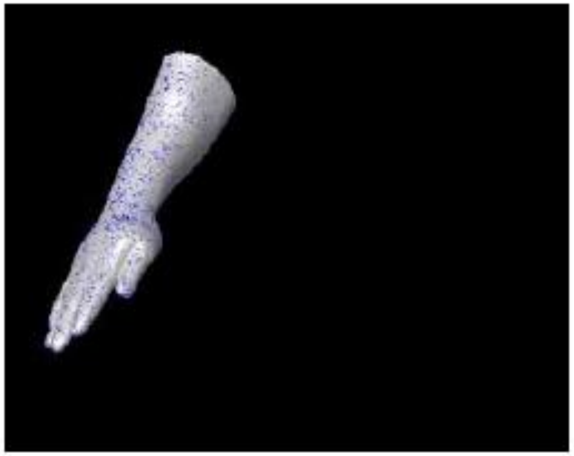
Application areas of Fluid Mechanics



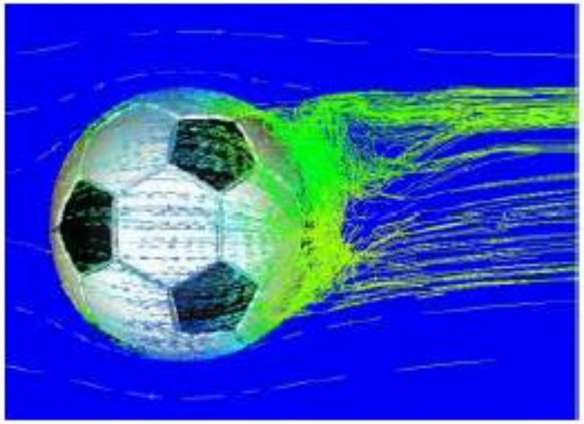
Ski Jumping



Golf

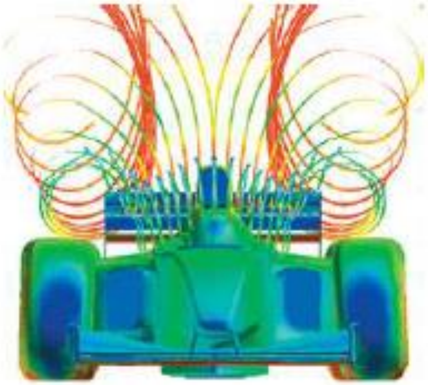


Swimming



Football

Cycling



Indy Car Racing

Classification of Fluid Flows

- There is a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups.

Viscous versus Inviscid Regions of Flow

- When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer.
- This internal resistance to flow is quantified by the fluid property *viscosity,* which is a measure of internal stickiness of the fluid.
- Viscosity is caused by cohesive forces between the molecules in liquids and by molecular collisions in gases.

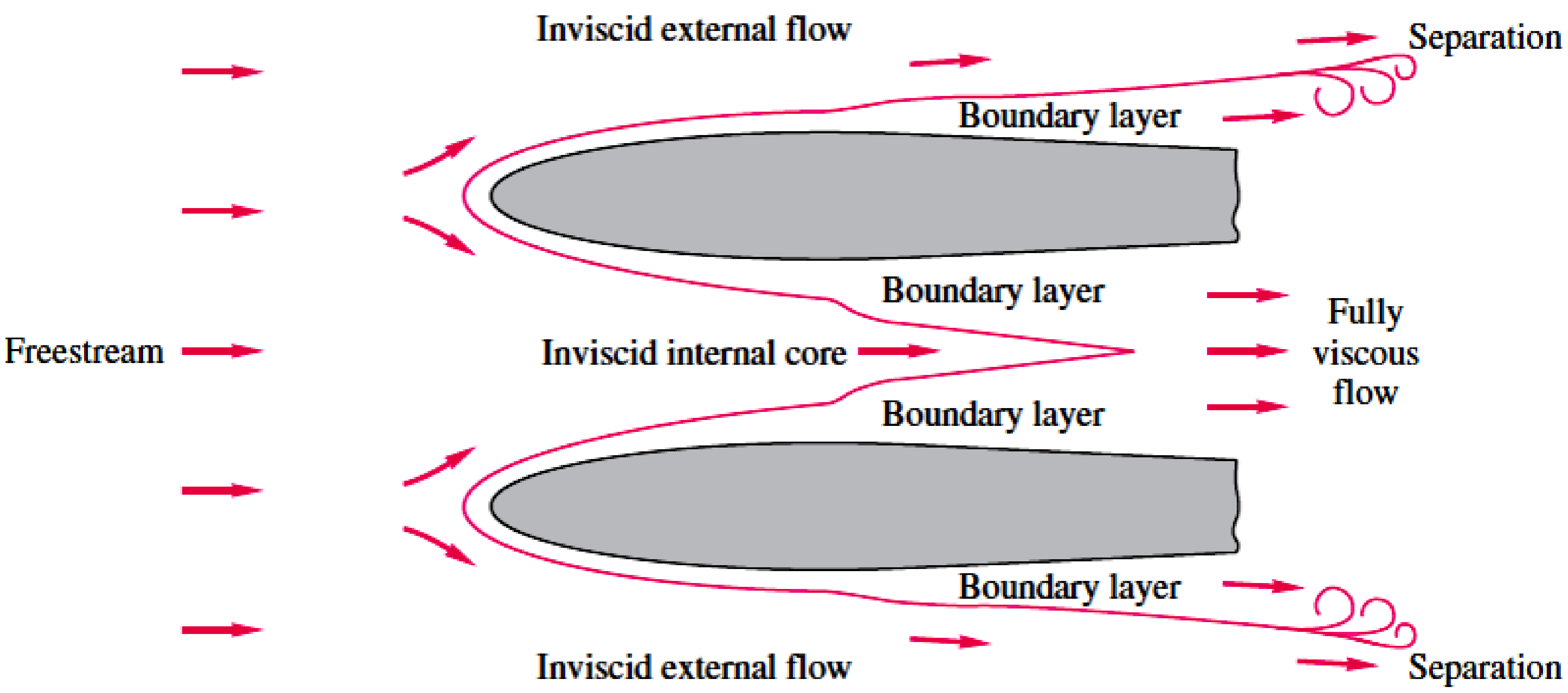
Classification of Fluid Flows

Viscous versus Inviscid Regions of Flow…

- There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree.
- Flows in which the frictional effects are significant are called **viscous flows.**
- However, in many flows of practical interest, there are *regions (typically regions not close to solid surfaces)* where viscous forces are negligibly small compared to inertial or pressure forces.
- Neglecting the viscous terms in such **inviscid** flow regions greatly simplifies the analysis without much loss in accuracy.

Classification of Fluid Flows

Viscous versus Inviscid Regions of Flow



Classification of Fluid Flows

Internal versus External Flow

- A fluid flow is classified as being internal or external, depending on whether the fluid is forced to flow in a confined channel or over a surface.
- The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow.**
- The flow in a pipe or duct is **internal flow** if the fluid is completely bounded by solid surfaces.
- Water flow in a pipe, for example, is internal flow, and airflow over a ball or over an exposed pipe during a windy day is external flow .

Classification of Fluid Flows

Compressible versus Incompressible Flow

- A flow is classified as being compressible or incompressible, depending on the level of variation of density during flow.
- Incompressibility is an approximation, and a flow is said to be **incompressible** if the density remains nearly constant throughout.
- Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow (or the fluid) is incompressible.
- The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually referred to as *incompressible substances*.

Classification of Fluid Flows

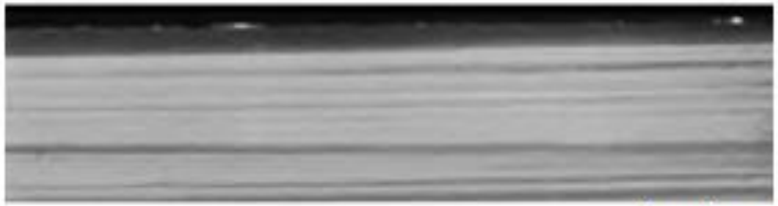
Compressible versus Incompressible Flow…

- A pressure of 210 atm, for example, causes the density of liquid water at 1 atm to change by just 1 percent.
- Gases, on the other hand, are highly compressible. A pressure change of just 0.01 atm, for example, causes a change of 1 percent in the density of atmospheric air.
- Gas flows can often be approximated as incompressible if the density changes are under about 5 percent.
- The compressibility effects of air can be neglected at speeds under about 100 m/s.

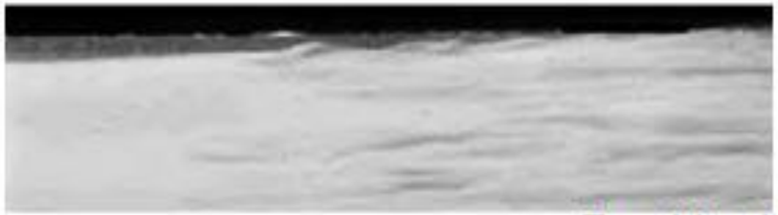
Classification of Fluid Flows

Laminar versus Turbulent Flow

- Some flows are smooth and orderly while others are rather chaotic.
- The highly ordered fluid motion characterized by smooth layers of fluid is called **laminar.**
- The flow of high-viscosity fluids such as oils at low velocities is typically laminar.
- The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called **turbulent .**



Laminar



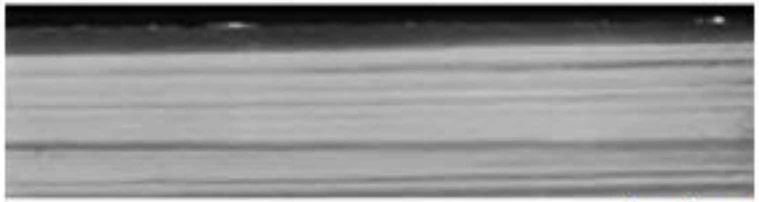
Transitional

Turbulent

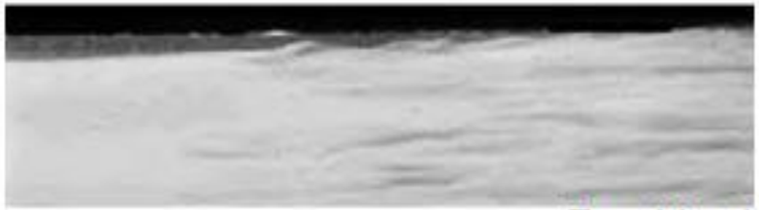
Classification of Fluid Flows

Laminar versus Turbulent Flow

- The flow of low-viscosity fluids such as air at high velocities is typically turbulent.
- A flow that alternates between being laminar and turbulent is called **transitional.**



Laminar



Transitional

Turbulent

Classification of Fluid Flows

Natural (or Unforced) versus Forced Flow

- A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated.
- In **forced flow,** a fluid is forced to flow over a surface or in a pipe by external means such as a **pump or a fan**.
- In **natural flows,** any fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid .
- In solar hot-water systems, for example, the thermosiphoning effect is commonly used to replace pumps by placing the water tank sufficiently above the solar collectors.

Classification of Fluid Flows

Steady versus Unsteady Flow

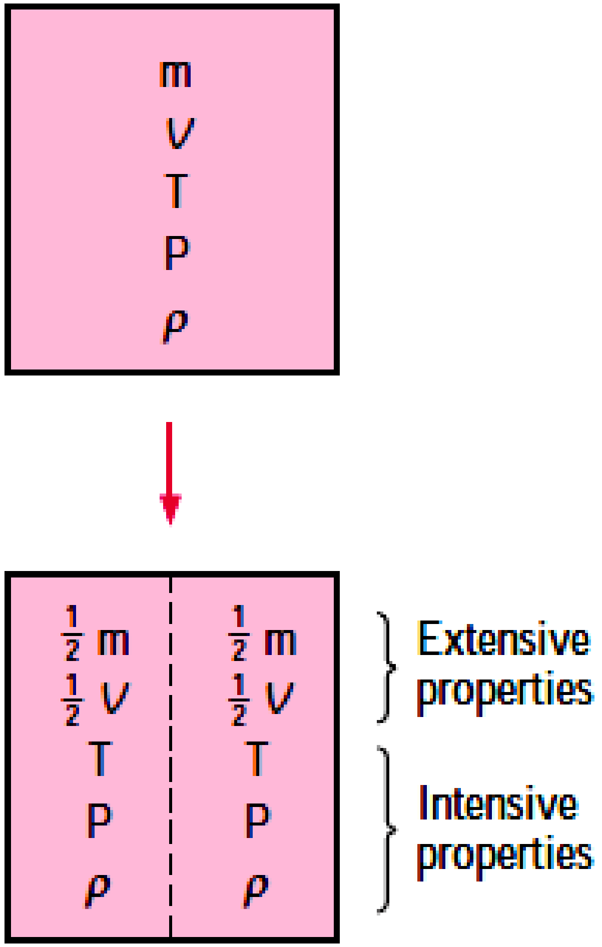
- The terms *steady and uniform are used frequently in engineering, and thus it* is important to have a clear understanding of their meanings.
- The term steady implies ***no change at a point with time*****.**
- The opposite of steady is **unsteady.**
- The term **uniform** implies ***no change with location*** *over a specified* region.

Properties of Fluids

- Any characteristic of a system is called a **property.**
- Some familiar properties are pressure *P*, temperature T, volume V, and mass m.
- Other less familiar properties include viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.
- Properties are considered to be either ***intensive or extensive***.
- ***Intensive*** properties are those that are independent of the mass of a system, such as temperature, pressure, and density.
- **Extensive** properties are those whose values depend on the size—or extent—of the system. Total mass, total volume *V*, and total momentum are some examples of extensive properties.

Properties of Fluids

- An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition.
- Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.



Properties of Fluids

Density or Mass Density

- Density or mass density of a fluid is defined as the ratio of the mass of a f1uid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic meter, i.e ., kg/m^3.
- The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.
- Mathematically mass density is written as.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

- The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

Properties of Fluids

Density or Mass Density

- The density of a substance, in general, depends on temperature and pressure.
- The density of most gases is proportional to pressure and inversely proportional to temperature.
- Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible.

Properties of Fluids

Specific weight or Weight Density

- Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.
- Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.
- Mathematically,

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{\text{(Mass of fluid) x Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid x g}}{\text{Volume of fluid}}$$

$$= \rho \text{ x g}$$

$$w = \rho g$$

Properties of Fluids

Specific Volume

- Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.
- Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

- Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg.
- It is commonly applied to gases.

Properties of Fluids

Specific Gravity.

- Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid.
- For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol *S*.

$$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

$$\text{Thus weight density of a liquid} = S \text{ x Weight density of water}$$

$$= S \text{ x } 1000 \text{ x } 9.81 \text{ N/m}^3$$

$$\text{Thus density of a liquid} = S \text{ x Density of water}$$

$$= S \text{ x } 1000 \text{ kg/m}^3$$

Properties of Fluids

Specific Gravity.

- If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water.
- For example the specific gravity of mercury is 13.6, hence density of mercury = 13.6 x 1000 = 13600 kg/m^3.

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Properties of Fluids

Example 1.

Calculate the specific weight, density and specific gravity of one liter of a liquid which weighs 7 N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \ \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3 \right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (*w*)

$$= \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \ N/m^3. \ Ans.}$$

(*ii*) Density (ρ)

$$= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \ kg/m^3. \ Ans.}$$

(*iii*) Specific gravity

$$= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \ \text{Density of water} = 1000 \text{ kg/m}^3\}$$

$$= \mathbf{0.7135. \ Ans.}$$

Properties of Fluids

Example 2. *Calculate the density, specific weight and weight of one liter of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(*ii*) *Specific weight* (*w*)

$$w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 = \mathbf{6867 \text{ N/m}^3}.$$ **Ans.**

(*iii*) *Weight* (*W*)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

∴ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

Properties of Fluids

Viscosity

- Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.
- When two layers of a fluid, a distance *'dy' apart move one* over the other at different velocities say u and u+ *du as shown in Fig. 1.1 , the viscosity together with* relative velocity causes a shear stress acting between the fluid layers:

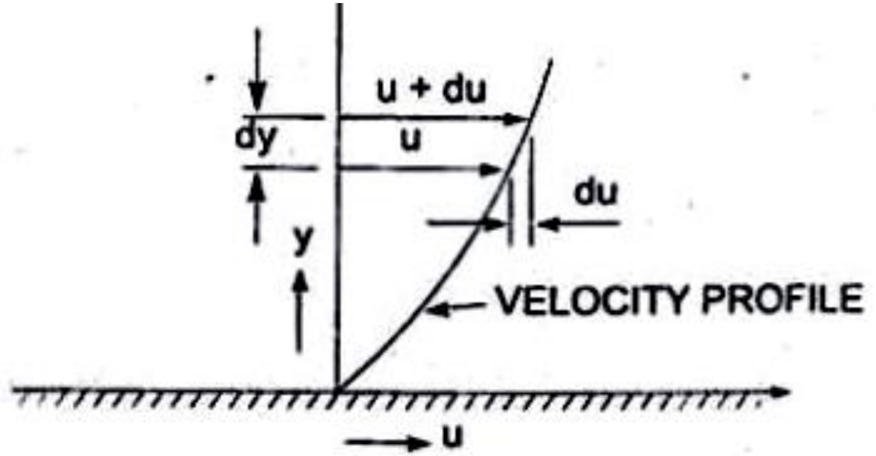


Fig. 1.1 ***Velocity variation near a solid boundary.***

Properties of Fluids

Viscosity

- The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.
- This shear stress is proportional to the rate of change of velocity with respect to *y. It is denoted by symbol* τ called Tau.
- Mathematically,

$$\tau \propto \frac{du}{dy}$$

- or

$$\tau = \mu \frac{du}{dy} \qquad (1.2)$$

Properties of Fluids

- where μ (called mu) is the constant of proportionality and is known as the coefficient of dynamic viscosity or only viscosity.
- $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.
- From equation (1.2) we have

$$\mu = \frac{\tau}{\frac{du}{dy}} \qquad (1.3)$$

- Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

Properties of Fluids

Unit of Viscosity.

- The unit of viscosity is obtained by putting the dimension of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

$$\text{SI unit of viscosity} = \frac{\text{Newton second}}{\text{m}^2} = \frac{\text{Ns}}{\text{m}^2}$$

Properties of Fluids

Kinematic Viscosity.

- It is defined as the ratio between the dynamic viscosity and density of fluid.lt is denoted by the Greek symbol (ν) called 'nu' . Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

- The SI unit of kinematic viscosity is m^2/s.

Newton's Law of Viscosity.

- It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient viscosity. Mathematically, it is expressed as given by equation (1 . 2).

Properties of Fluids

- Fluids which obey the above relation are known as Newtonian fluids and the fluids which do not obey the above relation are called Non-newtonian fluids.

Variation of Viscosity with Temperature

- Temperature affects the viscosity.
- The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and *molecular momentum transfer.*
- *In liquids the cohesive forces predominates the molecular momentum* transfer due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity.

Properties of Fluids

- But in the case of gases the cohesive force are small and molecular momentum transfer predominates. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids and gases are:

(i) For liquids, $\mu = \mu_o \left(\frac{1}{1 + \alpha t + \beta t^2} \right)$

where μ = Viscosity of liquid at t °C, in poise $\quad 1\,\text{poise} = \frac{1}{10} \frac{Ns}{m^2}$

μ_o = Viscosity of liquid at $0^o C$, in poise

α, β = are constants for the liquid

For water, $\mu_o = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = 0.000221$

(ii) For a gas, $\mu = \mu_o + \alpha t - \beta t^2$

where for air $\mu_o = 0.000017$, $\alpha = 0.000000056$, $\beta = 0.1189 \times 10^{-9}$

Types of Fluids

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.
2. Real fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids: in actual practice, are real fluids.
3. Newtonian Fluid. A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.
4. Non-Newtonian fluid. A real fluid, in which shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

Types of Fluids

5. Ideal Plastic Fluid.

A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

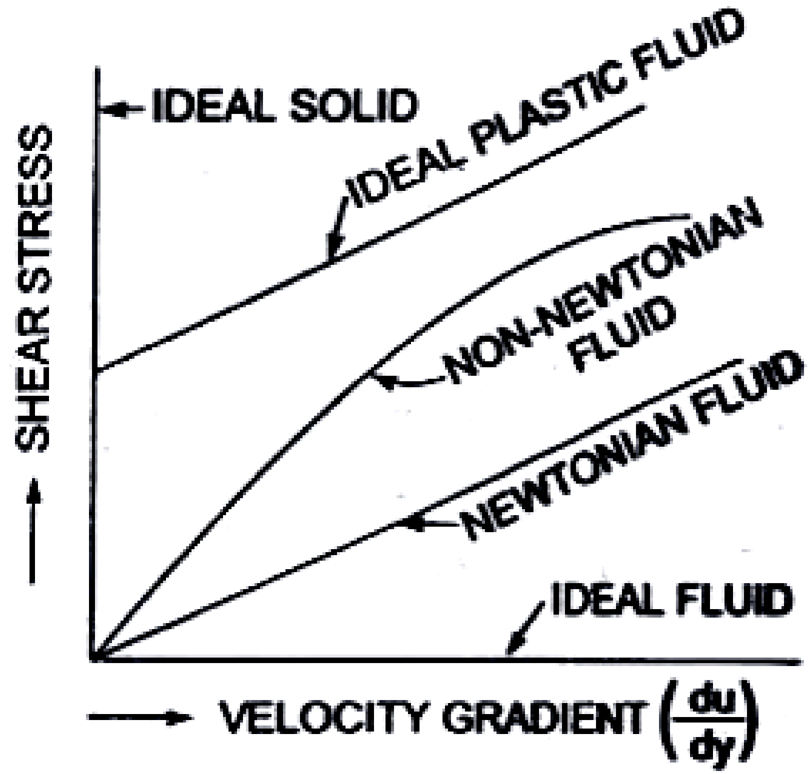


Fig. 1.2 *Types of fluids.*

Example 3

If the velocity distribution over a plate is given by

$$u = \frac{2}{3}y - y^2$$

in which u is velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y= 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.

Solution. Given : $u = \frac{2}{3}y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2.\ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2.\ Ans.}$$

Example 4

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30^o as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.

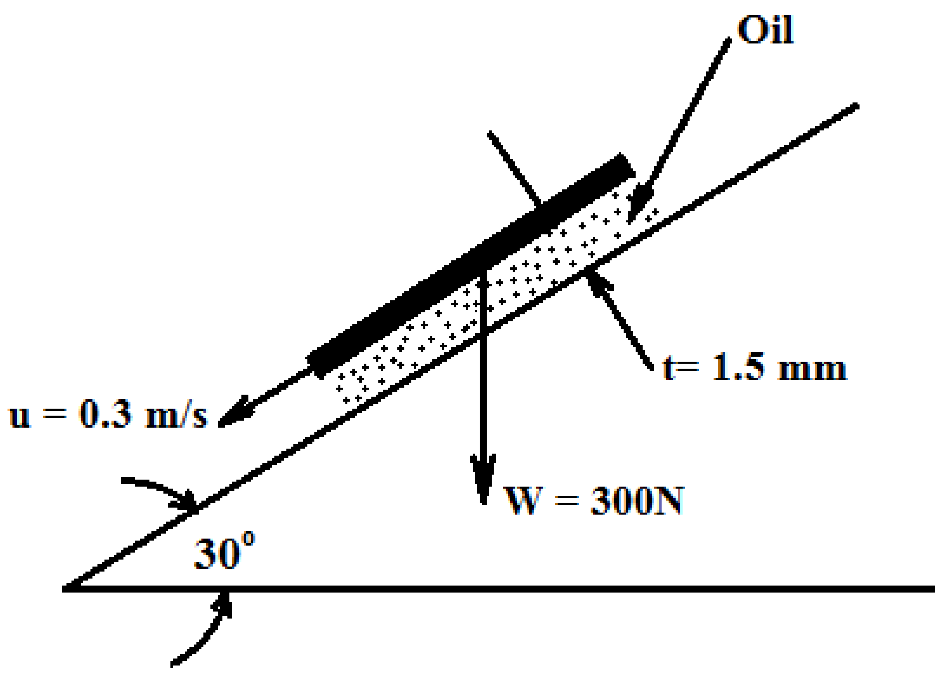


Fig.1.4

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300 \text{ N}$

Velocity of plate, $u = 0.3 \text{ m/s}$

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane $= W \cos 60° = 300 \cos 60° = 150 \text{ N}$

Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress, $\tau = \dfrac{F}{\text{Area}} = \dfrac{150}{0.64} \text{ N/m}^2$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity $= u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = 11.7 \text{ poise.}$$ **Ans.**

Example 5

The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed.
Determine : ·
i.the dynamic viscosity of the oil, and
ii.the kinematic viscosity of the oil if the specific gravity of the oil is 0.95.

Solution. Given:
Each side of a square plate = 60 cm = 0.6 m
Area A= 0.6 x 0.6 = 0.36 m^2
Thickness of oil film dy = 12.5 mm = 12.5 x 10^{-3} m
Velocity of upper plate u = 2.5 m/s

$\therefore$ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$\therefore$ Shear stress, $$\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1\ \text{N}}{0.36\ \text{m}^2}$$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $$\tau = \mu \frac{du}{dy} \text{ or } \frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2} \quad \textbf{Ans.}$$

(*ii*) Sp. gr. of oil, $S = 0.95$

Let ν = kinematic viscosity of oil

Using equation (1.1 A),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950\ \text{kg/m}^3$

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $$\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435\ \text{m}^2/\text{sec} \quad \textbf{Ans.}$$

Thermodynamic Properties

- Fluids consist of liquids or gases. But gases are compressible fluids and hence thermodynamic properties play an important role.
- With the change of pressure and temperature, the gases undergo large variation in density.
- The relationship between pressure (absolute), specific volume and temperature (absolute) of a gas is given by the equation of state as

$$p \forall = RT \text{ or } \frac{p}{\rho} = RT$$

where p = Absolute pressure of a gas in N/m^2

$\forall$ = Specific volume = $\frac{1}{\rho}$

R = Gas constant

T = Absolute temperature in °K

ρ = Density of a gas.

Thermodynamic Properties

- The value of gas constant R is $R = 287 \frac{J}{kg.K}$
- **Isothermal Process**. If the changes in density occurs at constant temperature, then the process is called isothermal and relationship between pressure (p) and density (ρ) is given by

$$\frac{p}{\rho} = \text{constant}$$

- **Adiabatic Process.** If the change in density occurs with no heat exchange to and from the gas, the process is called adiabatic. And if no heat is generated within the gas due to friction, the relationship between pressure and density is given by

$$\frac{p}{\rho^{k}} = \text{constant}$$

Thermodynamic Properties

- where *k* = *Ratio of specific heat of a gas at constant pressure and constant volume*.
- k = 1.4 for air

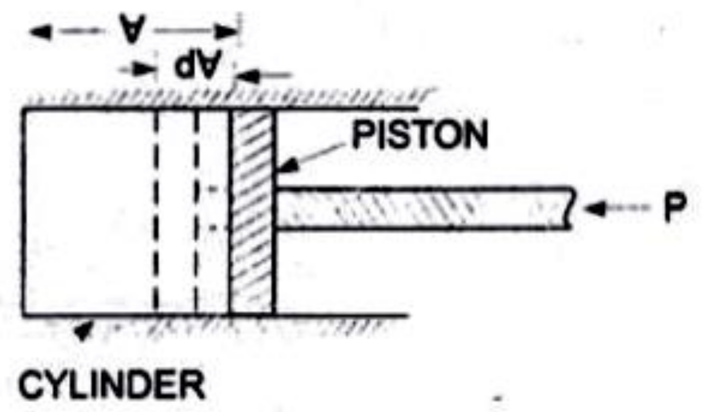
Compressibility and Bulk Modulus

- Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.
- Consider a cylinder fitted with a piston as shown in the Fig.
- Let *V= Volume of a gas enclosed in the cylinder*

  p =Pressure of gas when volume is V
- Let the pressure is increased *to p+ dp, the volume of gas* decreases from *V to V – dV.*
- Then increase in pressure = dp
- Decrease in volume = dV
- Volumetric strain = - dV/V

Compressibility and Bulk Modulus

- \- ve sign means the volume decreases with increase of pressure.



$$\therefore \text{Bulk modules} \quad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

$$= \frac{dp}{\frac{-dV}{V}} = -\frac{dp}{dV}V$$

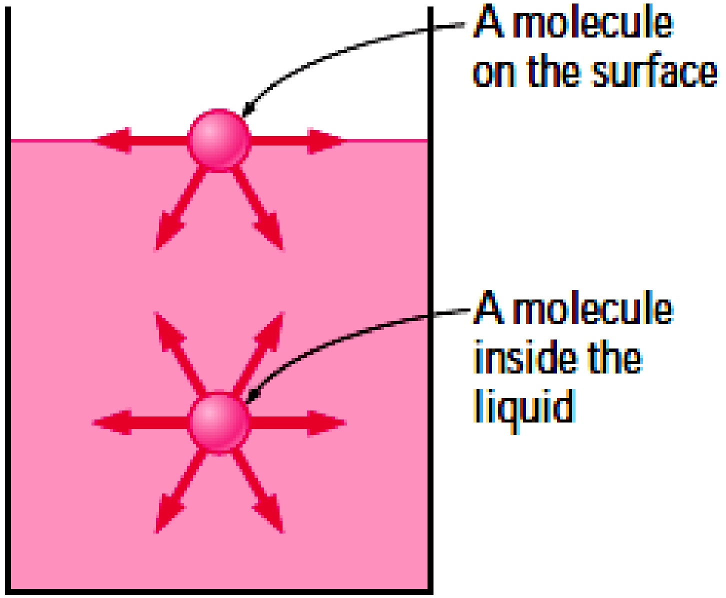
- Compressibility is given by = 1/K

Surface Tension and Capillarity

- Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.
- Surface tension is created due to the unbalanced cohesive forces acting on the liquid molecules at the fluid surface.
- Molecules in the interior of the fluid mass are surrounded by molecules that are attracted to each other equally.
- However, molecules along the surface are subjected to a net force toward the interior.
- The apparent physical consequence of this unbalanced force along the surface is to create the hypothetical skin or membrane.

Surface Tension and Capillarity

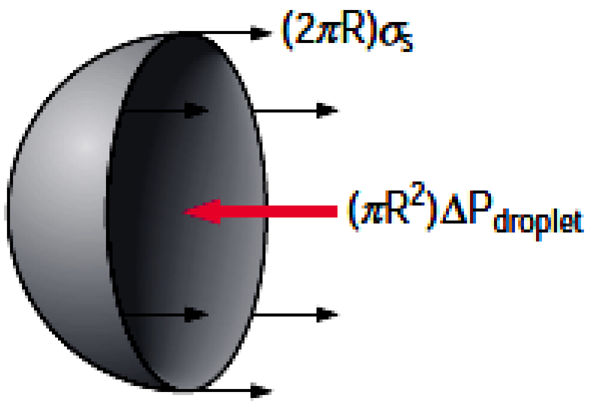
- A tensile force may be considered to be acting in the plane of the surface along any line in the surface.
- The intensity of the molecular attraction per unit length along any line in the surface is called the ***surface tension.***
- It is denoted by Greek letter σ (called sigma).
- The SI unit is N/m.



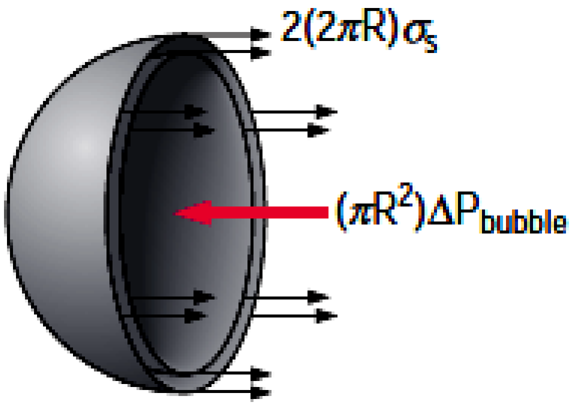
Surface Tension and Capillarity

Surface Tension on liquid Droplet and Bubble

- Consider a small spherical droplet of a liquid of radius *'R'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.*
- Let σ = surface tension of the liquid
- ΔP= Pressure intensity inside the droplet (in excess of the outside pressure intensity)
- *R= Radius of droplet.*
- Let the droplet is cut into two halves. The forces acting on one half (say left half) will be



(a) Half a droplet



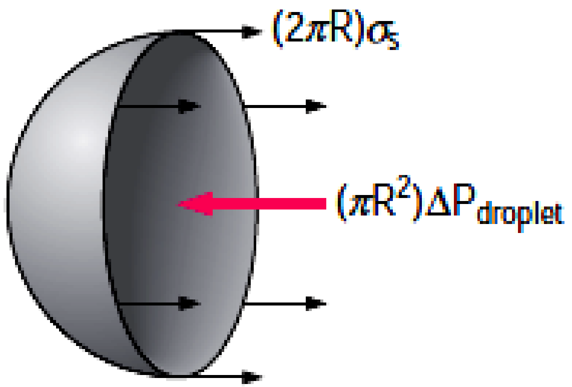
(b) Half a bubble

Surface Tension and Capillarity

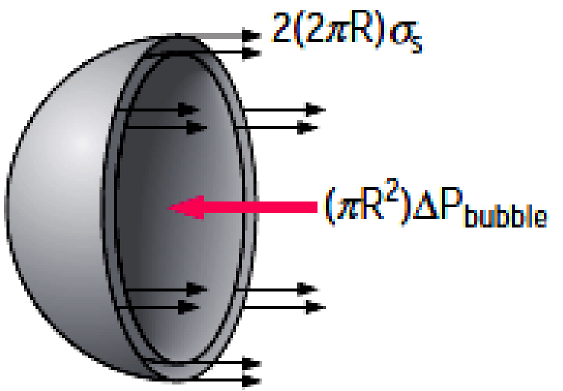
- (i) tensile force due to surface tension acting around the circumference of the cut portion as shown and this is equal to

  = σ x Circumference

  = σ x 2πR
- (ii) pressure force on the area $(\pi/4)d^2$ and
- = $\Delta P \, x \, \pi R^2$ *as shown*



(a) Half a droplet



(b) Half a bubble

Surface Tension and Capillarity

- These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

Droplet: $$(2\pi R)\sigma_s = (\pi R^2)\Delta P_{droplet} \rightarrow \Delta P_{droplet} = P_i - P_o = \frac{2\sigma_s}{R}$$

Bubble: $$2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{bubble} \rightarrow \Delta P_{bubble} = P_i - P_o = \frac{4\sigma_s}{R}$$

- A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected surface tension.

Surface Tension……. Example 1

- Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Solution. Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$

Pressure in excess of outside, $p = 2.5 \text{ N/m}^2$

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \mathbf{0.0125 \text{ N/m. Ans.}}$$

Surface Tension……. Example 2

- The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by

or $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

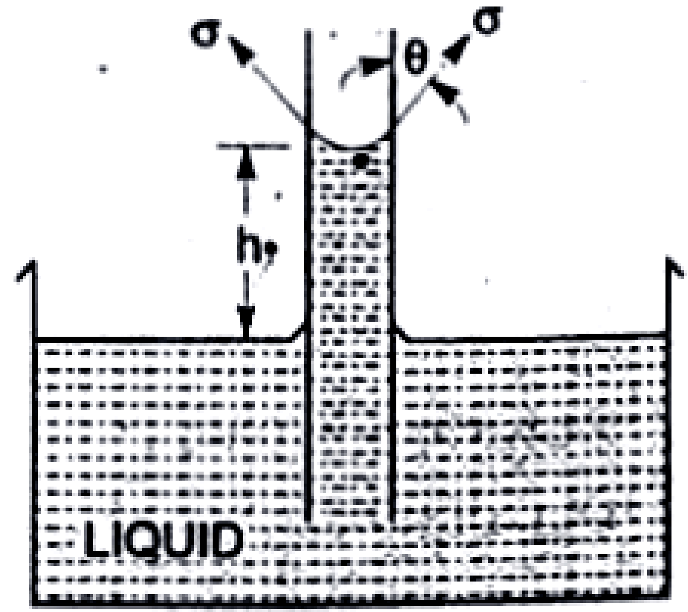
$= 0.725 + 10.32 =$ **11.045 N/cm^2. Ans.**

Surface Tension and Capillarity

Capillarity

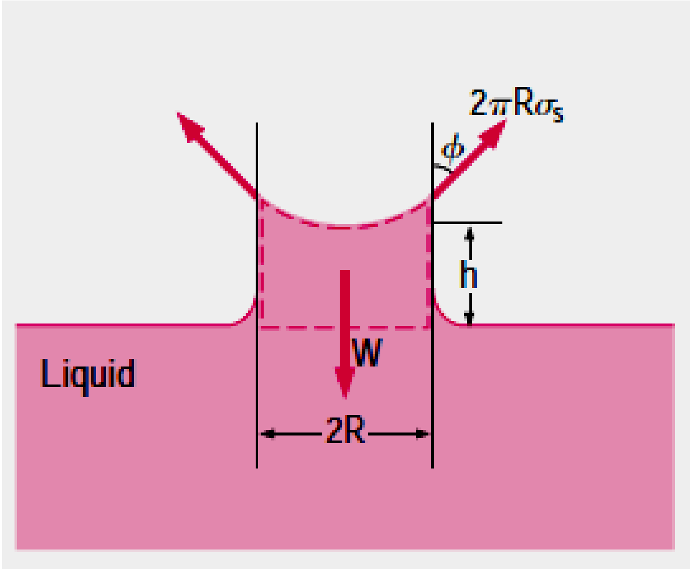
- Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.
- The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.
- The attraction (adhesion) between the wall of the tube and liquid molecules is strong enough to overcome the mutual attraction (cohesion) of the molecules and pull them up the wall. Hence, the liquid is said to *wet the solid surface.*
- It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Surface Tension and Capillarity



Expression for Capillary Rise

- Consider a glass tube of small diameter ‘d’ opened at both ends and is inserted in a liquid, say water.
- The liquid will rise in the tube above the level of the liquid.
- Let h = the height of the liquid in the tube . Under a state of equilibrium, the weight of the liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.



Expression for Capillary Rise…

- Let σ = Surface tension of liquid

  θ = Angle of contact between the liquid and glass tube

- The weight of the liquid of height h in the tube

  = (Area of the tube x h) x ρ x g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta$$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

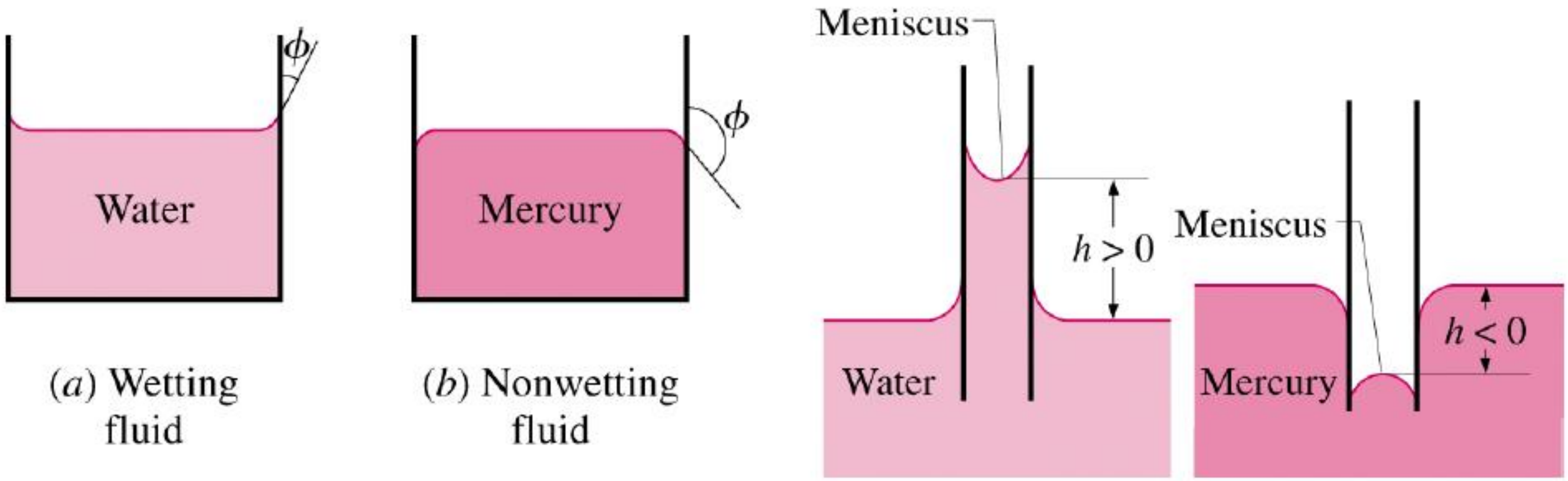
Expression for Capillary Rise…

- The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d}$$

- Contact angle depends on both the liquid and the solid.
- If θ is less than 90^o, the liquid is said to "wet" the solid. However, if θ is greater than 90^o, the liquid is repelled by the solid, and tries not to "wet" it.
- For example, water wets glass, but not wax. Mercury on the other hand does not wet glass.

Capillarity



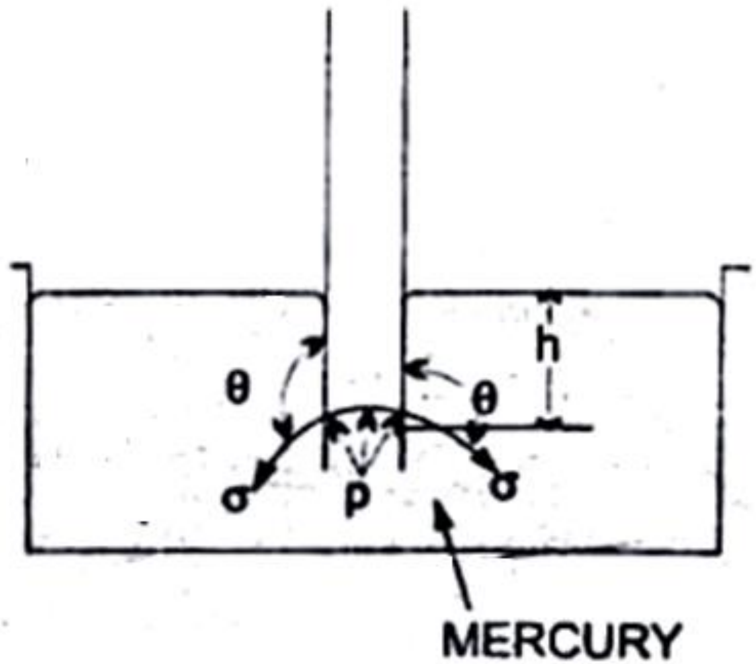
Expression for Capillary Fall

- lf the glass tube is dipped in mercury, the revel of mercury in the tube will be lower than the general level of the outside liquid as shown above.

Capillarity

Expression for Capillary Fall

- Let *h = Height of depression in tube.*
- Then in equilibrium, two forces arc acting on the mercury inside the tube.
- First one is due to surface tension acting in the downward direction and is equal to σ x πd x $\cos\theta$.
- Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth *'h' x Area*



Capillarity

Expression for Capillary Fall

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \quad \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

Value of θ for mercury and glass tube is 128^o

Capillarity…Example 1

- Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130^o.

Solution. Given :

Dia. of tube,	$d = 2.5$ mm $= 2.5 \times 10^{-3}$ m
Surface tenstion, σ for water	= 0.0725 N/m
σ for mercury	= 0.52 N/m
Sp. gr. of mercury	= 13.6

Capillarity…Example 1

∴ Density $= 13.6 \times 1000 \text{ kg/m}^3$.

(*a*) Capillary rise for water ($\theta = 0$)

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118 \text{ m} =$ **1.18 cm. Ans.**

(*b*) For mercury

Angle of constant between mercury and glass tube, $\theta = 130°$

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004 \text{ m} =$ **– 0.4 cm. Ans.**

The negative sign indicates the capillary depression.

Capillarity…Example 2

- Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Solution. Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3} \text{ m}$

Surface tension, $\sigma = 0.073575 \text{ N/m}$

Let dia. of tube $= d$

The angle θ for water $= 0$

The density for water, $\rho = 1000 \text{ kg/m}^3$

$$h = \frac{4\sigma}{\rho \times g \times d} \quad \text{or} \quad 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

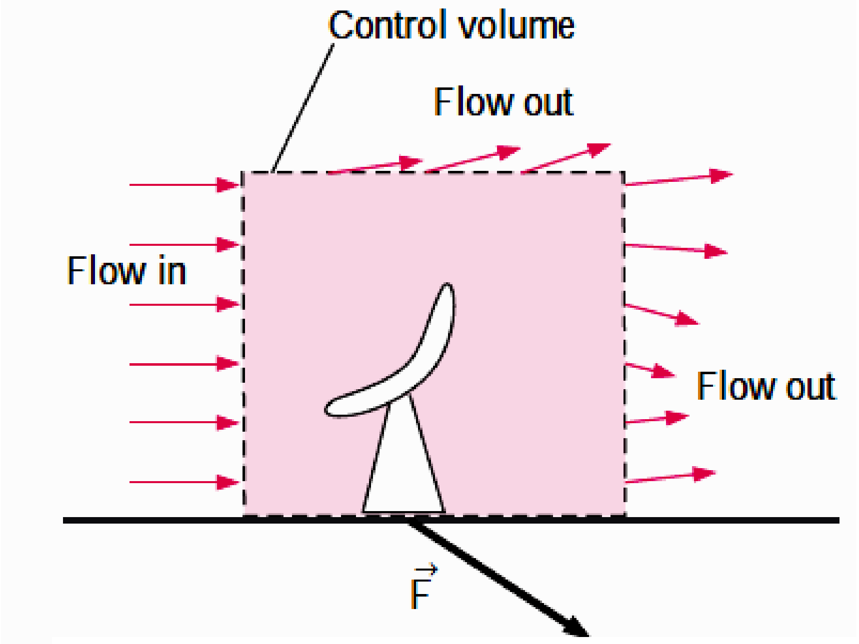
$$\therefore \quad d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5\ cm.\ Ans.}$$

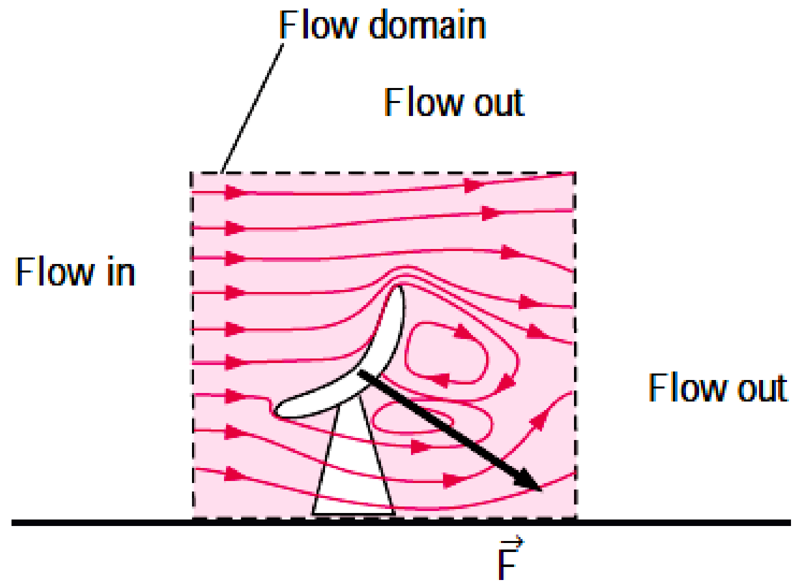
Thus minimum diameter of the tube should be 1.5 cm.

Flow Analysis Techniques

- In analyzing fluid motion, we might take one of two paths:

1. Seeking an estimate of gross effects (mass flow, induced force, energy change) over a finite region or control volume or
2. Seeking the point-by-point details of a flow pattern by analyzing an infinitesimal region of the flow.





Flow Analysis Techniques

- The control volume technique is useful when we are interested in the overall features of a flow, such as mass flow rate into and out of the control volume or net forces applied to bodies.
- Differential analysis, on the other hand, involves application of differential equations of fluid motion to *any and every point in the flow field over a region* called the flow domain.
- When solved, these differential equations yield details about the velocity, density, pressure, etc., at *every point* throughout the *entire flow domain.*

Flow Patterns

- Fluid mechanics is a highly visual subject. The patterns of flow can be visualized in a dozen different ways, and you can view these sketches or photographs and learn a great deal qualitatively and often quantitatively about the flow.
- Four basic types of line patterns are used to visualize flows:

1. A ***streamline*** is a line everywhere tangent to the velocity vector at a given instant.
2. A ***pathline*** is the actual path traversed by a given fluid particle.
3. A ***streakline*** is the locus of particles that have earlier passed through a prescribed point.
4. A ***timeline*** is a set of fluid particles that form a line at a given instant.

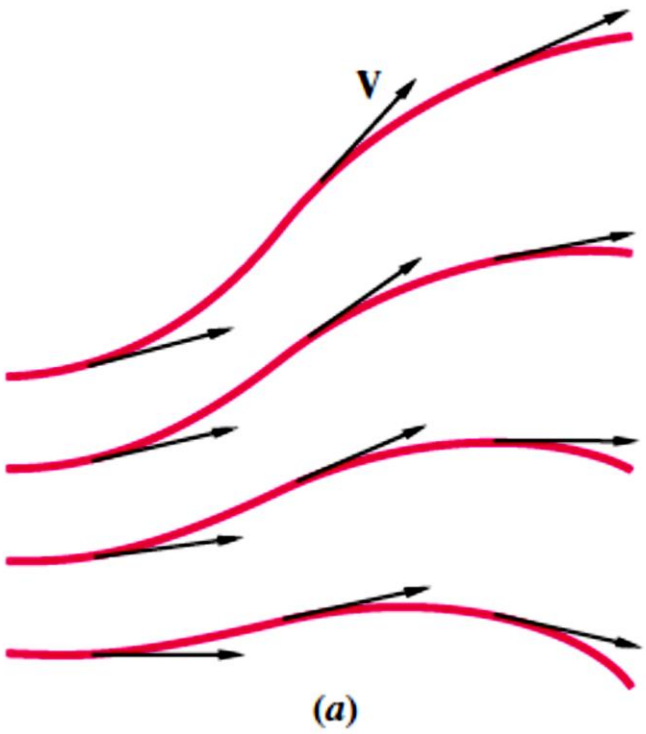
Flow Patterns

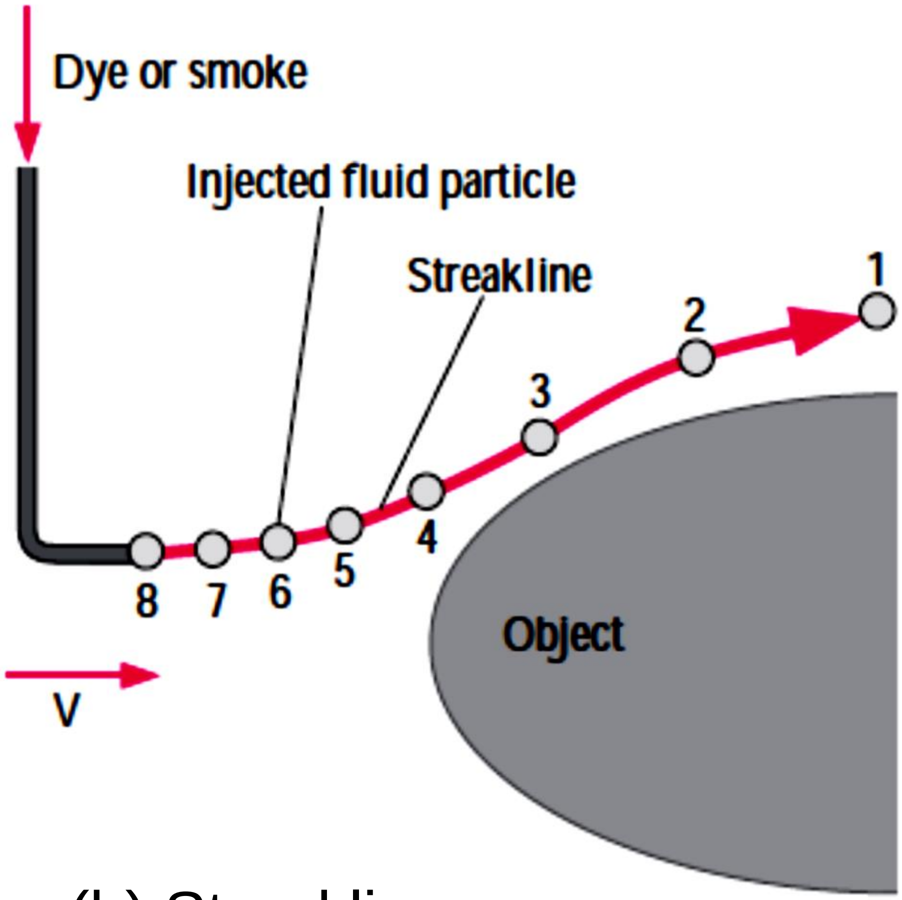
- The streamline is convenient to calculate mathematically, while the other three are easier to generate experimentally.
- Note that a streamline and a timeline are instantaneous lines, while the pathline and the streakline are generated by the passage of time.
- A ***streamline*** is a line that is everywhere tangent to the velocity field. If the flow is steady, nothing at a fixed point (including the velocity direction) changes with time, so the streamlines are fixed lines in space.
- For unsteady flows the streamlines may change shape with time.
- A pathline is the line traced out by a given particle as it flows from one point to another.

Flow Patterns

- A ***streakline*** consists of all particles in a flow that have previously passed through a common point. Streaklines are more of a laboratory tool than an analytical tool.
- They can be obtained by taking instantaneous photographs of marked particles that all passed through a given location in the flow field at some earlier time.
- Such a line can be produced by continuously injecting marked fluid (neutrally buoyant smoke in air, or dye in water) at a given location.
- If the flow is steady, each successively injected particle follows precisely behind the previous one forming a steady streakline that is exactly the same as the streamline through the injection point.

Flow Patterns



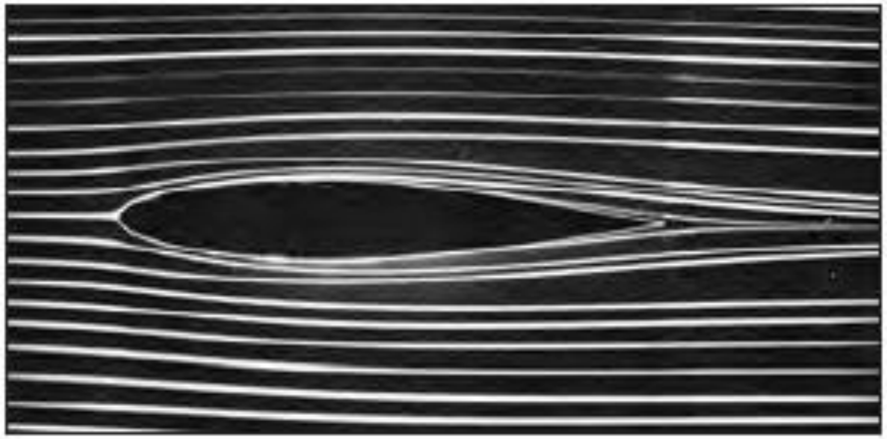


(a) Streamlines

(b) Streaklines

Flow Patterns

- Streaklines are often confused with streamlines or pathlines.
- While the three flow patterns are identical in steady flow, they can be quite different in unsteady flow.
- The main difference is that a streamline represents an *instantaneous* flow pattern at a given instant in time, while a streakline and a pathline are flow patterns that have some *age* and thus a *time history* associated with them.
- *If the flow is steady, streamlines, pathlines, and streaklines are identical*



Dimensions and Units

- Fluid mechanics deals with the measurement of many variables of many different types of units. Hence we need to be very careful to be consistent.

Dimensions and Base Units

- The *dimension of a measure is independent of any particular system of units. For* example, velocity may be inmetres per second or miles per hour, but dimensionally, it is always length per time, or $L/T = LT^{-1}$.
- The dimensions of the relevant base units of the Système International (SI) system are:

Dimensions and Units

Unit-Free		SI Units	
Dimension	**Symbol**	**Unit**	**Symbol**
Mass	M	kilogram	kg
Length	L	metre	m
Time	T	second	s
Temperature	θ	kelvin	K

Derived Units

Quantity	Dimension	SI Unit	
		Derived	**Base**
Velocity	LT^{-1}	m/s	$m\,s^{-1}$
Acceleration	LT^{-2}	m/s^2	$m\,s^{-2}$
Force	MLT^{-2}	Newton, N	$kg\,m\,s^{-2}$

Pressure Stress	$ML^{-1}T^{2}$	Pascal, Pa N/m^2	$kg\ m^{-1}\ s^{-2}$
Density	ML^{-3}	kg/m^3	$kg\ m^{-3}$
Specific weight	$ML^{-2}T^{-2}$	N/m^3	$kg\ m^{-2}\ s^{-2}$
Relative density	Ratio	Ratio	Ratio
Viscosity	$ML^{-1}T^{-1}$	Ns/m^2	$kg\ m^{-1}\ s^{-1}$
Energy (work)	$ML^{2}T^{-2}$	Joule, J Nm	$kg\ m^{2}\ s^{-2}$
Power	$ML^{2}T^{-3}$	Watt, W Nm/s	$kg\ m^{2}\ s^{-3}$

Note: The acceleration due to gravity will always be taken as 9.81 m/s^2.

Unit Table

Quantity	SI Unit	English Unit
Length (L)	Meter (*m*)	Foot (*ft*)
Mass (m)	Kilogram (*kg*)	Slug (slug) = $lb*sec^2/ft$
Time (T)	Second (*s*)	Second (*sec*)
Temperature (θ)	Celcius (^oC)	Farenheit (^oF)
Force	Newton $(N)=kg*m/s^2$	Pound (lb)

Dimensions and Units…

- 1 *Newton* – Force required to accelerate a 1 *kg* of mass to 1 m/s^2
- 1 *slug* – is the mass that accelerates at 1 ft/s^2 when acted upon by a force of 1 *lb*
- To remember units of a Newton use F=ma (Newton's 2nd Law)
  - $[F] = [m][a] = kg*m/s^2 = N$
- To remember units of a slug also use F=ma => m = F / a
- $[m] = [F] / [a] = lb / (ft / sec^2) = lb*sec^2 / ft$

End of Chapter 1

Next Lecture

Chapter 2: Fluid Statics