ELECTRO MAGNETIC FIELD

CHAPTER 1

VECTOR ALGEBRA

INTRODUCTION

- Electromagnetics (EM) may be regarded as the study of the interactions between electric charges at rest and in motion.
- It entails the analysis, synthesis, physical interpretation, and application of electric and magnetic fields.
- It is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied
- EM devices include transformers, electric relays, radio/TV, telephone, electric motors, transmission lines,

waveguides, antennas, optical fibers, radars, and lasers.

- The design of these devices requires thorough knowledge of the laws and principles of EM.

1.1 SCALARS AND VECTORS

- A scalar is a quantity that has only magnitude.
- A vector is a quantity that has both magnitude and direction.
- A field is a function that specifies a particular quantity everywhere in a region.
- If the quantity is scalar (or vector), the field is said to be a scalar (or vector) field.
- Examples of scalar fields are temperature distribution in a building, sound intensity in a theater, electric potential in a region, and refractive index of a stratified medium.
- The gravitational force on a body in space and the velocity of raindrops in the atmosphere are examples of vector fields.

1.2 UNIT VECTOR

- A vector A has both magnitude and direction. The magnitude of A is a scalar written as A or |A|.
- A unit vector aA along A is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along A, that is,

$$\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}$$

- Note that |aA| = 1. Thus we may write A as A = AaA,which completely specifies A in terms of it magnitude A and its direction aA.
- A vector A in Cartesian (or rectangular) coordinates may be represented as (Ax, Ay, Az) or Ayay + Azaz.

CONT’D

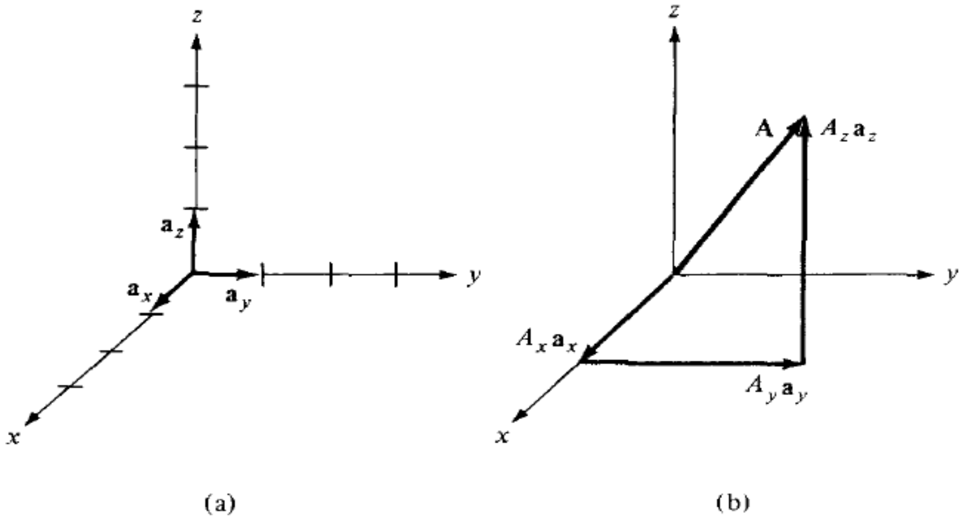


Figure 1.1 (a) Unit vectors ax, ay, and az, (b) components of A along ax, ay,, and az.

- where Ax, Ar and *Az* are called the *components of* A in the *x*, *y*, and z directions respectively; *ax*, *aT* and *az* are unit vectors in the *x*, *y*, and z directions, respectively.

CONT’D

The magnitude of vector A is given by;

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

and the unit vector along A is given by;

$$\mathbf{a}_A = \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

1.3 VECTOR ADDITION AND SUBTRACTION

- Two vectors A and B can be added together to give another vector C; that is,

C = A + B

- The vector addition is carried out component by component. Thus, if A = (Ax, Ay, Az) and

B = (Bx,By,Bz).   C = (Ax + Bx)ax + {Ay + By)ay + (Az + Bz)az

- Vector subtraction is similarly carried out as

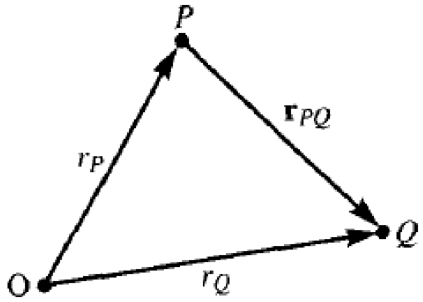
D = A - B = A + (-B)

= (Ax - Bx)ax + (Ay - By)ay + (Az - Bz)az

1.4 POSITION AND DISTANCE VECTORS

- The position vector r,. (or radius vector) of point P is as (he directed silancc from the origin () lo P: i.e..

$$rp = OP = Xax + Yay + Zaz$$



The **distance vector** is **thc** displacement from one point to another.

If two points *P* and *Q* are given by *(Px, Py, Pz)* and *(Qx, Qy, Qz),* the *distance vector* (or *separation vector)* is the displacement from *P* to *Q* as shown in; that is,

$$\boldsymbol{rPQ = Rq - rP}$$

$$= (xQ - xP)ax + (yQ - yP)\&y + (zQ - zP)az$$

CONT’D

EXAMPLE 1.1

- If A = 10ax - 4ay + 6az and B = 2ax + ay, find: (a) the component of A along ay, (b) the magnitude of 3A - B, (c) a unit vector along A + 2B.

Solution:

(a) The component of A along a_y is A_y = -4.

(b) 3A - B = 3(10, - 4 , 6) - (2, 1, 0)

= (30,-12,18) - (2, 1,0)

= (28,-13,18)

Hence = $|3\mathbf{A} - \mathbf{B}| = \sqrt{28^2 + (-13)^2 + (18)^2} = \sqrt{1277}$

35.74

(c) Let C = A + 2B = (10, - 4 , 6) + (4, 2, 0) = (14, - 2 , 6).

A unit vector along C is

$$\mathbf{a}_c = \frac{\mathbf{C}}{|\mathbf{C}|} = \frac{(14, -2, 6)}{\sqrt{14^2 + (-2)^2 + 6^2}}$$

ac = 0.91 13ax - 0.1302ay + 0.3906az

Note that |ac| = 1 as expected.

1.5 VECTOR MULTIPLICATION

- There are two types of vector multiplication:

1. Scalar (or dot) product: A • B
2. Vector (or cross) product: A X B

1.Dot Product

The dot product of two vectors A and B, written as A • B. is defined geometrically as the product of the magnitudes of A and B and the cosine of the angle between them.

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta_{AB}$$

2. Cross Product

The **cross product** of two vectors A and B. written as A X B. is a vector quantity whose magnitude is the area of the parallelopiped formed by A and It and is in the direction of advance of a right-handed screw as A is turned into B.

CONT'D

$$\mathbf{A} \times \mathbf{B} = AB \sin \theta_{AB} \mathbf{a}_n$$

The above vector multiplication of is called cross product due to the cross sign; it is also called vector product because the result is a vector. If A = (Ax,Ay,Az) B = (Bx, By, Bz) then

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

=(AyBz- AzBy)ax + (AZBX - AxBz)ay + (AxBy - AyBx)az

CHAPTER TWO

COORDINATE SYSTEMS AND TRANSFORMATION

INTRODUCTION

- In general, the physical quantities we shall be dealing with in EM are functions of space and time.
- In order to describe the spatial variations of the quantities, we must be able to define all points uniquely in space in a suitable manner.
- This requires using an appropriate coordinate system. A point or vector can be represented in any curvilinear coordinate system, which may be orthogonal or nonorthogonal.
- An orthogonal system is one in which the coordinates arc mutually perpendicular.
- Nonorthogonal systems are hard to work with and they are of little or no practical use.

2.1 CARTESIAN COORDINATES (X, Y, Z)

- As mentioned in Chapter 1, a point P can be represented as (x, y, z) as illustrated in Figure 1.1. The ranges of the coordinate variables x, y, and z are

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$

- A vector A in Cartesian (otherwise known as rectangular) coordinates can be written as A_x, A_y, A_z or $A_x a_x + A_y a_y + A_z a_z$
- where a_x, a_y, and a_z are unit vectors along the x-, y-, and z-directions as shown in Figure 1.1.

2.2 CIRCULAR CYLINDRICAL COORDINATES (ρ, ϕ, z)

A point P in cylindrical coordinates is represented as (ρ, ϕ, z) and is as shown in Figure 2.1.

p is the radius of the cylinder passing through P or the radial distance from the z-axis: ϕ, called the azimuthal angle, is measured from the x-axis in the xy-plane; and z is the same as in the Cartesian system.

CONT’D

- The ranges of the variables are:

$$0 \le \rho < \infty$$
$$0 \le \phi < 2\pi$$
$$-\infty < z < \infty$$

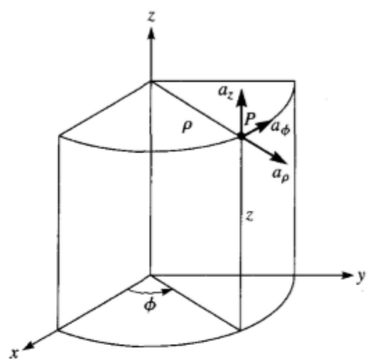


Figure 2.1 Point P and unit vectors in the cylindrical coordinate system

- A vector A in cylindrical coordinates can be written as

(Aρ, Aϕ,, Az) or Aρaρ +Aϕaϕ+Azaz

- the relationships between the variables (x, y, z) of the Cartesian coordinate system and those of the cylindrical system (ρ, ϕ, z) are easily obtained from Figure 2.2

CONT'D

$$\rho = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}, \qquad z = z$$

$$x = \rho\cos\phi, \qquad y = \rho\sin\phi, \qquad z = z$$

- ❖ Whereas the first is for transforming a point from Cartesian (x, y, z) to cylindrical (p, ϕ, z) coordinates, second eq. is for (p, ϕ, z) _(x, y, z) transformation.

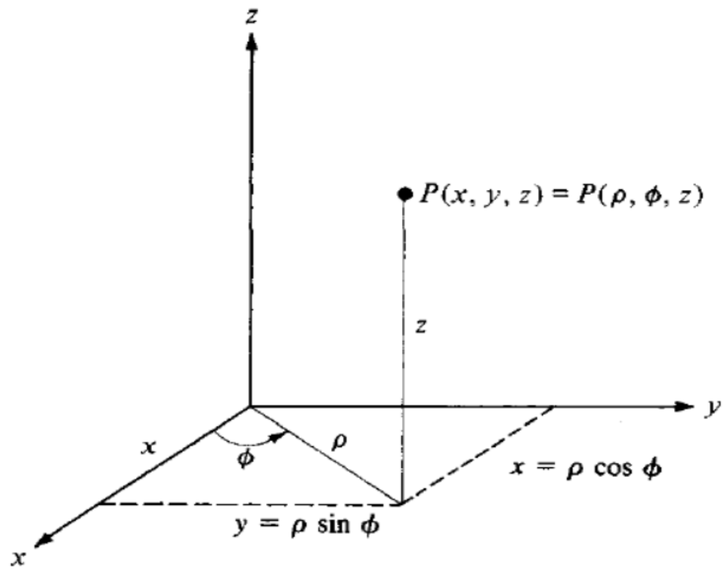


Figure 2.2 Relationship between (x, y, z) and (P, ϕ, z).

CONT'D

- The relationships between (ax, ay, az) and (ap, aϕ, az) are obtained geometrically from Figure 2.3:

$$\mathbf{a}_x = \cos\phi\,\mathbf{a}_\rho - \sin\phi\,\mathbf{a}_\phi$$
$$\mathbf{a}_y = \sin\phi\,\mathbf{a}_\rho + \cos\phi\,\mathbf{a}_\phi$$
$$\mathbf{a}_z = \mathbf{a}_z$$

$$\mathbf{a}_\rho = \cos\phi\,\mathbf{a}_x + \sin\phi\,\mathbf{a}_y$$
$$\mathbf{a}_\phi = -\sin\phi\,\mathbf{a}_x + \cos\phi\,\mathbf{a}_y$$
$$\mathbf{a}_z = \mathbf{a}_z$$

CONT'D

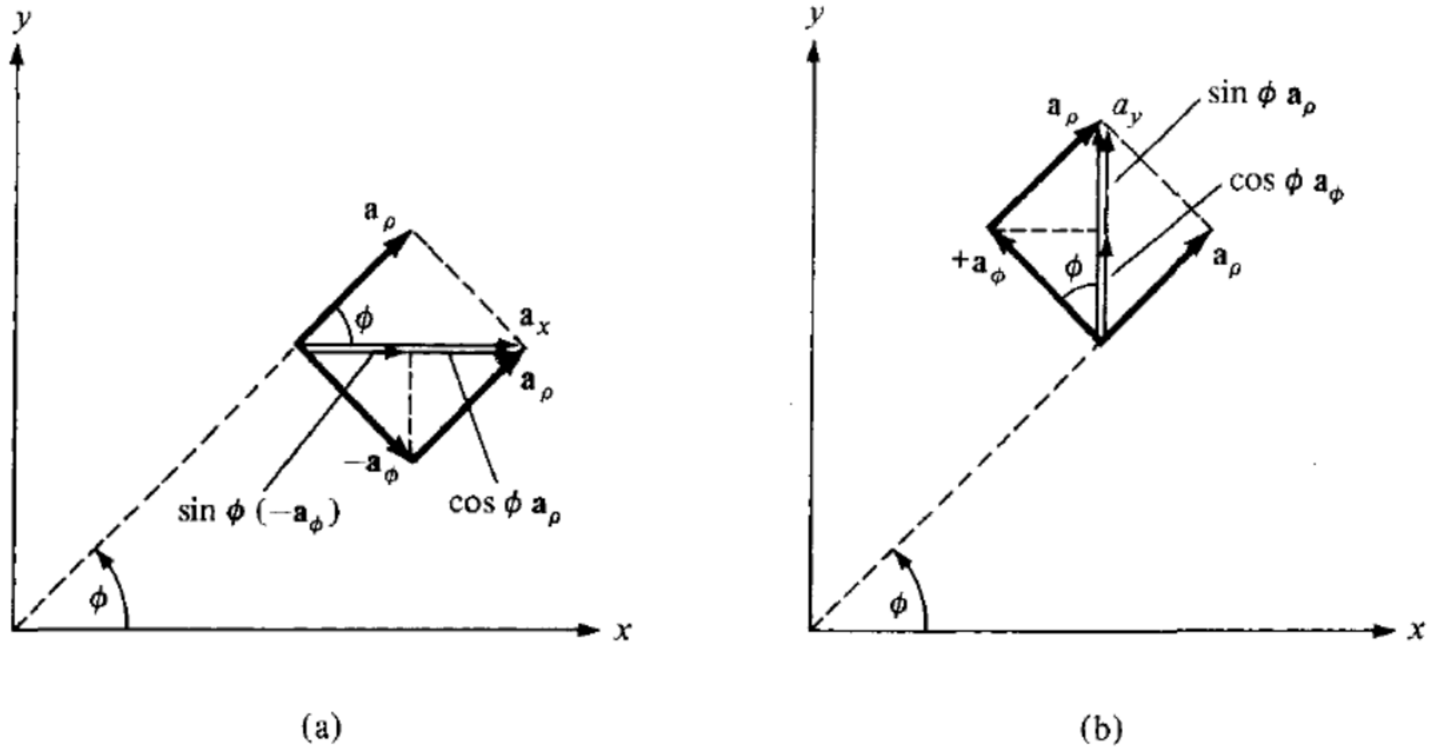


Figure 2.3 Unit vector transformation: **(a)** cylindrical components of $\mathbf{a}_x$, **(b)** cylindrical components of $\mathbf{a}_y$.

CONT'D

- ❖ Finally, the relationships between (Ax, Ay, Az) and (Ap, Aϕ, Az) are

$$\mathbf{A} = (A_x \cos\phi + A_y \sin\phi)\mathbf{a}_\rho + (-A_x \sin\phi + A_y \cos\phi)\mathbf{a}_\phi + A_z\mathbf{a}_z$$

$$A_\rho = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$A_z = A_z$$

In matrix form,

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

CONT'D

The inverse of the transformation is

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

2.3 SPHERICAL COORDINATES (r, ϕ, z)

- From Figure 2.4, we notice that r is defined as the distance from the origin to point P or the radius of a sphere centered at the origin and passing through P; θ (called the colatitude) is the angle between the z-axis and the position vector of P; and ϕ is measured from the x-axis (the same azimuthal angle in cylindrical coordinates). According to these definitions, the ranges of the variables are

CONT'D

$$0 \leq r < \infty$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \phi < 2\pi$$

A vector **A** in spherical coordinates may be written as

$$(A_r, A_\theta, A_\phi) \qquad \text{or} \qquad A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$$

where ar, aθ, and aϕ are unit vectors along the r, θ, and ϕ directions.

CONT’D

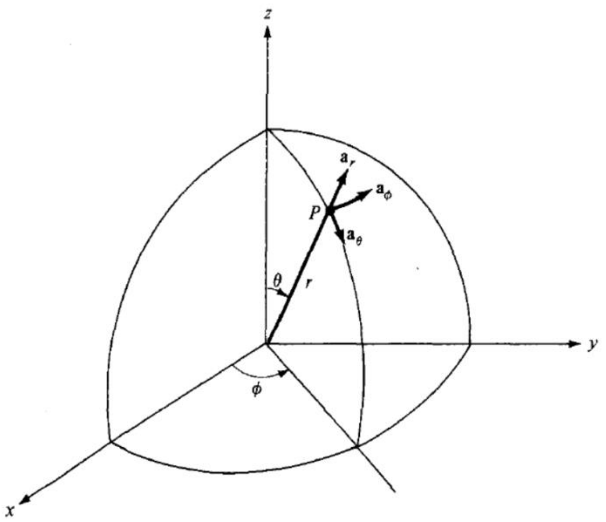


Figure 2.4 Point P and unit vectors in spherical coordinates.

- The space variables (x, y, z) in Cartesian coordinates can be related to variables (r, θ, ϕ) of a spherical coordinate system. From Figure 2.5 it is easy to notice that

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

or

$$x = r\sin\theta\cos\phi, \qquad y = r\sin\theta\sin\phi, \qquad z = r\cos\theta$$

- The unit vectors ax, ay, az and ar, aθ, aϕ are related as follows:

$$\mathbf{a}_x = \sin\theta\cos\phi\,\mathbf{a}_r + \cos\theta\cos\phi\,\mathbf{a}_\theta - \sin\phi\,\mathbf{a}_\phi$$

$$\mathbf{a}_y = \sin\theta\sin\phi\,\mathbf{a}_r + \cos\theta\sin\phi\,\mathbf{a}_\theta + \cos\phi\,\mathbf{a}_\phi$$

$$\mathbf{a}_z = \cos\theta\,\mathbf{a}_r - \sin\theta\,\mathbf{a}_\theta$$

or

$$\mathbf{a}_r = \sin\theta\cos\phi\,\mathbf{a}_x + \sin\theta\sin\phi\,\mathbf{a}_y + \cos\theta\,\mathbf{a}_z$$

$$\mathbf{a}_\theta = \cos\theta\cos\phi\,\mathbf{a}_x + \cos\theta\sin\phi\,\mathbf{a}_y - \sin\theta\,\mathbf{a}_z$$

$$\mathbf{a}_\phi = -\sin\phi\,\mathbf{a}_x + \cos\phi\,\mathbf{a}_y$$

CONT'D

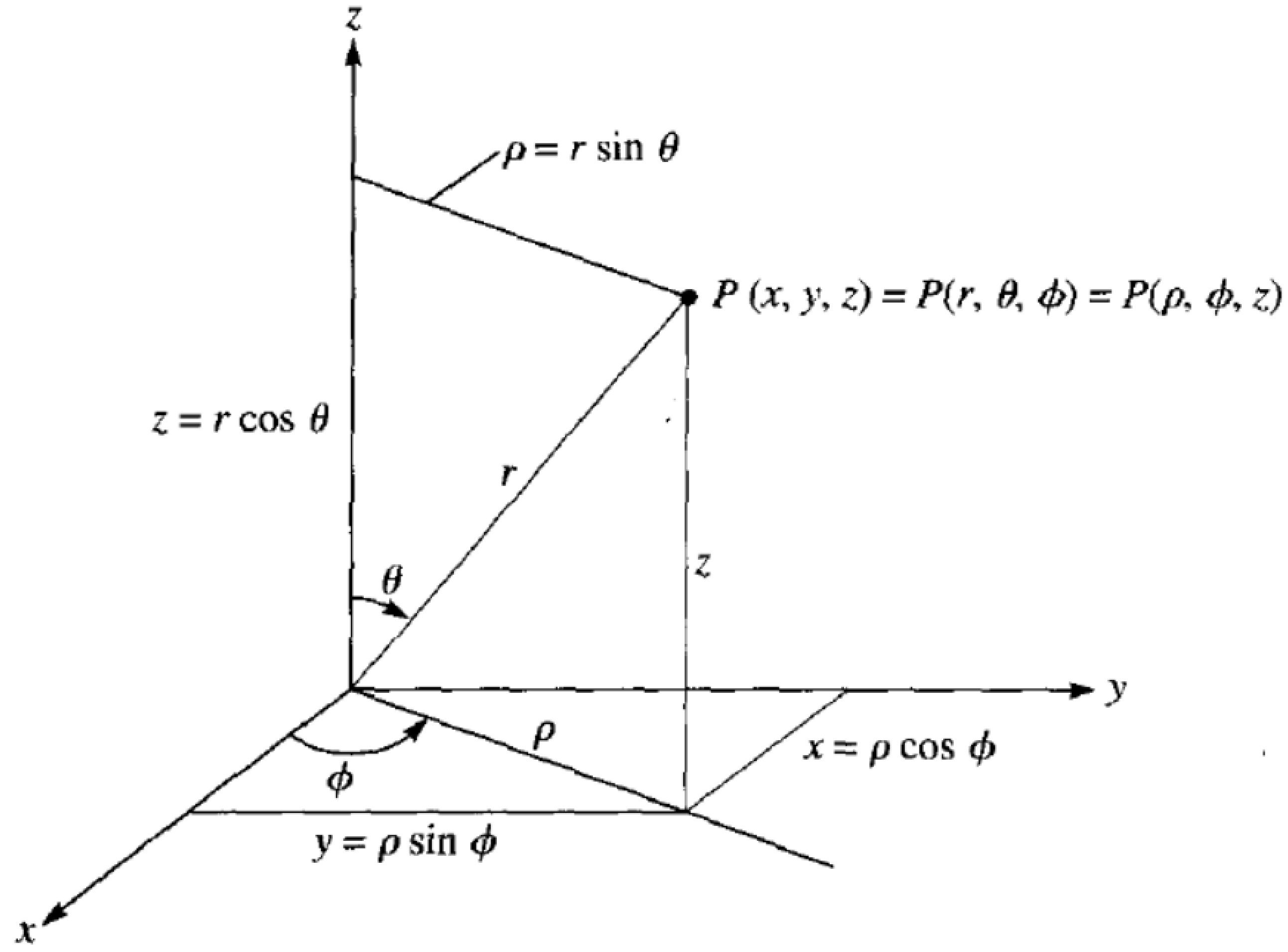


Figure 2.5 Relationships between space variables (x, y, z), (r, θ,ϕ) and (p, ϕ, z).

CHAPTER 3

VECTOR CALCULUS

3.1 DIFFERENTIAL LENGTH, AREA, AND VOLUME

❖ Differential elements in length, area, and volume are useful in vector calculus.

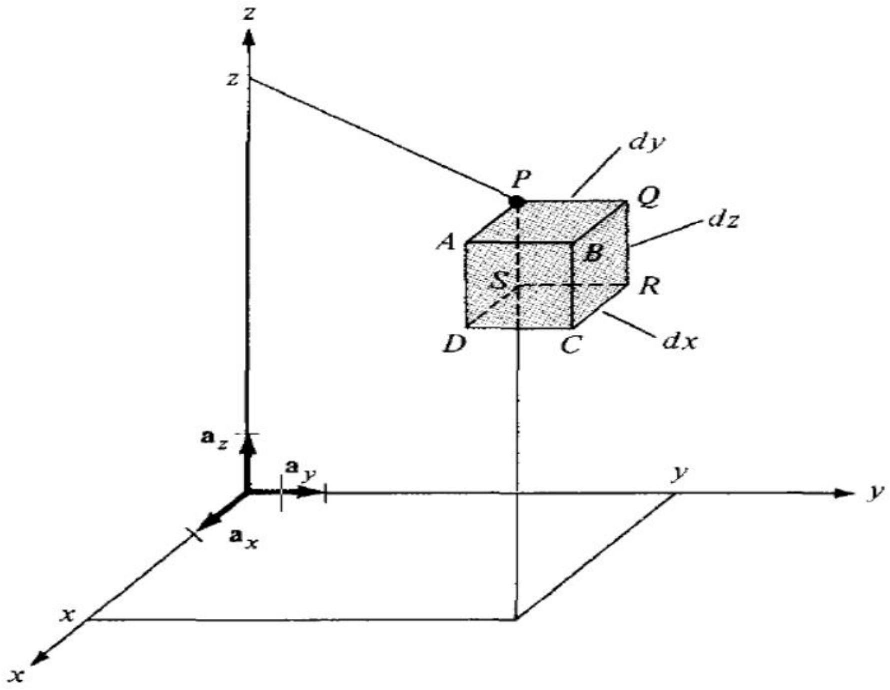
They are defined in the Cartesian,cylindrical, and spherical coordinate systems.

A**. Cartesian Coordinates**

From Figure 3.1, we notice that

(1) Differential displacement is given by

$$dl = dx\, a_x + dy\, a_y + dz\, a_z$$



Afigure 3.1 Differential elements in the right-handed Cartesian coordinate system.

CONT’D

(2) Differential normal area is given by

$dS = dy\, dz a_x$

$= dx dz a_y$

$= dz dy a_z$

(3) Differential volume is given by
$dv = dx\, dy\, dz$

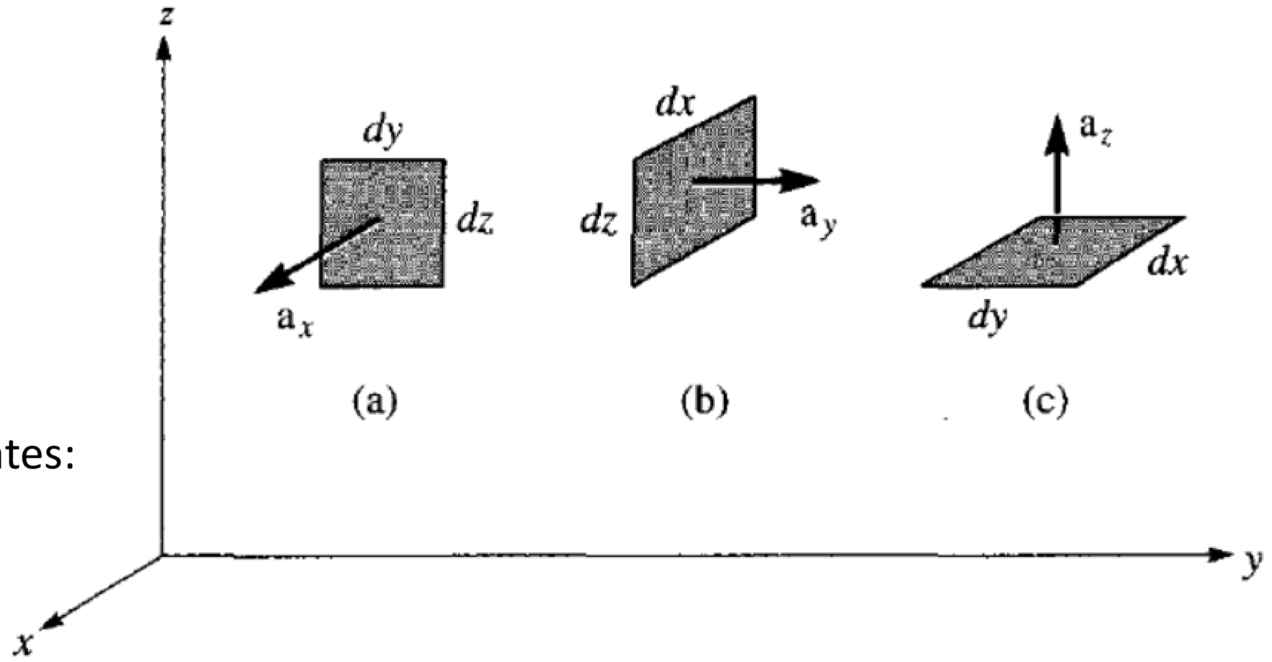


Figure 3.2 Differential normal areas in Cartesian coordinates:
(a) $dS = dy\, dz\, a_x$, (b) $dS = dxdz\, a_y$, (c) $dS = dx\, dy\, a_z$,

B. Cylindrical Coordinates

- From Figure 3.3 that in cylindrical coordinates, differential elements can be found as follows:

(1) Differential displacement is given by

dl = dpap + p dϕaϕ + dzaz

(2) Differential normal area is given by

$$d\mathbf{S} = \begin{matrix} \rho\, d\phi\, dz\, \mathbf{a}_\rho \\ d\rho\, dz\, \mathbf{a}_\phi \\ \rho\, d\phi\, d\rho\, \mathbf{a}_z \end{matrix}$$

and illustrated in Figure 3.4.

(3) Differential volume is given by

$$dv = \rho\, d\rho\, d\phi\, dz$$

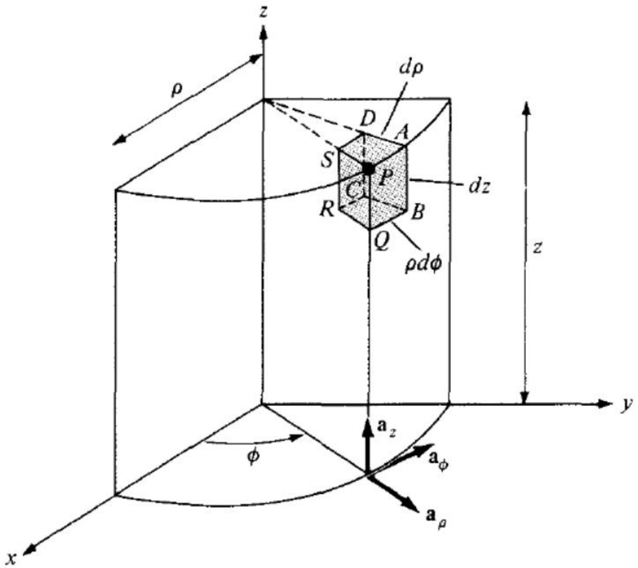


Figure 3.3 Differential elements in cylindrical coordinates

C. Spherical Coordinates

From Figure 3.5, we notice that in spherical coordinates,

(1) The differential displacement is

dl = drar + rdθaθ + r sin θ dφaφ

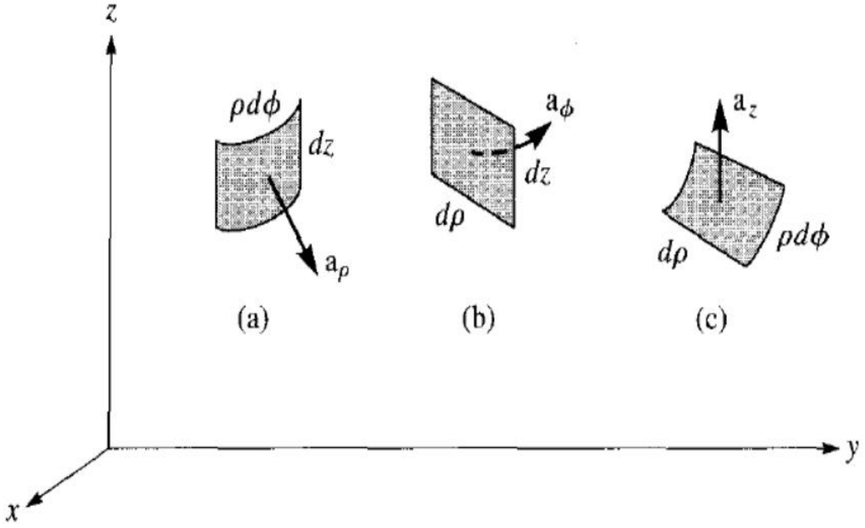


Figure 3.4 Differential normal areas in cylindrical coordinates: **(a)** $d\mathbf{S} = \rho\, d\phi\, dz\, \mathbf{a}_\rho$, **(b)** $d\mathbf{S} = d\rho\, dz\, \mathbf{a}_\phi$, **(c)** $d\mathbf{S} = \rho\, d\phi\, d\rho\, \mathbf{a}_z$

CONT'D

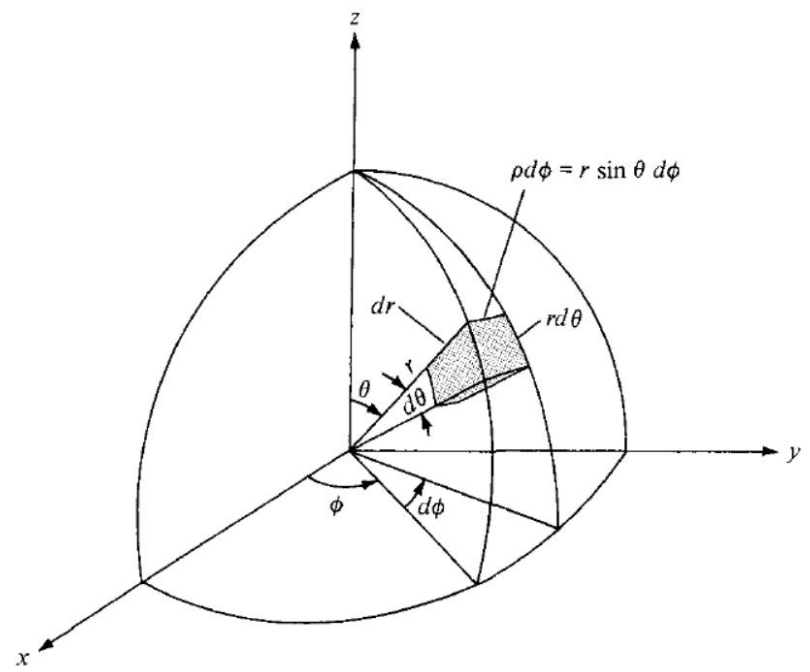


Figure 3.5 Differential elements in the spherical coordinate system.

(2) The differential normal area is

$$d\mathbf{S} = r^2 \sin\theta \, d\theta \, d\phi \, \mathbf{a}_r$$
$$r \sin\theta \, dr \, d\phi \, \mathbf{a}_\theta$$
$$r \, dr \, d\theta \, \mathbf{a}_\phi$$

CONT'D

(3) The differential volume is

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

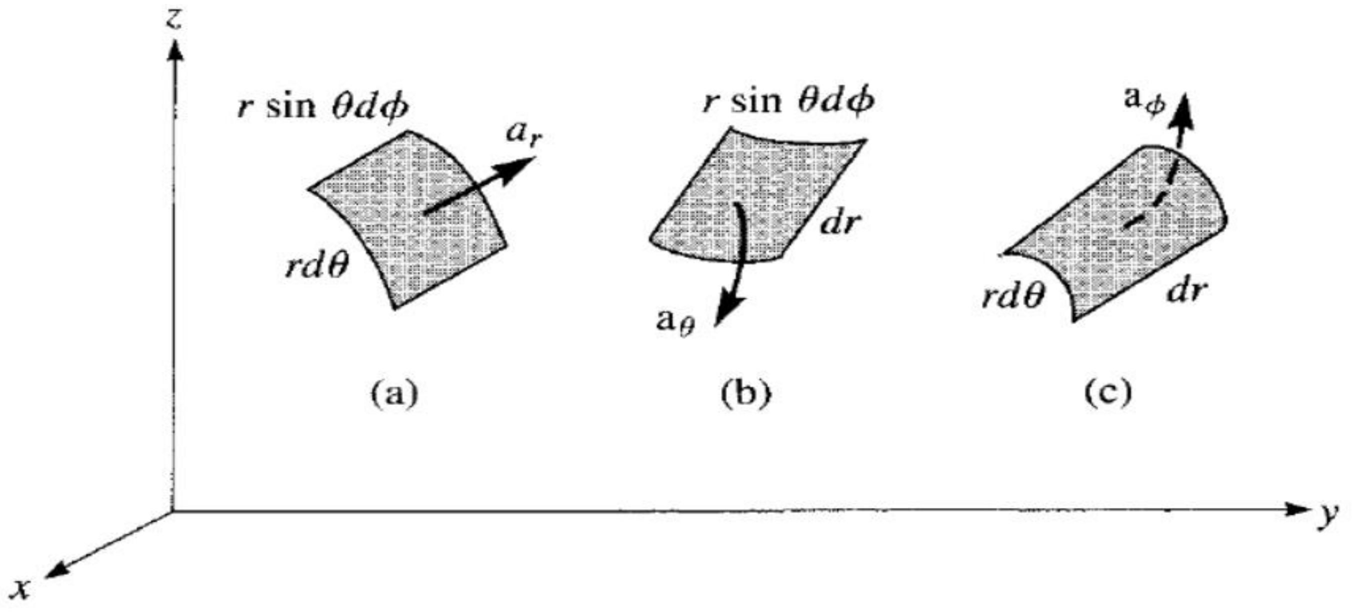


Figure 3.6 Differential normal areas in spherical coordinates: **(a)** $d\mathbf{S} = r^2 \sin\theta \, d\theta \, d\phi \, \mathbf{a}_r$, **(b)** $d\mathbf{S} = r \sin\theta \, dr \, d\phi \, \mathbf{a}_\phi$, **(c)** $d\mathbf{S} = r \, dr \, d\theta \, \mathbf{a}_\phi$

3.2 LINE, SURFACE, AND VOLUME INTEGRALS

- The line integral Is the integral of the tangential component of A along curve L.

$$\int_L \mathbf{A} \cdot d\mathbf{l}$$

- Given a vector field A and a curve L, we define the integral

$$\int_L \mathbf{A} \cdot d\mathbf{l} = \int_a^b |\mathbf{A}| \cos\theta \, dl$$

- If the path of integration is a closed curve such as abca .

$$\oint_L \mathbf{A} \cdot d\mathbf{l}$$

- which is called the circulation of A around L.

Given a vector field A, continuous in a region containing the smooth surface S, we
define the surface integral or the flux of A through S as

CONT'D

$$\Psi = \int_S |\mathbf{A}| \cos\theta \, dS = \int_S \mathbf{A} \cdot \mathbf{a}_n \, dS$$

or simply

$$\boxed{\Psi = \int_S \mathbf{A} \cdot d\mathbf{S}}$$

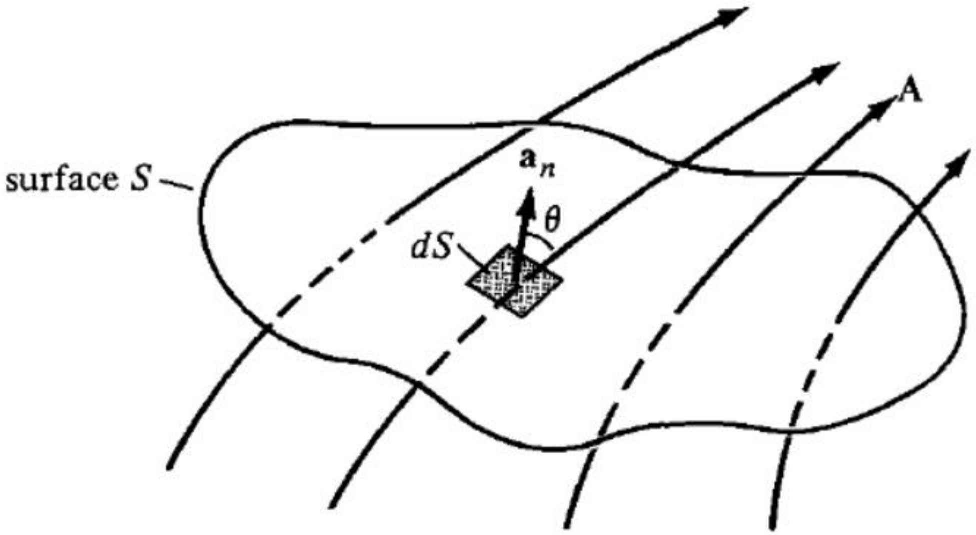


Figure 3.1 The flux of a vector field A through surface S.

CONT'D

- ❖ Notice that a closed path defines an open surface whereas a closed surface defines a volume .

We define the integral

$$\int_v \rho_v \, dv$$

- ❖ as the volume integral of the scalar pv over the volume v. The physical meaning of a line, surface, or volume integral depends on the nature of the physical quantity represented by A or pv.

3.3 DEL OPERATOR

- The del operator, written V, is the vector differential operator. In Cartesian coordinates,

$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$

- This vector differential operator, otherwise known as the gradient operator, is not a vector in itself, but when it operates on a scalar function, for example, a vector ensues. The operator is useful in defining

1. The gradient of a scalar V, written as ∇V
2. The divergence of a vector $\mathbf{A}$, written as $\nabla \cdot \mathbf{A}$
3. The curl of a vector $\mathbf{A}$, written as $\nabla \times \mathbf{A}$
4. The Laplacian of a scalar V, written as $\nabla^2 V$

CONT’D

To obtain ∇ in terms of ρ, ϕ, and z

$$\rho = \sqrt{x^2 + y^2}, \qquad \tan\phi = \frac{y}{x}$$

Hence

$$\frac{\partial}{\partial x} = \cos\phi \frac{\partial}{\partial \rho} - \frac{\sin\phi}{\rho}\frac{\partial}{\partial \phi}$$

$$\frac{\partial}{\partial y} = \sin\phi \frac{\partial}{\partial \rho} + \frac{\cos\phi}{\rho}\frac{\partial}{\partial \phi}$$

In cylindrical coordinates as

$$\nabla = \mathbf{a}_\rho \frac{\partial}{\partial \rho} + \mathbf{a}_\phi \frac{1}{\rho}\frac{\partial}{\partial \phi} + \mathbf{a}_z \frac{\partial}{\partial z}$$

CONT'D

Similarly, to obtain ∇ in terms of r, θ, and ϕ, we use

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad \tan\theta = \frac{\sqrt{x^2 + y^2}}{z}, \qquad \tan\phi = \frac{y}{x}$$

to obtain

$$\frac{\partial}{\partial x} = \sin\theta\cos\phi\frac{\partial}{\partial r} + \frac{\cos\theta\cos\phi}{r}\frac{\partial}{\partial\theta} - \frac{\sin\phi}{\rho}\frac{\partial}{\partial\phi}$$

$$\frac{\partial}{\partial y} = \sin\theta\sin\phi\frac{\partial}{\partial r} + \frac{\cos\theta\sin\phi}{r}\frac{\partial}{\partial\theta} + \frac{\cos\phi}{\rho}\frac{\partial}{\partial\phi}$$

$$\frac{\partial}{\partial z} = \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}$$

in spherical coordinates:

$$\nabla = \mathbf{a}_r\frac{\partial}{\partial r} + \mathbf{a}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \mathbf{a}_\phi\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$$

CONT'D

3.4 GRADIENT OF A SCALAR

- The gradient of a scalar field V is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.
- A mathematical expression for the gradient can be obtained by evaluating the difference in the field dV between points P_1 and $P_2$of Figure 3.2 where V1,, V2, and V3 are contours on which V is constant. From calculus,

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$
$$= \left(\frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z\right) \cdot (dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z)$$

For convenience, let

$$\mathbf{G} = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

CONT'D

Then $$dV = \mathbf{G} \cdot d\mathbf{l} = G \cos\theta \, dl$$

or $$\frac{dV}{dl} = G \cos\theta$$

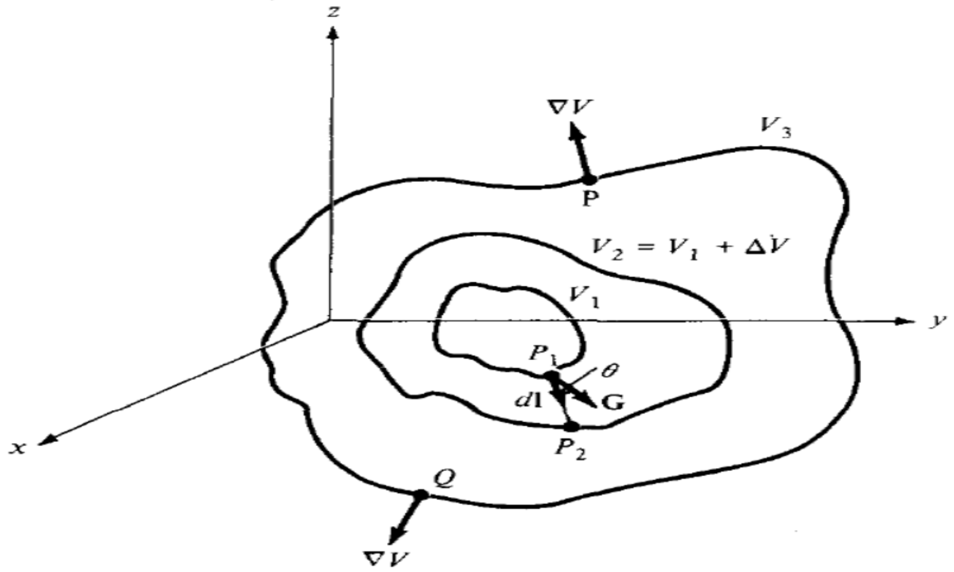


Figure 3.2 Gradient of a scalar.

CONT'D

- ❖ where dli s the differential displacement from P1, to P2 and θ is the angle between G and dl.
- ❖ NOTE that dV/dl is a maximum when θ = 0, that is, when dL is in the direction of G. Hence,

$$\left.\frac{dV}{dl}\right|_{max} = \frac{dV}{dn} = G$$

- ❖ where dV/dn is the normal derivative. Thus G has its magnitude and direction as those of the maximum rate of change of V. By definition, G is the gradient of V. Therefore:

$$\text{grad } V = \nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

CONT’D

- The gradient of V can be expressed in Cartesian, cylindrical, and spherical coordinates. For Cartesian coordinates

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

for cylindrical coordinates,

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

and for spherical coordinates,

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

CONT'D

- The following computation formulas on gradient, which are easily proved, should be noted:

$$\text{(a)}\quad \nabla(V + U) = \nabla V + \nabla U$$

$$\text{(b)}\quad \nabla(VU) = V\nabla U + U\nabla V$$

$$\text{(c)}\quad \nabla\left[\frac{V}{U}\right] = \frac{U\nabla V - V\nabla U}{U^2}$$

$$\text{(d)}\quad \nabla V^n = nV^{n-1}\nabla V$$

where U and V are scalars and n is an integer.

- Also take note of the following fundamental properties of the gradient of a scalar field V:

1. The magnitude of ∇V equals the maximum rate of change in V per unit distance.
2. ∇V points in the direction of the maximum rate of change in V.
3. ∇V at any point is perpendicular to the constant V surface that passes through that

3.5 DIVERGENCE OF A VECTOR AND DIVERGENCE THEOREM

- The divergence of A at a given point P is ihc outward Flux per unit volume as the volume shrinks about P.

Hence,

$$\operatorname{div} \mathbf{A} = \nabla \cdot \mathbf{A} = \lim_{\Delta v \to 0} \frac{\oint_S \mathbf{A} \cdot d\mathbf{S}}{\Delta v}$$

- Suppose we wish to evaluate the divergence of a vector field A at point P(xo,yo, zo); we let the point be enclosed by a differential volume as in Figure 3.3. The surface integral is obtained from

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \left(\int_{\text{front}} + \int_{\text{back}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{top}} + \int_{\text{bottom}} \right) \mathbf{A} \cdot d\mathbf{S}$$

CONT'D

- ❖ A three-dimensional Taylor series expansion of A_x about P is

$$A_x(x, y, z) = A_x(x_o, y_o, z_o) + (x - x_o)\left.\frac{\partial A_x}{\partial x}\right|_P + (y - y_o)\left.\frac{\partial A_x}{\partial y}\right|_P + (z - z_o)\left.\frac{\partial A_x}{\partial z}\right|_P + \text{higher-order terms}$$

For the front side, $x = x_o + dx/2$ and $d\mathbf{S} = dy\,dz\,\mathbf{a}_x$. Then,

$$\int_{\text{front}} \mathbf{A} \cdot d\mathbf{S} = dy\,dz\left[A_x(x_o, y_o, z_o) + \frac{dx}{2}\left.\frac{\partial A_x}{\partial x}\right|_P\right] + \text{higher-order terms}$$

For the back side, $x = x_0 - dx/2$, $d\mathbf{S} = dy\,dz(-\mathbf{a}_x)$. Then,

$$\int_{\text{back}} \mathbf{A} \cdot d\mathbf{S} = -dy\,dz\left[A_x(x_o, y_o, z_o) - \frac{dx}{2}\left.\frac{\partial A_x}{\partial x}\right|_P\right] + \text{higher-order terms}$$

CONT'D

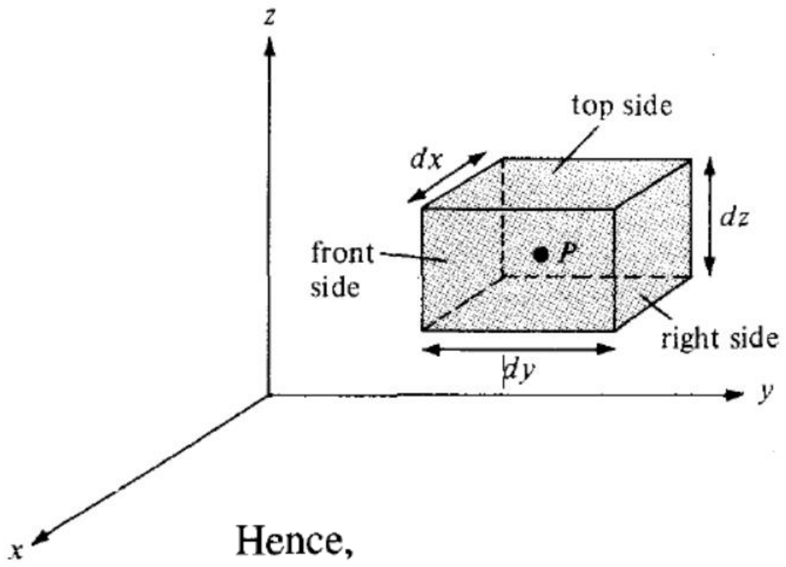


Figure 3.15 Evaluation of V • A at point P(x0, yo,Zo)

Hence,

$$\int_{\text{front}} \mathbf{A} \cdot d\mathbf{S} + \int_{\text{back}} \mathbf{A} \cdot d\mathbf{S} = dx\, dy\, dz \left.\frac{\partial A_x}{\partial x}\right|_P + \text{higher-order terms}$$

By taking similar steps, we obtain

$$\int_{\text{left}} \mathbf{A} \cdot d\mathbf{S} + \int_{\text{right}} \mathbf{A} \cdot d\mathbf{S} = dx\, dy\, dz \left.\frac{\partial A_y}{\partial y}\right|_P + \text{higher-order terms}$$

and

$$\int_{\text{top}} \mathbf{A} \cdot d\mathbf{S} + \int_{\text{bottom}} \mathbf{A} \cdot d\mathbf{S} = dx\, dy\, dz \left.\frac{\partial A_z}{\partial z}\right|_P + \text{higher-order terms}$$

CONT'D

- Note that Av = dx dy dz, we get

$$\lim_{\Delta v \to 0} \frac{\oint_S \mathbf{A} \cdot d\mathbf{S}}{\Delta v} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) \Bigg|_{\text{at } P}$$

- because the higher-order terms will vanish as Av —> 0. Thus, the divergence of A at point P(xo, yo, zo) in a Cartesian system is given by

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- In cylindrical coordinates,

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

CONT'D

❖ divergence of A in spherical coordinates as

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Note the following properties of the divergence of a vecto field:

1. It produces a scalar field (because scalar product is involved).
2. The divergence of a scalar V, div V, makes no sense.
3. $\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$
4. $\nabla \cdot (V\mathbf{A}) = V\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla V$

CONT’D

- ❖ The divergence theorem states that the total outward flux of a vector field A through the closed surface S is the same as the volume integral of the divergence of A.

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A} \, dv$$

3.6 CURL OF A VECTOR AND STOKES'S THEOREM

- ❖ The curl of A is an axial (or rotational) vector whose magnitude is the maximum circulation of A per unit area as the area lends to zero and whose direction is the normal direction of the area when the area is oriented so as to make the circulation maximum.''

CONT'D

That is

$$\text{curl}\,\mathbf{A} = \nabla \times \mathbf{A} = \left(\lim_{\Delta S \to 0} \frac{\oint_L \mathbf{A} \cdot d\mathbf{l}}{\Delta S} \right)_{\max} \mathbf{a}_n$$

In Cartesian coordinates the curl of A is easily found using

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right] \mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right] \mathbf{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right] \mathbf{a}_z$$

CONT'D

The curl of A in cylindrical coordinates as:

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\boxed{\begin{aligned} \nabla \times \mathbf{A} = &\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\mathbf{a}_y \\ &+ \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\mathbf{a}_z \end{aligned}}$$

and in spherical coordinates as

$$\nabla \times \mathbf{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \mathbf{a}_r & r\,\mathbf{a}_\theta & r\sin\theta\,\mathbf{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$\boxed{\begin{aligned} \nabla \times \mathbf{A} = &\frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi \sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_r \\ &+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\mathbf{a}_\phi \end{aligned}}$$

CONT’D

Stokes's theorem states that the circulation of a vector field A around a (closed) path L is equal lo the surface integral of the curl of A over the open surface S bounded by l provided that A and V X A are continuous on V.

$$\oint_L \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

3.7 LAPLACIAN OF A SCALAR

The Laplacian of a scalar field V, written as V2V. is the divergence of the gradient of V.

Thus, in Cartesian coordinates,

$$\text{Laplacian } V = \nabla \cdot \nabla V = \nabla^2 V$$

CONT’D

$$= \left[\frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z\right] \cdot \left[\frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z\right]$$

that is,

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}$$

Notice that the Laplacian of a scalar field is another scalar field.

The Laplacian of V in other coordinate systems can be obtained from by transformation. In cylindrical coordinates,

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}}$$

CONT'D
and in spherical coordinates,

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

3.8 CLASSIFICATION OF VECTOR FIELDS

A vector field A is said to be solenoidal (or divergenceless) if V • A = 0.

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\,dv = 0$$

CONT’D

A vector field A is said to be irrotational (or potential) if V X A =0.

$$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l} = 0$$

CHAPTER FOUR

ELECTROSTATICS

4.1 COULOMB'S LAW AND FIELD INTENSITY

Coulomb's law states that the force F between two point charges Q1 and *Q2* is:

1. Along the line joining them
2. Directly proportional to the product *Q1Q2* of the charges
3. Inversely proportional to the square of the distance *R* between them

Expressed mathematically,

$$F = \frac{k\,Q_1Q_2}{R^2}$$

CONT'D

- If point charges Q1 and Q2 are located at points having position vectors r1 and r2, then the force F12 on Q2 due to Q1,

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\,\mathbf{a}_{R_{12}}$$

where

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1$$

$$R = |\mathbf{R}_{12}|$$

$$\mathbf{a}_{R_{12}} = \frac{\mathbf{R}_{12}}{R}$$

$$\mathbf{F}_{12} = \frac{Q_1 Q_2 (\mathbf{r}_2 - \mathbf{r}_1)}{4\pi\varepsilon_o |\mathbf{r}_2 - \mathbf{r}_1|^3}$$

CONT’D

- The electric field intensity (or electric field strength) K is the force per unit charge when placed in the electric field.

Thus

$$\mathbf{E} = \lim_{Q\to 0} \frac{\mathbf{F}}{Q}$$

or simply

$$\boxed{\mathbf{E} = \frac{\mathbf{F}}{Q}}$$

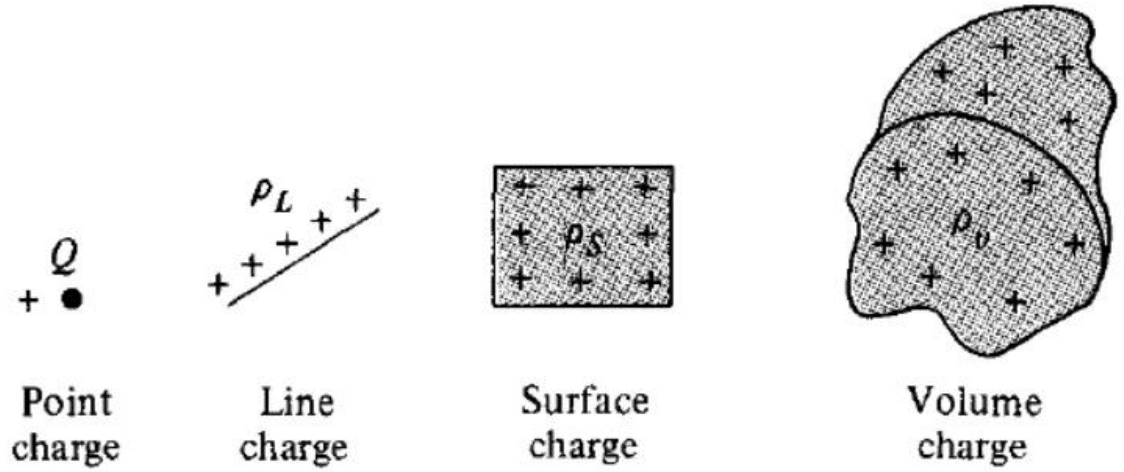
- The electric field intensity E is obviously in the direction of the force F and is measured in newtons/coulomb or volts/meter.

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o R^2}\,\mathbf{a}_R = \frac{Q(\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}'|^3}$$

4.2 ELECTRIC FIELDS DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

- ❖ So far we have only considered forces and electric fields due to point charges, which are essentially charges occupying very small physical space. It is also possible to have continuous charge distribution along a line, on a surface, or in a volume.
- ❖ The charge element dQ and the total charge Q due to these charge distributions are obtained from Figure 2.1 as

$$dQ = \rho_L \, dl \rightarrow Q = \int_L \rho_L \, dl \qquad \text{(line charge)}$$



$$dQ = \rho_S \, dS \rightarrow Q = \int_S \rho_S \, dS \qquad \text{(surface charge)}$$

$$dQ = \rho_v \, dv \rightarrow Q = \int \rho_v \, dv \qquad \text{(volume charge)}$$

Thus by replacing Q in the above eq. with charge element dq = pL dl, ps dS, or pv dv and integrating, we get

$$\mathbf{E} = \int \frac{\rho_L \, dl}{4\pi\varepsilon_o R^2} \mathbf{a}_R \qquad \text{(line charge)}$$

$$\mathbf{E} = \int \frac{\rho_S \, dS}{4\pi\varepsilon_o R^2} \mathbf{a}_R \qquad \text{(surface charge)}$$

$$\mathbf{E} = \int \frac{\rho_v \, dv}{4\pi\varepsilon_o R^2} \mathbf{a}_R \qquad \text{(volume charge)}$$

A. **Line Charge**

- Consider a line charge with uniform charge density pL extending from A to B along the z-axis as shown in Figure 2.2. The charge element dQ associated with element dl = dz of the line is

$$dQ = \rho_L \, dl = \rho_L \, dz$$

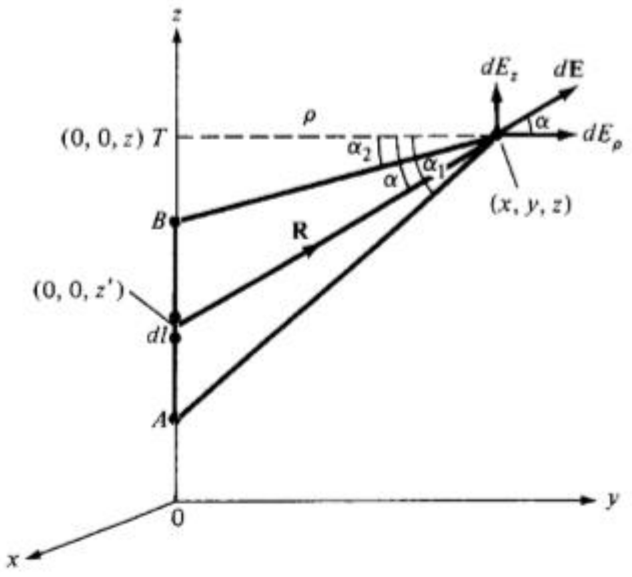


Figure 4.1Evaluation of the E field due to a line charge

Thus E=

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o\rho}\mathbf{a}_\rho$$

B. A Surface Charge

❖ Consider an infinite sheet of charge in the xy-plane with uniform charge density ρs. The charge associated with an elemental area dS is,

$$dQ = \rho_S \, dS$$

and hence the total charge is

$$Q = \int \rho_S \, dS$$

❖ the contribution to the E field at point P(0, 0, h) by the elemental surface shown in Figure 2.3 is

$$d\mathbf{E} = \frac{dQ}{4\pi\varepsilon_o R^2} \mathbf{a}_R$$

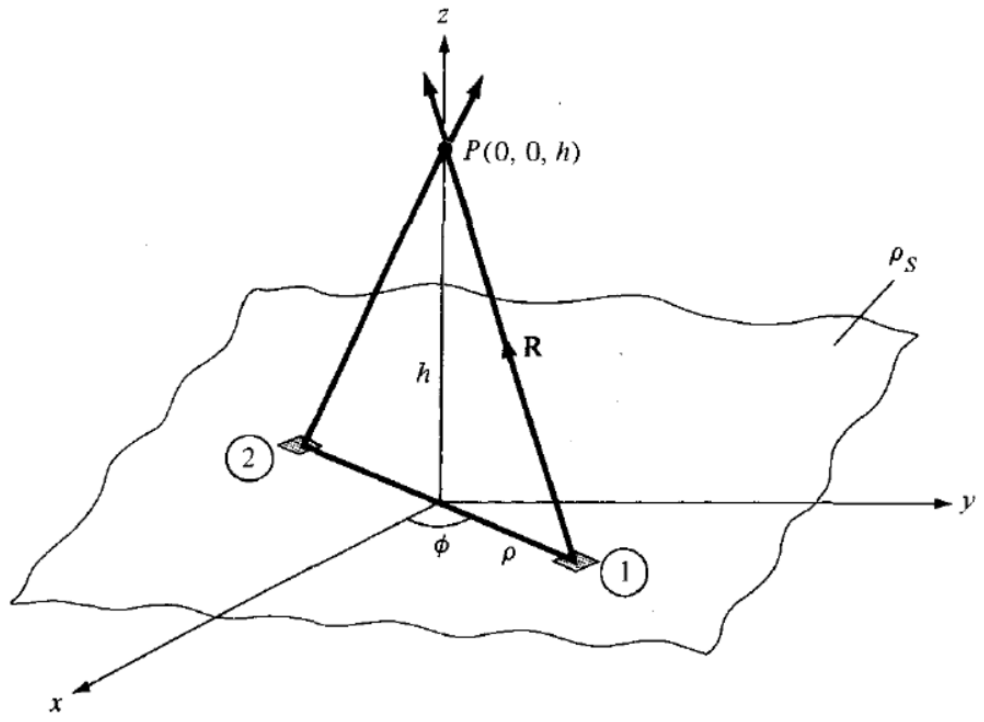


Figure 4.2 Evaluation of the E field due to an infinite sheet of charge.

CONT'D

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\,\mathbf{a}_z$$

- that is, E has only z-component if the charge is in the xy-plane. In general, for an infinite sheet of charge

$$\boxed{\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\,\mathbf{a}_n}$$

where an is a unit vector normal to the sheet.

C. A Volume Charge

Let the volume charge distribution with uniform charge density pv be as shown in Figure 2.4. The charge dQ associated with the elemental volume dv is

$$dQ = \rho_v\, dv$$

CONT’D

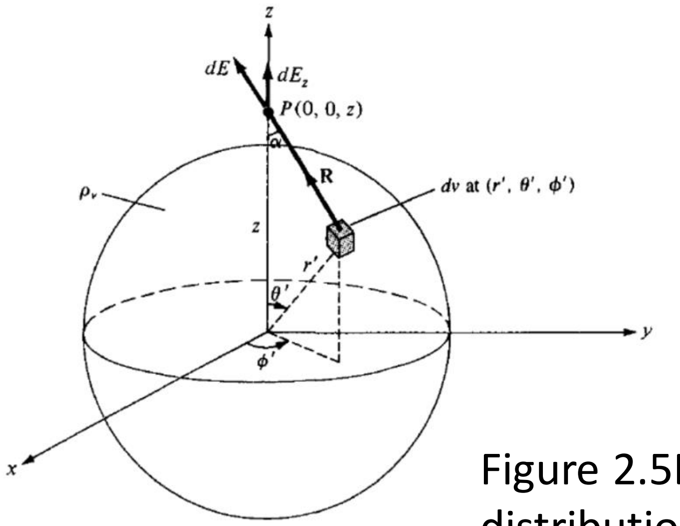


Figure 2.5Evaluation of the E field due to a volume charge distribution.

and hence the total charge in a sphere of radius a is

$$Q = \int \rho_v \, dv = \rho_v \int dv = \rho_v \frac{4\pi a^3}{3}$$

- The electric field dE at P(0, 0, z) due to the elementary volume charge is

$$d\mathbf{E} = \frac{\rho_v \, dv}{4\pi\varepsilon_o R^2} \mathbf{a}_R$$

4.3 ELECTRIC FLUX DENSITY

- A vector field D independent of the medium ψis defined by

$$\boxed{\mathbf{D} = \varepsilon_o \mathbf{E}}$$

- We define electric flux f in terms of D using

$$\Psi = \int \mathbf{D} \cdot d\mathbf{S}$$

and for a volume charge distribution

$$\mathbf{D} = \int \frac{\rho_v \, dv}{4\pi R^2} \mathbf{a}_R$$

4.4 GAUSS'S LAW—MAXWELL'S EQUATION

- Gauss's law states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface.

Thus

$$\Psi = Q_{enc}$$

that is,

$$\Psi = \oint d\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

$$= \text{Total charge enclosed } Q = \int \rho_v \, dv$$

or

$$\boxed{Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv}$$

CONT’D

By applying divergence theorem to the middle term,

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{D}\, dv$$

Comparing the two volume integrals

$$\boxed{\rho_v = \nabla \cdot \mathbf{D}}$$

which is the first of the four Maxwell's equations to be derived.

4.4 **ELECTRIC POTENTIAL**

- The electric field intensity E due to a charge distribution can be obtained from Coulomb's law in general or from Gauss's law when the charge distribution is symmetric.
- Another way of obtaining E is from the electric scalar potential V to be defined in this section. In a sense, this way of obtaining E is easier because it is easier to handle scalars than vectors.
- Suppose we wish to move a point charge Q from point A to point B in an electric field E as shown in Figure 2.6. From Coulomb's law, the force on Q is F = QE so that the work done in displacing the charge by dl is

$$dW = -\mathbf{F} \cdot d\mathbf{l} = -Q\mathbf{E} \cdot d\mathbf{l}$$

- The negative sign indicates that the work is being done by an external agent. Thus the total work done, or the potential energy required, in moving Q from A to B is

$$\boxed{W = -Q \int_{A}^{B} \mathbf{E} \cdot d\mathbf{l}}$$

CONT’D

Gives the potential energy per unit charge. This quantity, denoted by V_{AB}, is known as the potential difference between points A and B. Thus

$$V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$$

2. 5 RELATIONSHIP BETWEEN E AND V—MAXWELL'S EQUATION

- The potential difference between points A and B is independent of the path taken. Hence,

$$V_{BA} = -V_{AB}$$

CONT'D

that is,

$$V_{BA} + V_{AB} = \oint \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{l} = 0}$$

- This shows that the line integral of E along a closed path as shown in Figure 2.6 must be zero. Physically, this implies that no net work is done in moving a charge along a closed path in an electrostatic field.

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = 0$$

or

$$\boxed{\nabla \times \mathbf{E} = 0}$$

CONT'D

From the way we defined potential, $V = -\int \mathbf{E} \cdot d\mathbf{l},$

it follows that $dV = -\mathbf{E} \cdot d\mathbf{l} = -E_x\,dx - E_y\,dy - E_z\,dz$

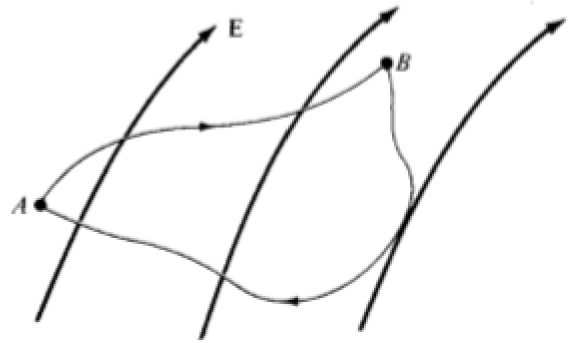


Figure 4.3 Conservative nature of an electrostatic field

But

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

Comparing the two expressions for dV, we obtain

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}$$

Thus:

$$\boxed{\mathbf{E} = -\nabla V}$$

CONT'D

- That is, the electric field intensity is the gradient of V. The negative sign shows that the direction of E is opposite to the direction in which V increases; E is directed from higher to lower levels of V.

4.5 AN ELECTRIC DIPOLE AND FLUX LINES

- An electric dipole is formed when two poim charges of equal magnitude but opposite Sign are separated by a small distance.

$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q}{4\pi\varepsilon_o}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

CONT’D

where r1, and r2 are the distances between P and +Q and P and -Q, respectively If r » d,r2- r1, = d cos θ, r2rx1- r2,

$$V = \frac{Q}{4\pi\varepsilon_o}\frac{d\cos\theta}{r^2}$$

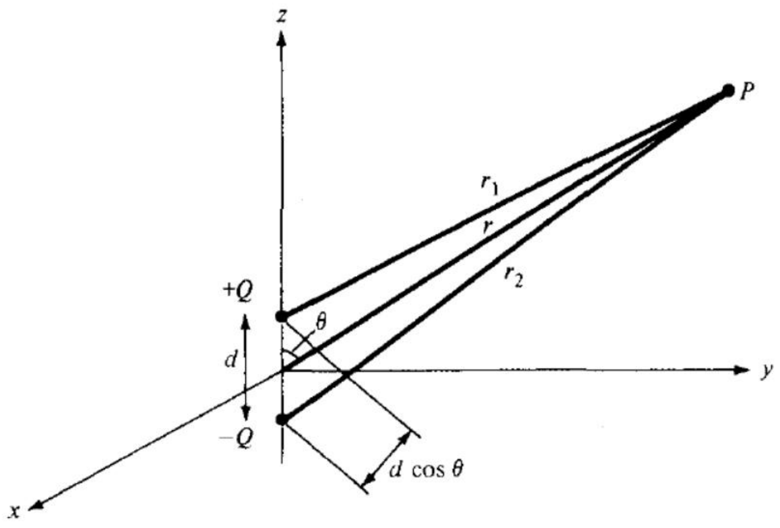


Figure 4.4 An electric dipole.

Since d cos θ = d • ar, where d = daz, if we define

$$\mathbf{p} = Q\mathbf{d}$$

as the dipole moment, may be written as

$$V = \frac{\mathbf{p}\cdot\mathbf{a}_r}{4\pi\varepsilon_o r^2}$$

CONT’D

- Note that the dipole moment p is directed from — Q to +Q.

  If the dipole center is not at the origin but at r', becomes

$$V(\mathbf{r}) = \frac{\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o |\mathbf{r} - \mathbf{r}'|^3}$$

- The electric field due to the dipole with center at the origin, can be obtained readily as

$$\mathbf{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta\right]$$

$$= \frac{Qd\cos\theta}{2\pi\varepsilon_o r^3}\mathbf{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_o r^3}\mathbf{a}_\theta$$

- An electric flux line is an imaginary path or line drawn in such a way that its direction at any point is the direction of the electric field at that point.

CHAPTER FIVE

ELECTRIC FIELDS IN MATERIAL SPACE

- Just as electric fields can exist in free space, they can exist in material media. Materials are broadly classified in terms of their electrical properties as conductors and nonconductors.
- Nonconducting materials are usually referred to as insulators or dielectrics.

5.1 CONVECTION AND CONDUCTION CURRENTS

- The current (in amperes) through a given area is the electric charge passing through the area per unit time.
- Thus in a current of one ampere, charge is being transferred at a rate of one columb per second. That is,

$$I = \frac{dQ}{dt}$$

CONT'D

- We now introduce the concept of current density J. If

  current ΔI flows through a surface ΔS, the current density is

$$J_n = \frac{\Delta I}{\Delta S}$$

$$\Delta I = J_n \Delta S$$

- assuming that the current density is perpendicular to the surface. If the current density is not normal to the surface,

$$\Delta I = \mathbf{J} \cdot \Delta S$$

Thus, the total current flowing through a surface S is

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

CONT’D

- If there is a flow of charge, of density pv, at velocity

u = ay**ay**, the current through the filament is.

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \, \Delta S \frac{\Delta \ell}{\Delta t} = \rho_v \, \Delta S \, u_y$$

- The current density at a given point is thc current through a unit normal area at that point.

5.2 CONDUCTORS

- A perfect conductor cannot contain an electrostatic field within it.
- A conductor is called an equipotential body, implying that the potential is the same everywhere in the conductor. This is based on the fact that

$$\mathbf{E} = -\nabla V = 0.$$

5.3 DIELECTRIC CONSTANT AND STRENGTH

$$\mathbf{D} = \varepsilon_o(1 + \chi_e)\,\mathbf{E} = \varepsilon_o\varepsilon_r\mathbf{E}$$

or

$$\boxed{\mathbf{D} = \varepsilon\mathbf{E}}$$

where

$$\boxed{\varepsilon = \varepsilon_o\varepsilon_r}$$

and

$$\boxed{\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_o}}$$

- ❖ The dielectric constant (or relative permittivity) Ɛr, is the ratio of the permittivity of the dielectric to that of free space.