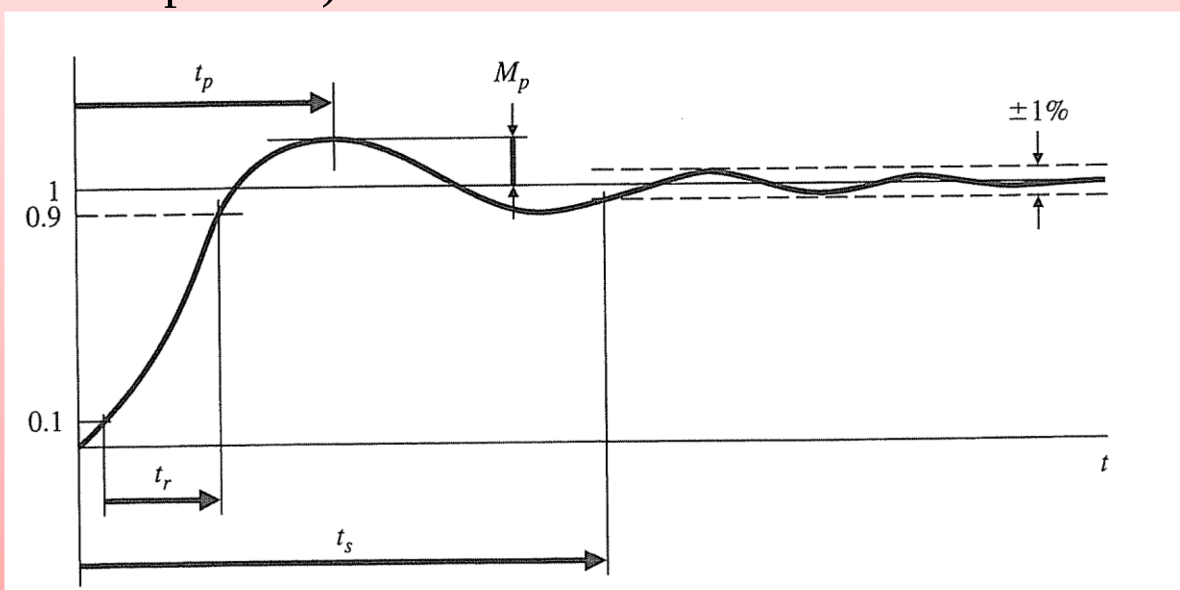
Digital Control Systems (DCS)

Lecture-4&5

Control specifications and digital control design techniques

Control specifications and digital control design techniques

- Control specifications
  - ✓ Rise time t_r Time to reach the vicinity of its new set point (90%)
  - ✓ Settling time t_s :Time for the decay of transient (inside 1% of steady state (final) value)
  - ✓ Overshoot M_p: maximum overshoot from the final vale (usually in percent)



$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

ζ is damping ratio
ω_n is the undamped natural frequency

Control specifications

- The rise time is $t_r \approx \dfrac{1.8}{\omega_n}$
- The settling time $t_s \approx \dfrac{4.6}{\zeta\omega_n} = \dfrac{4.6}{\sigma}$
- Maximum overshoot $M_p \approx e^{-\pi\zeta / \sqrt{1-\zeta^2}}$ $\quad 0 \le \zeta < 1$
- Note:
  - The requirement on the natural frequency is obtained from the rise time
  - The requirement on the magnitude of the real part of the pole is obtained from the settling time
  - The requirement the damping ratio is obtained from the overshoot

Control specifications

- ***Example 1.5***

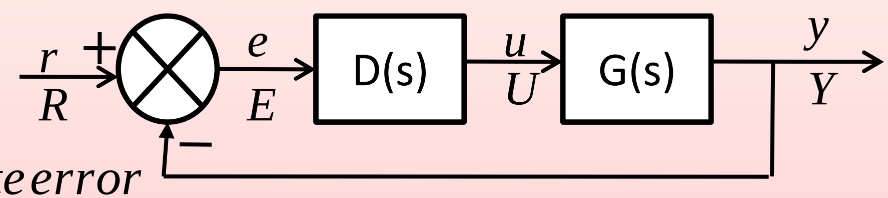
Find the acceptable pole region on s-plane of the following specifications

1. Over shoot less than 20% and settling time less than 3sec
2. Over shoot less than 10% , settling time less than 10sec and rise time less than 5sec

Specifications on steady state error

- For a system with unity feedback and forward transfer function *D(s)G(s)* shown below the error *e* becomes

$$E(s) = \frac{R(s)}{1 + D(s)G(s)}$$



From final value theorem the steady state error

$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1 + D(s)G(s)}R(s)$$

- Lets first define system type number.

Consider the following forward transfere function

$$D(s)G(s) = \frac{K\prod_{i=1}^{M}(s - z_i)}{s^N \prod_{k=1}^{Q}(s - p_k)}, \prod \text{ denotes the product, the zeros } z_i \neq 0 \text{ and poles } p_i \neq 0$$

N defines the type number of the system.

steady state error cont..

- IF N=0,its type zero system: If N=1,its type one system: If N=2,it's type two system and so on
- It is important to determine the steady state error for the three standard test inputs (step, ramp and acceleration) for the unity feedback system shown.
  - Step input: Step input of magnitude A is given as *R(s)=A/s*.So the steady state error becomes

$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+D(s)G(s)}R(s) = \lim_{s\to 0}\frac{A}{1+D(s)G(s)}$$

$$Define\ position\ constant\ as\ K_p = \lim_{s\to 0} D(s)G(s)$$

$$So \quad e_{ss} = \frac{A}{1+K_p}\ , \quad Where\ K_p = \lim_{s\to 0}\frac{K\prod_{i=1}^{M}(s-z_i)}{s^N\prod_{k=1}^{Q}(s-p_k)} = \infty \quad for\ N \geq 1$$

steady state error cont…

- Ramp input. A ramp (velocity) input of slope A is given as *r(t)=At* or *R(s)=A/s²*.So the steady state error becomes

$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+D(s)G(s)}R(s) = \lim_{s\to 0}\frac{A}{s+sD(s)G(s)} = \lim_{s\to 0}\frac{A}{sD(s)G(s)}$$

$$Define\, velocity\, constant\, as\;\; K_v = \lim_{s\to 0} sD(s)G(s)$$

$$So \quad e_{ss} = \frac{A}{K_v}\;,\;\; Where\, K_v = \lim_{s\to 0} sD(s)G(s) = \lim_{s\to 0}\frac{K\prod_{i=1}^{M}(s-z_i)}{s^{N-1}\prod_{k=1}^{Q}(s-p_k)} = \infty \;\; for\, N \ge 2.$$

$$And\; e_{ss} = 0 \;\; for\, N \ge 2.$$

$$Note: If\; the\, tf\;\, has\, more\, than\, one\, integrator, the\, error\, will\, be\, zero.$$

steady state error cont…

- Acceleration input: $r(t)=At^2/2$ or $R(s)=A/s^3$

$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+D(s)G(s)}R(s)$$

$$= \lim_{s\to 0} \frac{A}{s^2 + s^2 D(s)G(s)} = \lim_{s\to 0} \frac{A}{s^2 D(s)G(s)}$$

$$Define\,acceleration\,error\,cons\tan t\,as\;\; K_v = \lim_{s\to 0} s^2 D(s)G(s)$$

$$So \quad e_{ss} = \frac{A}{K_a}\;,\;\; Where\,K_a = \lim_{s\to 0} s^2 D(s)G(s) = \lim_{s\to 0} \frac{K\prod_{i=1}^{M}(s-z_i)}{s^{N-2}\prod_{k=1}^{Q}(s-p_k)} = \infty \;\; for\, N \ge 3.$$

$$And\; e_{ss} = 0\;\; for\, N \ge 3.$$

steady state error cont…

Number of integrators in D(s)G(s) or type number	Input		
	Step input $r(t)=A$ or $R(s)=A/s$	Ramp input $r(t)=At$ or $R(s)=A/s^2$	Acceleration input $r(t)=At^2/2$ or $R(s)=A/s^3$
0	$e_{ss}= A/(1+K_p)$	$e_{ss}= \infty$	$e_{ss}= \infty$
1	$e_{ss}=0$	$e_{ss}= A/K_v$	$e_{ss}= \infty$
2	$e_{ss}=0$	$e_{ss}= 0$	$e_{ss}= A/K_a$

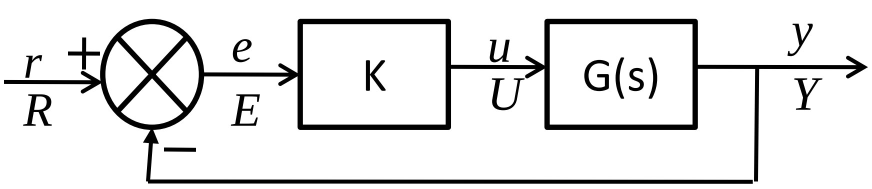
Example

- ***Example 1.6***

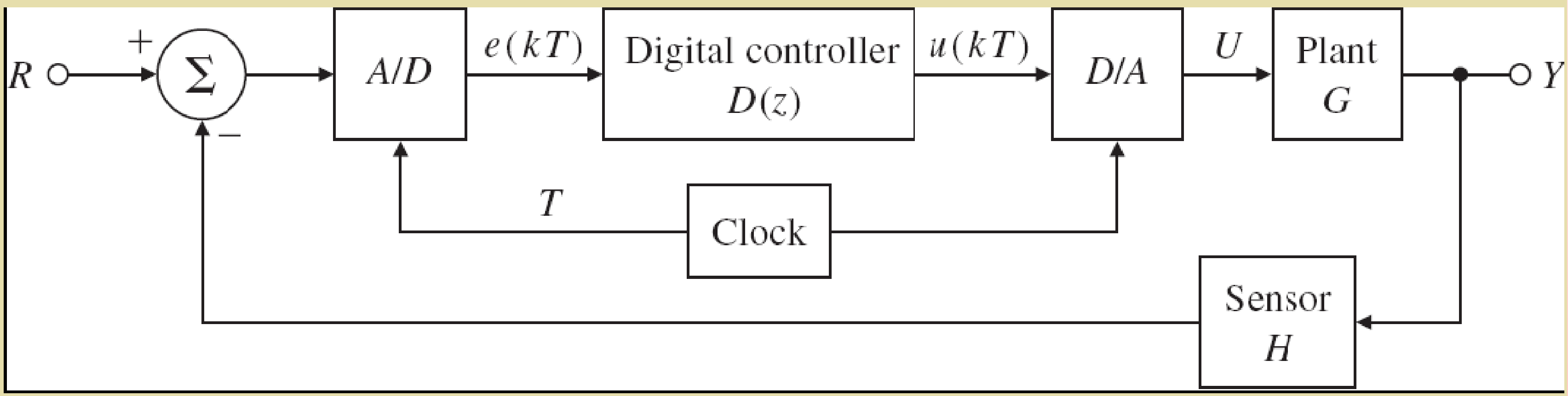
Find the value of K for the following unity feedback system to have

1. Unit step steady state error<2%
2. Unit ramp input steady state error<1%

$G(s)=(s+1)/(s^3+5s^2+6s)$



Digital controller design techniques

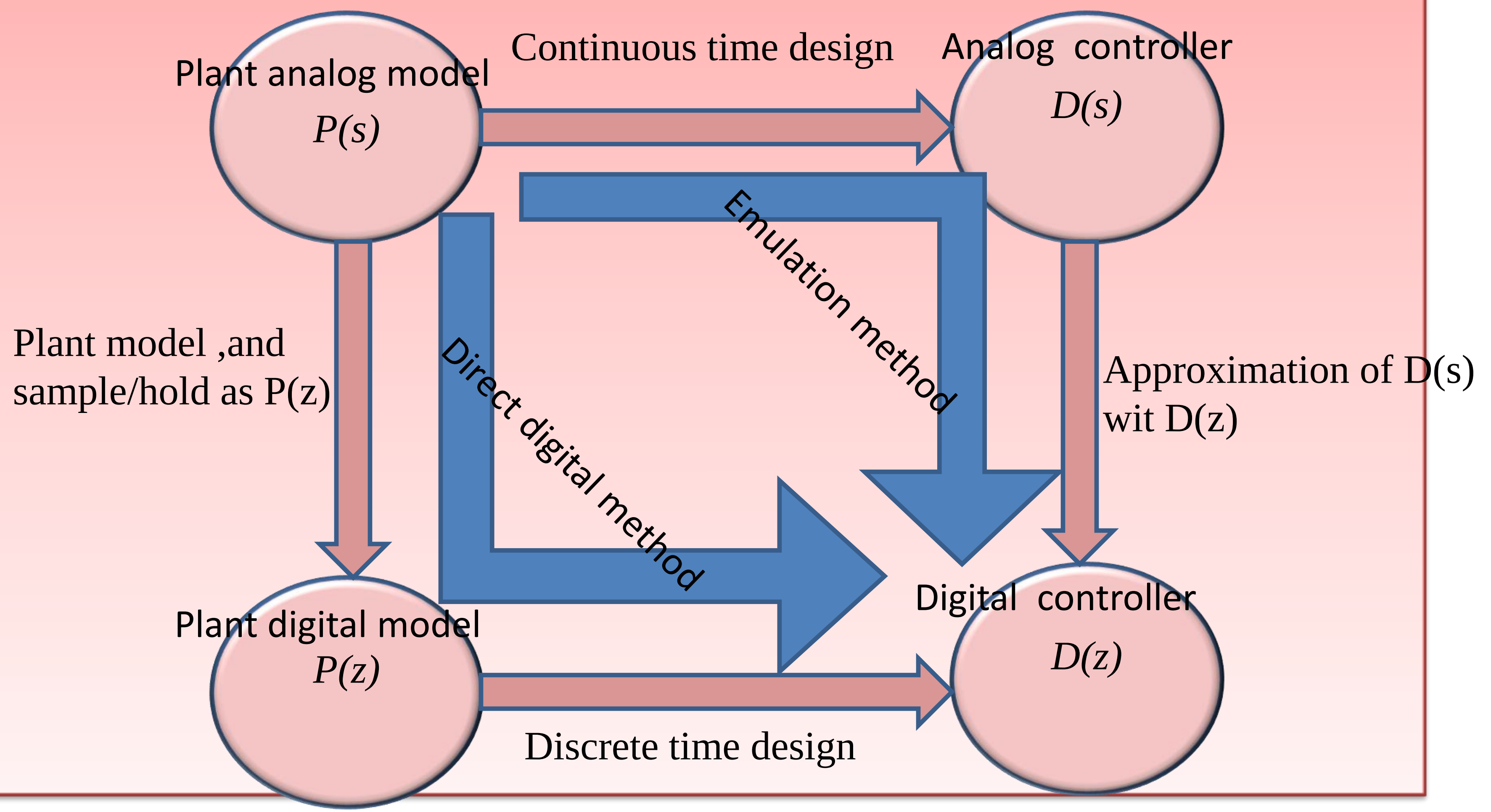


Block Diagram of a Digital Controller

Digital controller design techniques

- Two approaches may be used in the design of digital compensators.
- Emulation: an analog compensator may be designed and then converted by some approximation procedures to a digital compensator,
- Direct digital methods of designing digital compensators: as compared to the approximate methods of converting analog compensators to digital compensators.

Sampled data control system

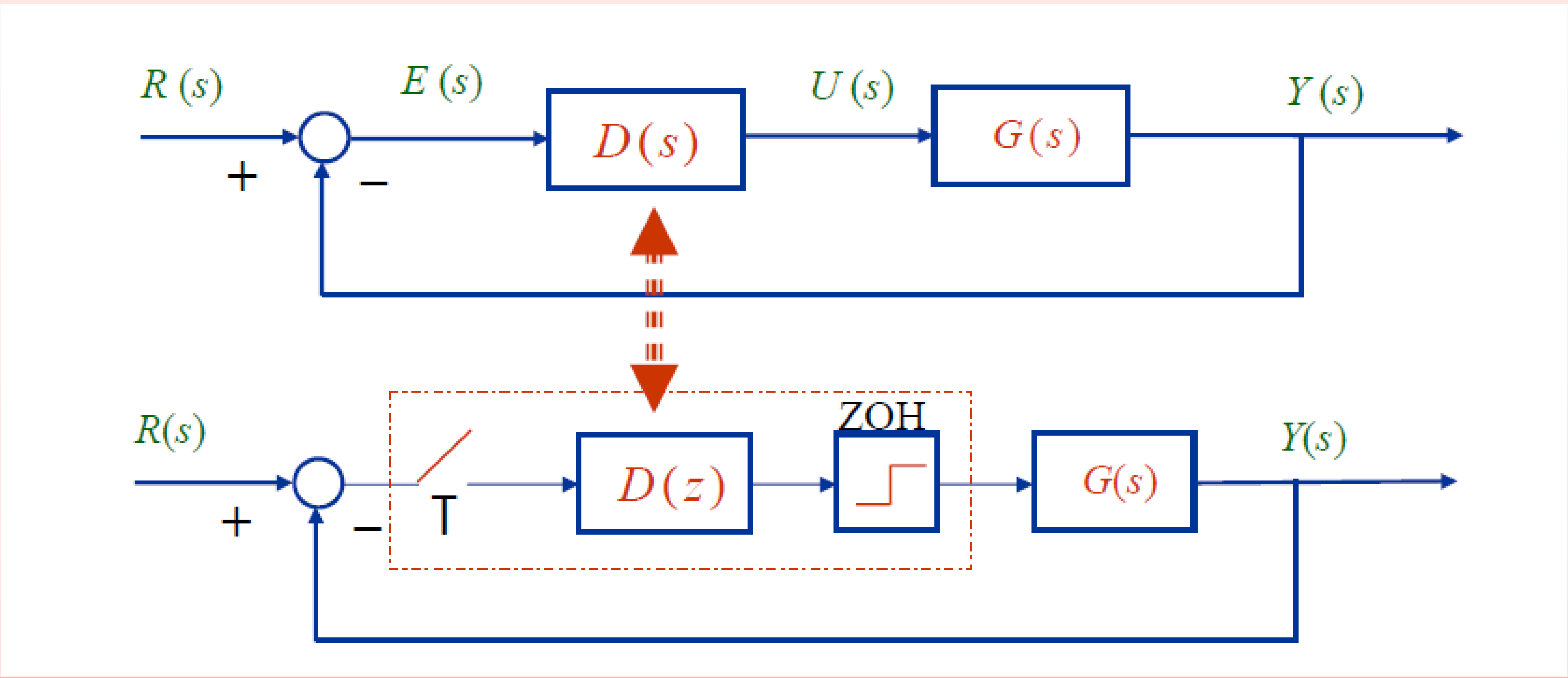


Emulation Design vs Direct Digital Control

- Emulation
  - ✓ Can use continuous time methods (well developed)
  - ✓ Few new tools needed
  - ✓ Works well if sampling fast
  - ✓ Mapping of control law from continuous time to discrete time is not exact
  - ✓ Ignore continuous system response between sampling times

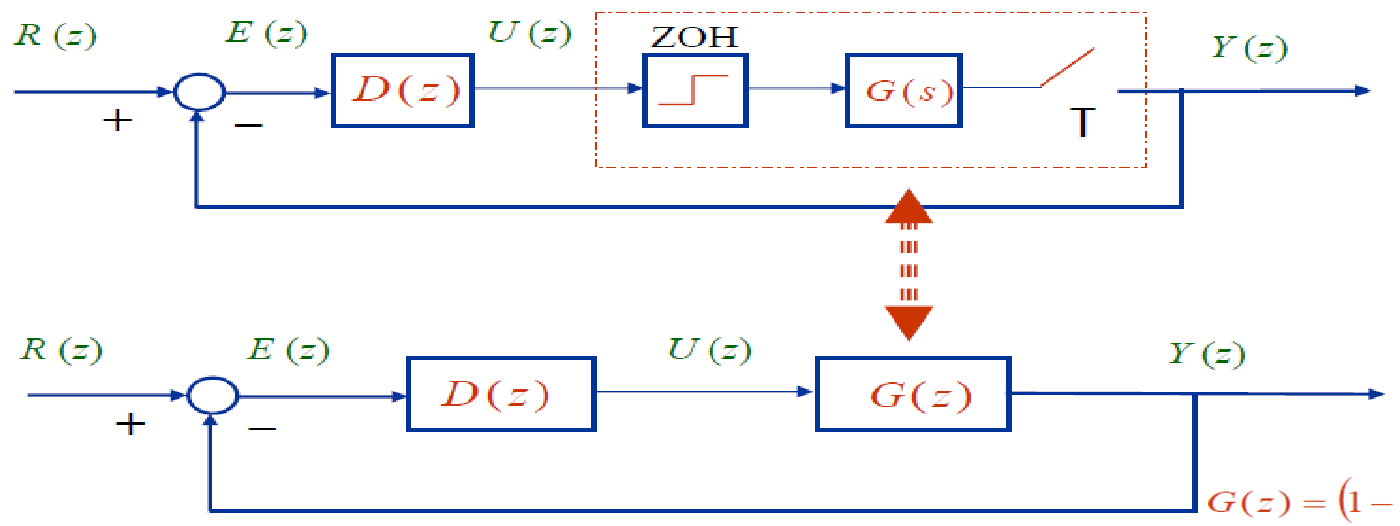
Sampled data control system

- Emulation



Sampled data control system

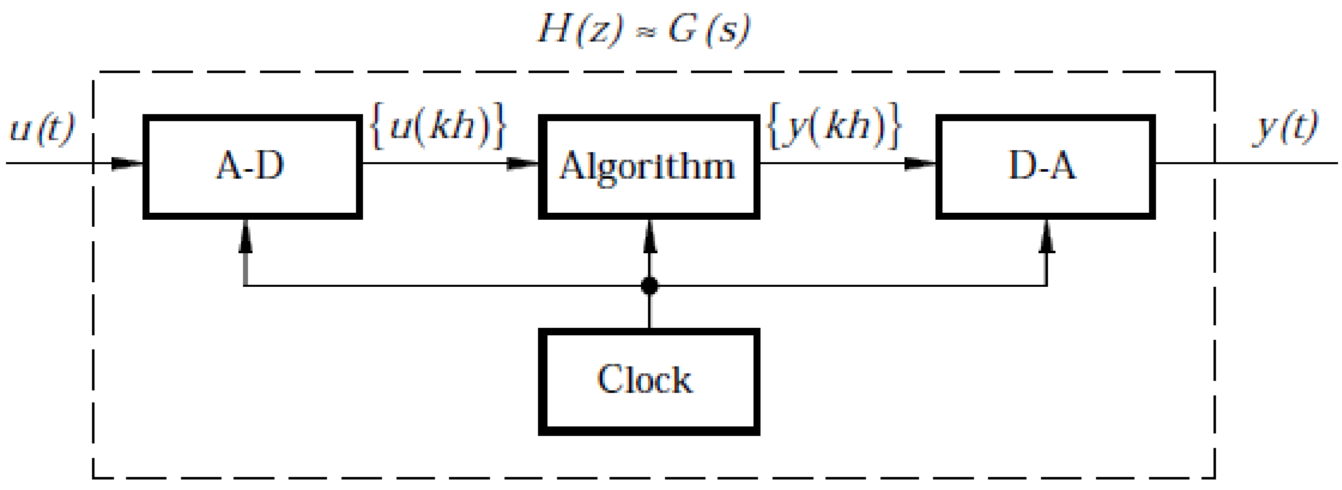
- Direct digital control
  - ✓Design of discrete time control law (and thus digital closed loop system) is exact for any sampling rate
  - ✓ Ignore continuous system response between sampling times



$$G(z) = (1 - z^{-1})\mathrm{Z}\left\{\frac{G(s)}{s}\right\}$$

Discretization of continuous-time controllers

- Basic idea: Reuse the analog design



- Want to get:
  - A/D + Algorithm + D/A≈ $G(s)$
- Methods:
  - Approximate s, i.e., $H(z) = G(s')$
  - Other discretization methods (Matlab)

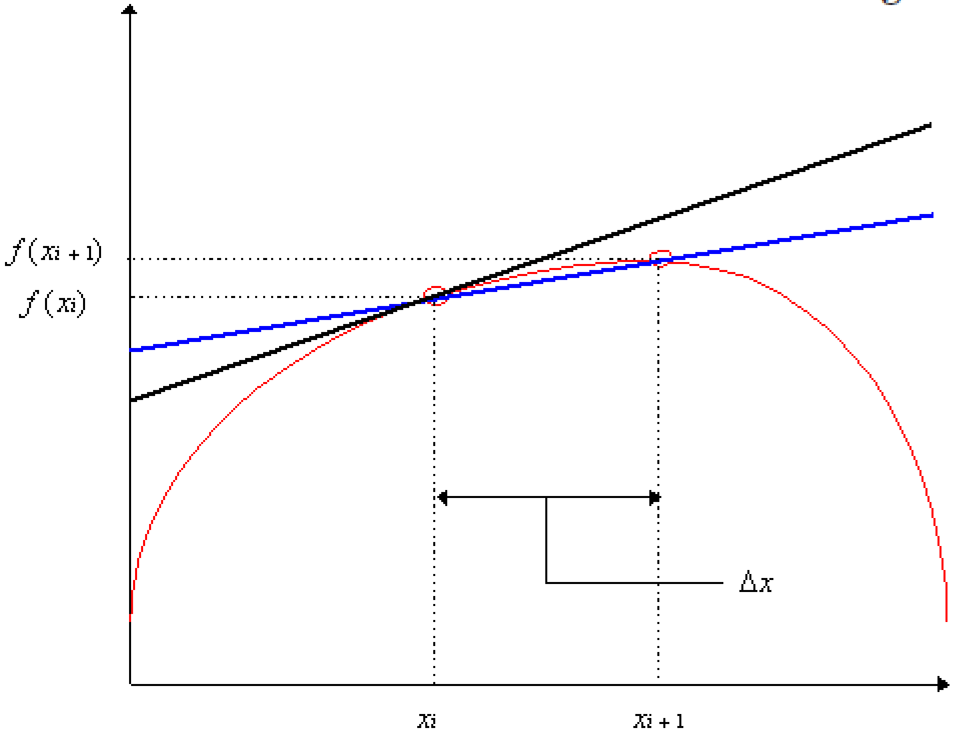
Discretization of continuous-time controllers

- Approximation Methods

Forward Difference (Euler's method):

$$\frac{dx(t)}{dt} \approx \frac{x(t_{k+1}) - x(t_k)}{h}$$

$$s' = \frac{z-1}{h}$$



$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{\Delta x}$$

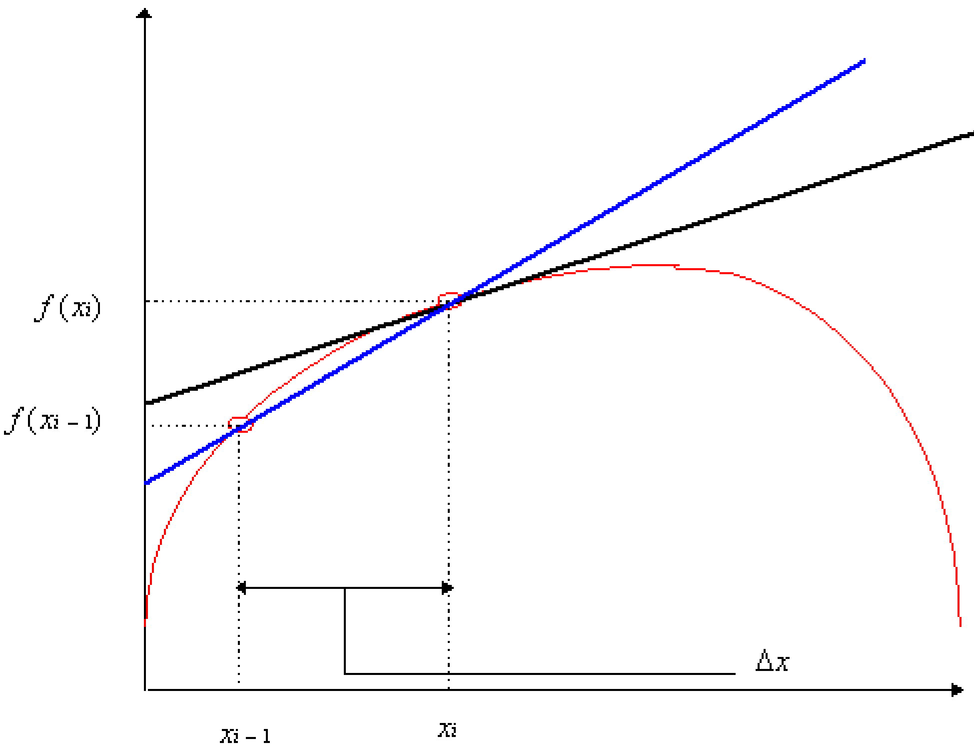
Discretization of continuous-time controllers

- Approximation Methods

Backward Difference:

$$\frac{dx(t)}{dt} \approx \frac{x(t_k) - x(t_{k-1})}{h}$$

$$s' = \frac{z-1}{zh}$$



$$f'(x_i) \cong \frac{f(x_i) - f(x_{i-1})}{\Delta x}$$

Discretization of continuous-time controllers

- Approximation Methods

Tustin:

$$\frac{\frac{dx(t)}{dt} + \frac{dx(t_{k+1})}{dt}}{2} \approx \frac{x(t_{k+1}) - x(t_k)}{h}$$

$$s' = \frac{2}{h}\,\frac{z-1}{z+1}$$

Example-1

- Using the three approximation methods to find the discrete-time equivalent of a lead compensator.

$$G(s) = \frac{10s + 1}{s + 1}$$

- Compare the approximation result by plotting the frequency response of the continuous-time controller and the discrete-time approximation for sampling periods T = 1, 0.5 and 0.1.

Example-1

- Solution: The approximations give the following relations:

Forward Difference (Euler's method): $s' = \frac{z-1}{h}$

Backward Difference: $s' = \frac{z-1}{zh}$

Tustin: $s' = \frac{2}{h}\frac{z-1}{z+1}$

- Using Euler's approximation method

$$G(z) = \frac{10(\frac{z-1}{T}) + 1}{\frac{z-1}{T} + 1} = \frac{10(z-1) + T}{Z - 1 + T}$$

Example-1

- Solution: The approximations give the following relations:

Forward Difference (Euler's method): $s' = \frac{z-1}{h}$

Backward Difference: $s' = \frac{z-1}{zh}$

Tustin: $s' = \frac{2}{h}\frac{z-1}{z+1}$

- Using Backward Difference approximation method

$$G(z) = \frac{10(\frac{z-1}{zT}) + 1}{\frac{z-1}{zT} + 1} = \frac{10(z-1) + zT}{Z - 1 + zT}$$

Example-1

- Solution: The approximations give the following relations:

Forward Difference (Euler's method): $s' = \frac{z-1}{h}$

Backward Difference: $s' = \frac{z-1}{zh}$

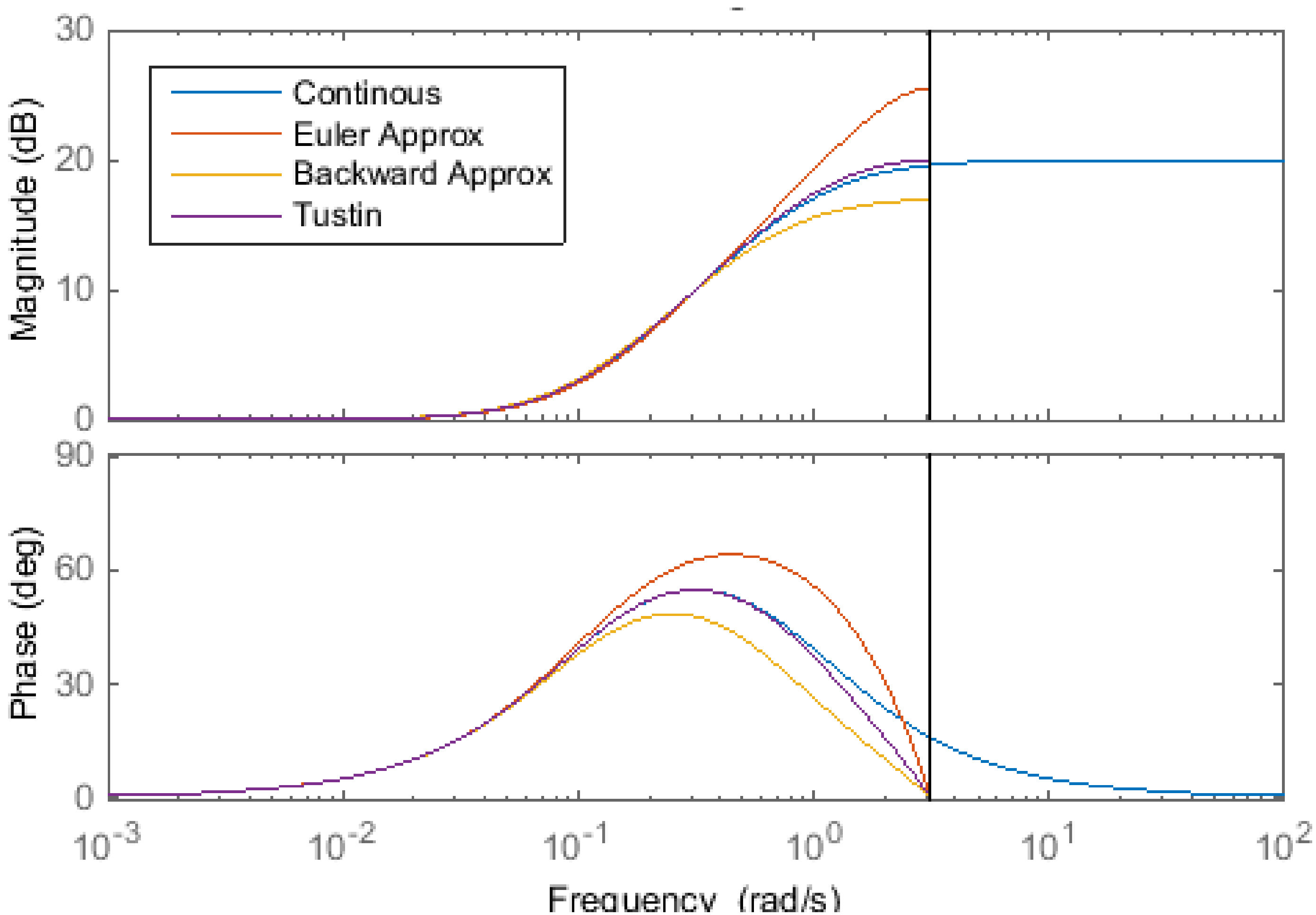
Tustin: $s' = \frac{2}{h}\frac{z-1}{z+1}$

- Using Tustin's approximation method

$$G(z) = \frac{10(\frac{2}{T}\frac{z-1}{z+1}) + 1}{\frac{2}{T}\frac{z-1}{z+1} + 1} = \frac{20(z-1) + T(z+1)}{2(Z-1) + T(z+1)}$$

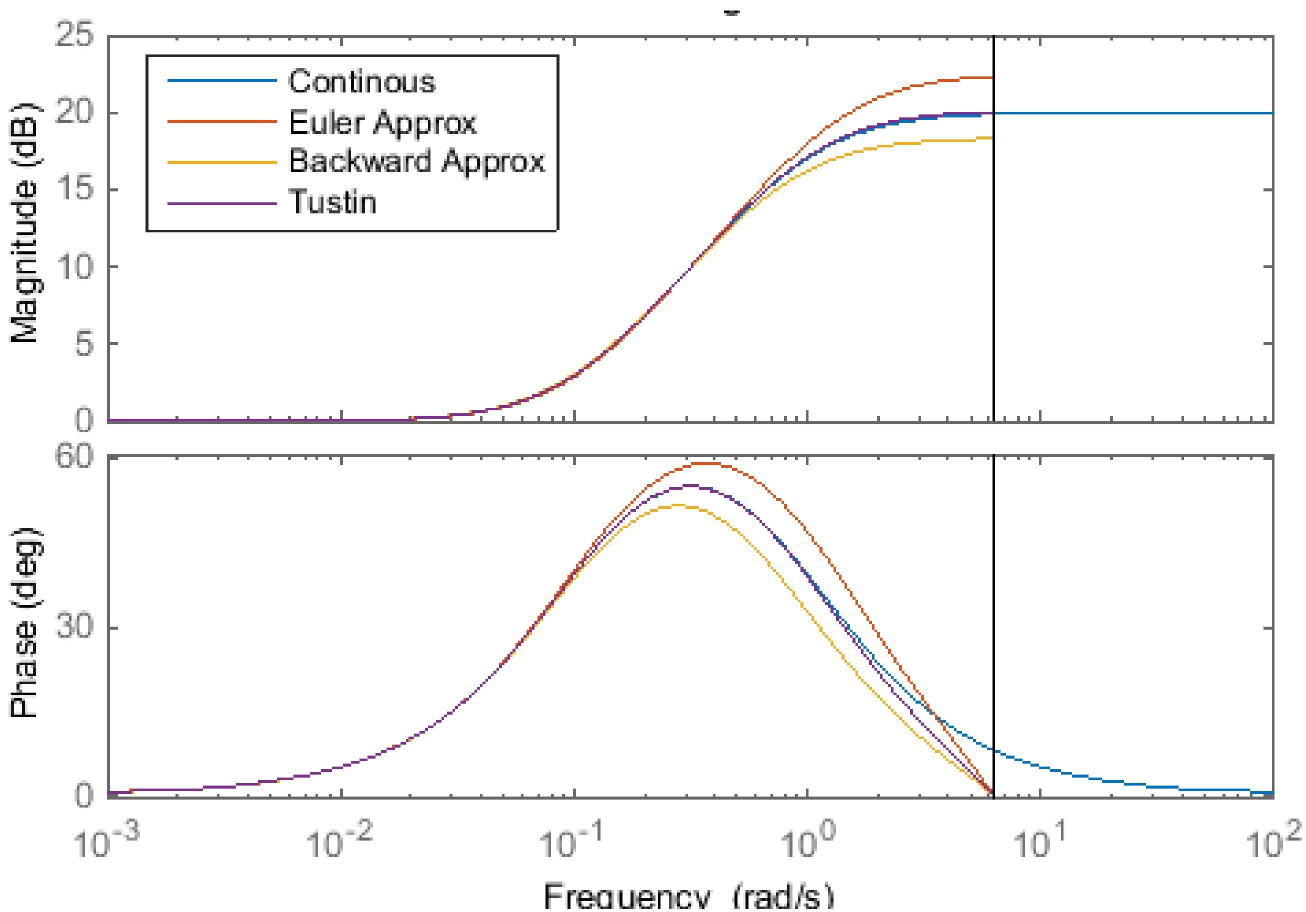
Example-1

- Frequency Response @ T=1



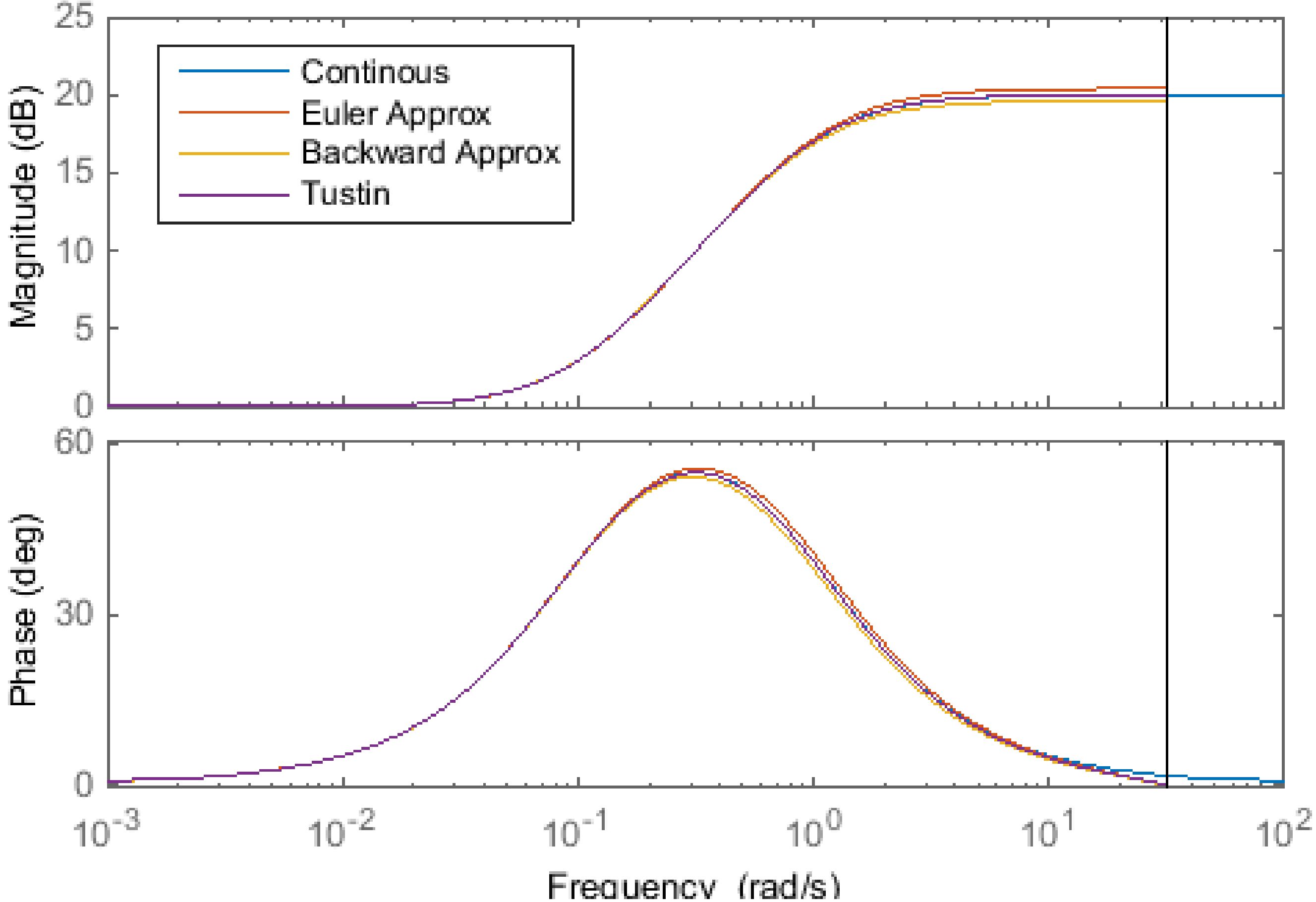
Example-1

- Frequency Response @ T=0.5



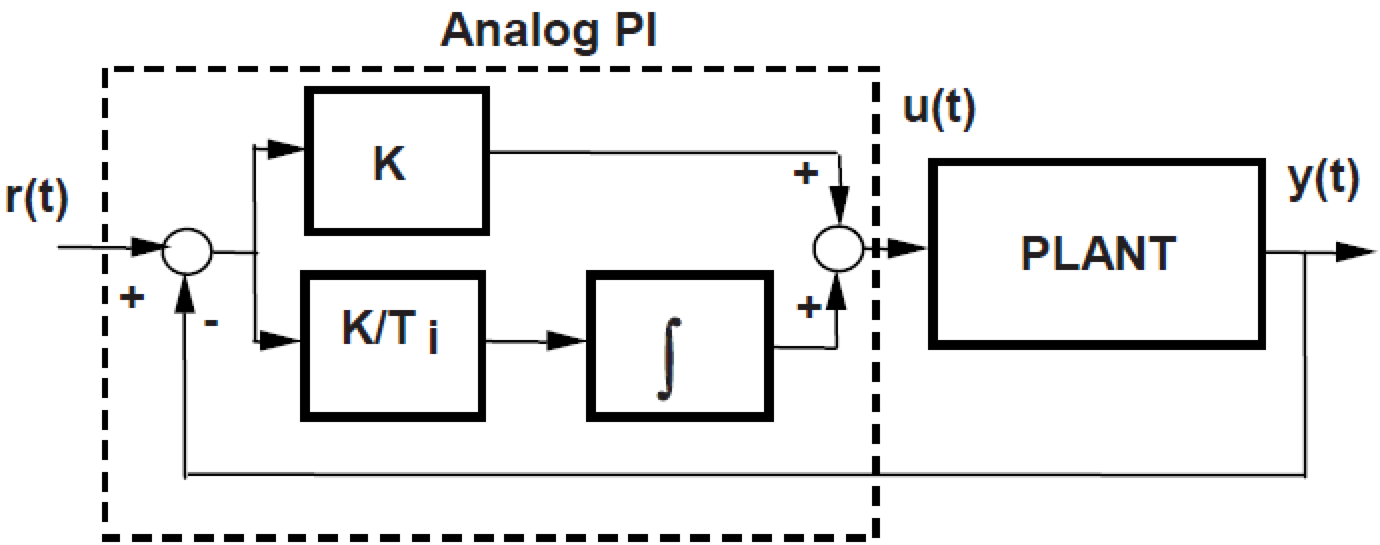
Example-1

- Frequency Response @ T=0.1



PI Controller

- Figure shows the diagram of a PI type analog controller.



- The controller contains two channels (a proportional channel and an integral channel) that process the error between the reference signal and the output.

Digital PI Controller

- Digital PI control law can even be obtained by the discretization of a PI analog controller.

- The control law for an analog PI controller is given by

$$C(s) = K\left[1 + \frac{1}{T_i s}\right]$$

- Using Tustin's Approximation method

$$i.e \quad s = \frac{2}{T}\frac{z-1}{z+1}$$

$$C(z) = K\left[1 + \frac{1}{T_i \frac{2}{T}\frac{z-1}{z+1}}\right]$$

Digital PID Controller

- Many practical control problems are solved by PID controllers or their variants.

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_{o}^{t} e(t)\, dt + T_d \frac{de(t)}{dt} \right]$$

- The continuous-time transfer function of a PID controller can be obtained by taking the Laplace transform of above eq

$$C_{pid}(s) = \frac{K_p(T_i T_D s^2 + T_i s + 1)}{T_i s}$$

- PID controller is non-causal and cannot, and should not, be implemented.

Digital PID Controller

$$C_{pid}(s) = \frac{K_p(T_i T_D s^2 + T_i s + 1)}{T_i s}$$

- The main reason is that the derivative term is non-causal and that it amplifies high frequency noise in the measured signals.
- Hence, the gain of the derivative action must be limited.
- This can be achieved by introducing an additional low-pass filter to the derivative action:

$$K_D s \approx \frac{K_D s}{\tau_L s + 1}$$

Digital PID Controller

$$K_D s \approx \frac{K_D s}{\tau_L s + 1} \qquad C_{pid}(s) = \frac{K_p(T_i T_D s^2 + T_i s + 1)}{T_i s}$$

- With the augmentation of a low pass filter, the modified continuous-time PID controller can be written as

$$C_{pid}(s) = \frac{K_p(T_i T_D s^2 + T_i s + 1)}{T_i s(\tau_L s + 1)}$$

- which introduced two zeros, a pole at the origin and another "fast" pole.

- Any of the previous approximation methods can be used to approximate the PID controller.