Chapter Seven

Frequency Response Analysis.

Introduction.

In the preceding chapters the response and performance of a system have been described as interms of the complex frequency Variables s and the location of the poles and zeros on the s-plane. A very practical and important alternative approach to the analysis and design of a system is the frequency response method.

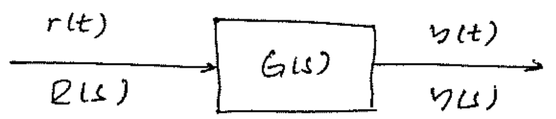
A frequency response of a system is defined as the steady-state response of the system to a sinusoidal input signal. The sinusoidal is unique input signal, and the resulting output signal for a linear system, as well as signals throughout the system, is sinusoidal in the steady state; it differs from the input wave form only in amplitude and phase angle.

Various standard test signals used to study the performance of control systems. (step, impulse ramp input)

The frequency response is one of the fundamental test mechanism in in describing the performance and spesfying design criterion.

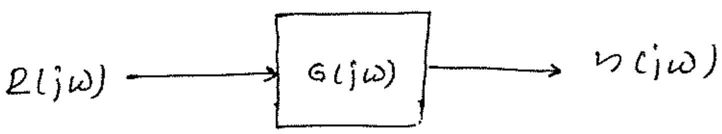
The frequency response test on a system or a component is normally performed by keeping the magnitude A fixed and determing B and ϕ for a suitable range of frequencies.

Let us consider the LTI system shown below



In linear-time-invariant system (LTI), the frequency response is independent of amplitude and phase of the input signal.

for $s = j\omega$



Let this input $r(t) = A\sin(\omega t)$, and then the output $y(t)$ can be given as

$$y(t) = B\sin(\omega t + \phi)$$

where $B = A|G(j\omega)|$ amplitude of the output.

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\text{imaginary part of } G(j\omega)}{\text{Real part of } G(j\omega)}\right]$$

7.2 Frequency Respons plots.

The frequency response is easily evaluated from the sinusoidal transfer function which can be obtained simply by replacing s with $j\omega$ in the system transfer function $G(s)$. The transfer function $G(j\omega)$ thus obtained, is a complex function of frequency and has both a magnitude and a phase angle. These characteristics are conveniently represented by graphical plots. The various graphical techniques are

1. Polar plot or Nyquiest plot
2. Bode plot or Logaritmic plot
3. ~~Magn~~ Log-magnitude versus phase plot.

7-2.1 Polar Plot. (also called Nyquiest plot)

The polar-plot of a sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity. Thus, the polar plot is the locus of vectors $|G(j\omega)| \angle G(j\omega)$ as ω varies from zero to infinity.

Note: in polar plots a positive (negative) phase angle is measured counter clockwise (clockwise) from the positive real axis.

Each point on the polar plot of $G(j\omega)$ represents the terminal point of a vector at a particular value of ω.

Example.

Draw the polar plot of for the transfer function of an RC-filter where $G(s) = \dfrac{1}{1+sT}$

Sol/n

Sinusoidal transfer function is given by

$$G(j\omega) = \frac{1}{1+j\omega T} \quad \text{for } s = j\omega$$

The sinusoidal transfer function can be represented by a magnitude $|G(j\omega)|$ and a phase angle of $\phi(j\omega)$ $\quad G(j\omega) = \dfrac{1}{1+j\omega T} * \dfrac{1-j\omega T}{1-j\omega T}$

1. Magnitude of $G(j\omega)$

$$|G(j\omega)| = \frac{1}{\sqrt{1+(\omega T)^2}}$$

Define $G(j\omega) = x + jy$

$$= \frac{1}{1+\omega^2T^2} - j\frac{\omega T}{1+\omega^2 T^2}$$

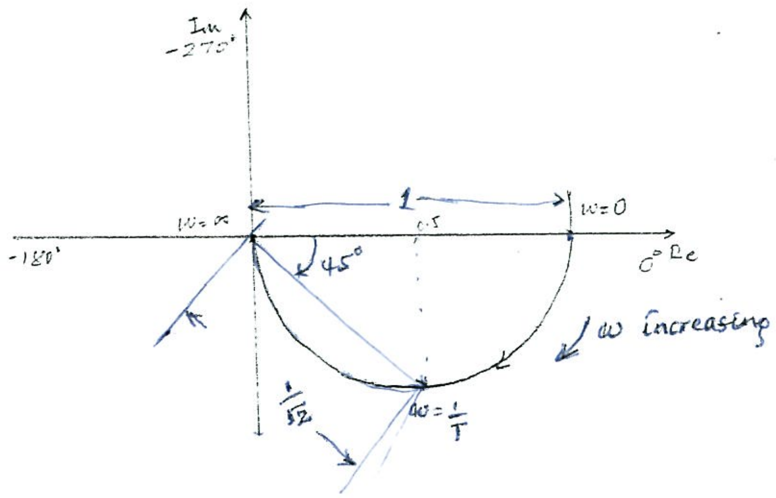
2. Phase angle $= \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}(-\omega T)$

ω	vector $\lvert G(j\omega)\rvert$	$\phi = \angle G(j\omega)$
0	1	0°
$\frac{1}{T}$	$\frac{\sqrt{2}}{2}$ $\frac{1}{\sqrt{2}}$	-45°
∞	0	-90

$(X-\frac{1}{2})^2 + Y^2$

$= \left(\frac{1}{2}\,\frac{1-\omega^2T^2}{1+\omega^2T^2}\right)^2 + \left(\frac{-\omega T}{1+\omega^2T^2}\right)^2 = \left(\frac{1}{2}\right)^2$

The polar plot is a semi circle with center located at 0.5 on the real axis and radius of 0.5



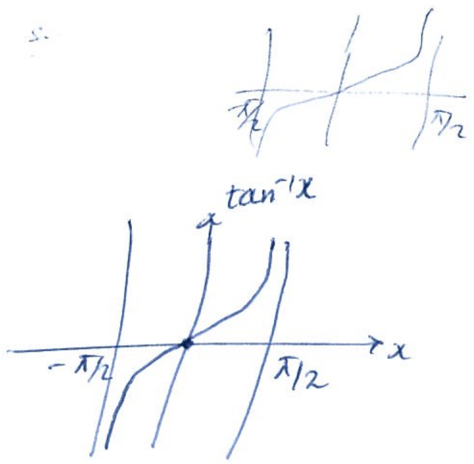
Example 2.

Consider the transfer function

$$G(s) = \frac{1}{s(1+sT)}$$

sinusoidal trans. function

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$



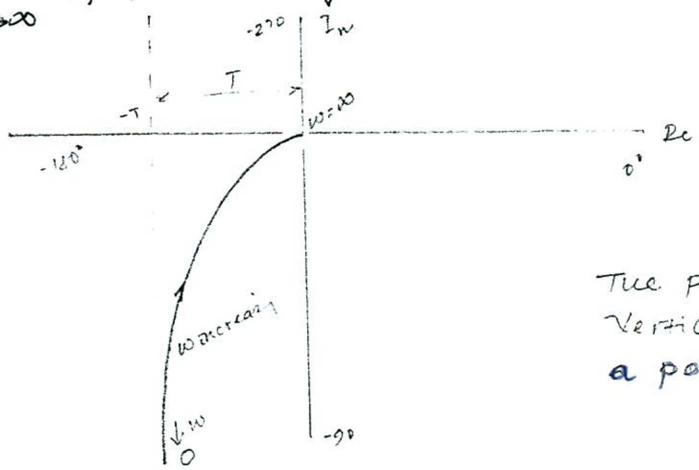
Rearrange

$$G(j\omega) = \frac{-T}{1+\omega^2T^2} - j\,\frac{1}{\omega(1+\omega^2T^2)}$$

as

$$\lim_{\omega\to 0} G(j\omega) = -T - j\infty \Rightarrow \infty \angle -90^\circ$$

$$\lim_{\omega\to\infty} G(j\omega) = -0 - j0 \Rightarrow 0 \angle -180^\circ$$



The plot is asymptotic to the vertical line passing through a point $(-T, 0)$

The major advantage of the polar plot lies in stability study of systems. N. Nyquist (in 1932) relate the stability of a system to the form of these plots. (Next Chapter → Chap 8, freq. stability)

Example.

The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$

Sketch the polar plot of this transfer function.

sol/n

The sinusoidal transfer function is given by

$$G(j\omega) = G(s)\Big|_{s=j\omega} = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

Which can be re-written as

$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)} * \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1+j\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

$$= \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1+j\omega T_1)(1-j\omega T_1)(1+j\omega T_2)(1+j\omega T_2)}$$

$$= \frac{1-\omega^2 T_1 T_2}{[(1+\omega^2T_1^2)(1+\omega^2T_2^2)]} - \frac{j\,\omega(T_1+T_2)}{[(1+\omega^2T_1^2)(1+\omega^2T_2^2)]}$$

1. The low frequency portion of the polar plot

$$\lim_{\omega\to 0} G(j\omega) = 1 - j0 = 1\angle 0^\circ$$

2. High-frequency portion of the polar plot

$$\lim_{\omega\to\infty} G(j\omega) = -0 - j0 = 0\angle -180^\circ$$

3. The point of intersection with the imaginary axis is obtained by setting the real part of $G(j\omega) = 0$

$$\frac{1-\omega^2 T_1 T_2}{[(1+\omega^2T_1^2)(1+\omega^2T_2^2)]} = 0$$

$$1-\omega^2 T_1 T_2 = 0$$

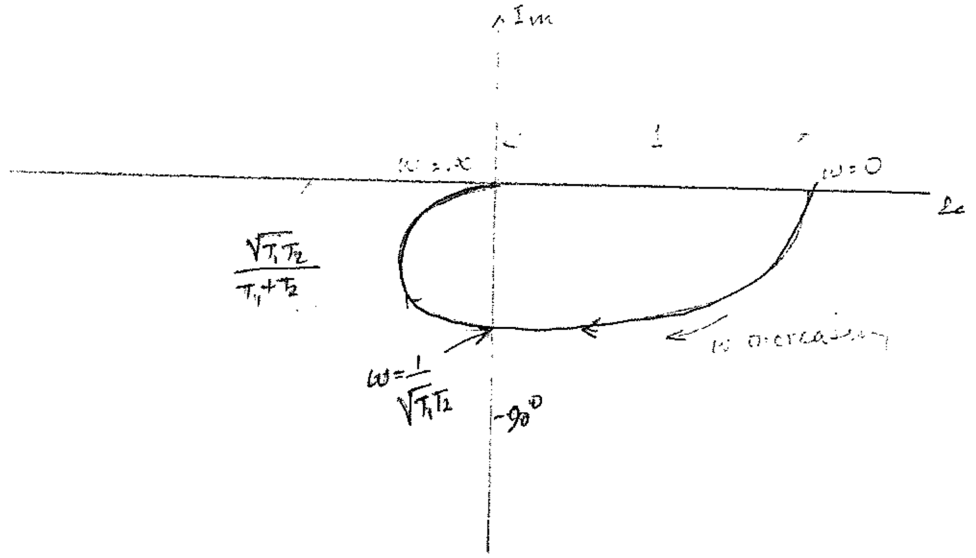
$$\omega = \frac{1}{\sqrt{T_1 T_2}}$$

Substituting $\omega = \frac{1}{\sqrt{T_1 T_2}}$

$$G(j\omega) = 0 + \frac{\dfrac{T_1 + T_2}{\sqrt{T_1 T_2}}}{\left[\left(1+\dfrac{T_1^2}{T_1 T_2}\right)\left(1+\dfrac{T_2^2}{T_1 T_2}\right)\right]}$$

$$= \frac{\sqrt{T_1 T_2}}{T_1 + T_2}$$

4. The general shape of the polar plot of $G(j\omega)$ is shown below.



Exercise

Sketch the polar plot for

a) $G(s) = \dfrac{1+sT_1}{1+sT_2}$, $T_1 > T_2$

b) $G(s) = \dfrac{1}{s(1+sT_1)(1+sT_2)}$

c) $G(s) = \dfrac{10(s+0.2)}{s^2(s+0.1)(s+0.5)}$

7.2.2 Bode Plot or Logarithmic Plot.

~~The~~ A sinusoidal transfer function $G(j\omega)$ may represented by two separate plots, (magnitude Vs frequency and phase angle (in degree) Vs freq

A Bode diagram consists of two graphs:

1- a plot the logarithm of the magnitude of sinusoidal transfer function

2- a plot of the phase angle;

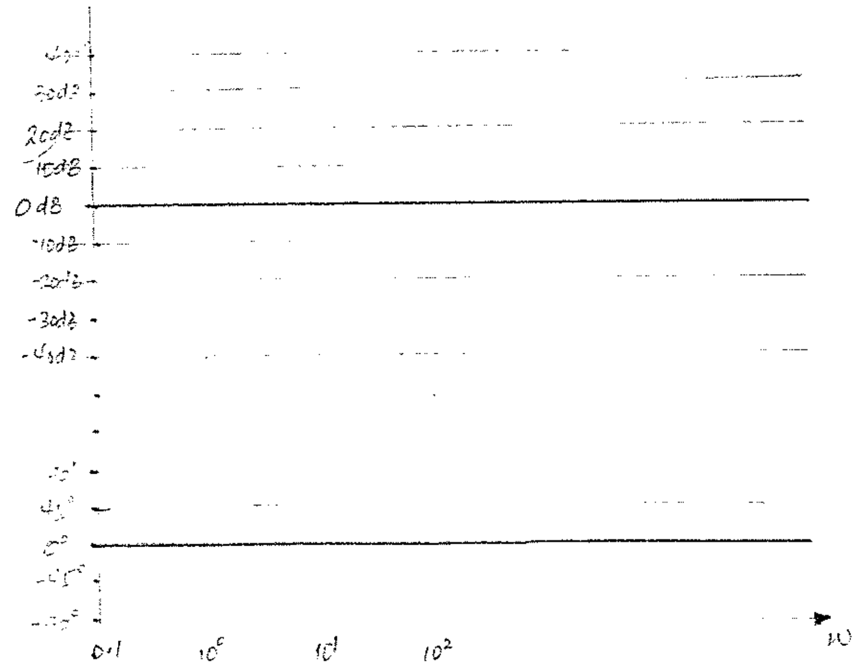
where both are plotted against the frequency in logarithmic scale.

The standard procedure is to plot

$20 \log_{10} |G(j\omega)|$ and phase angle $\phi(\omega)$ Vs $\log \omega$ i.e frequency on logarithmic scale.

Where → the unit of magnitude $20 \log_{10} |G(j\omega)|$ is decibel (db)

→ The curve are normally drawn on a semilog paper using log scale for frequency and linear scale for magnitude in db and phase angle in degrees.



The main advantage of using the bode plot is that multiplication of magnitude can be converted into addition.

Furthermore, a simple method for polting a variable based on the asymptotic approximation by a straight line.

Basic Factors of $G(j\omega)$ ~~$H(j\omega)$~~

The basic factors that very frequently occur in an arbitrary transfer function $G(j\omega)$ ~~$H(j\omega)$~~ are:

1- Constant gain k

2- poles at the origion $\frac{1}{(j\omega)^r}$

3 - Pole on real axis $\frac{1}{(1+j\omega)}$

4. - Zero on the real axis $1+j\omega T$

5. complex conjugate poles $\frac{1}{[1+j2\xi(\omega/\omega_n)+(j\omega/\omega_n)^2]}$

6. complex conjugate zeros may also present.

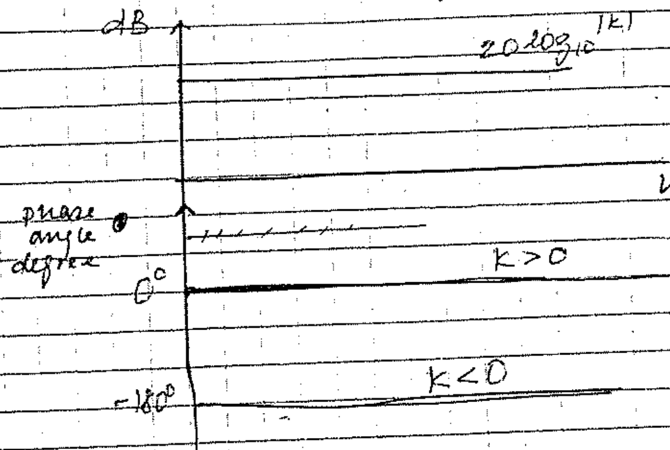
Once we become familiar with the logarithmic plots of these basic factors, it is possible to utilize them in constructing a composite logarithmic plot for any general form of $G(j\omega)$ ~~$H(j\omega)$~~

1- Constant gain K

The log-magnitude for a constant gain K is Horizontal line at the magnitude of

$20\log_{10} K$ = constant in dB

The constant K has magnitude $|K|$, a phase angle of 0° of K +ve, -180° of K -ve



- The effect of Varying the gain K in transfer function is that it raises or lowers the log-magnitude curve of the transfer function.

2. Pole or zero at origin

- consider the transfer function $G(j\omega) = \frac{1}{j\omega}$ (pole at origin)

a) The magnitude of $G(j\omega)$ in dB

$= 20\log\left|\frac{1}{j\omega}\right|$

$= 20\log \omega^{-1}$

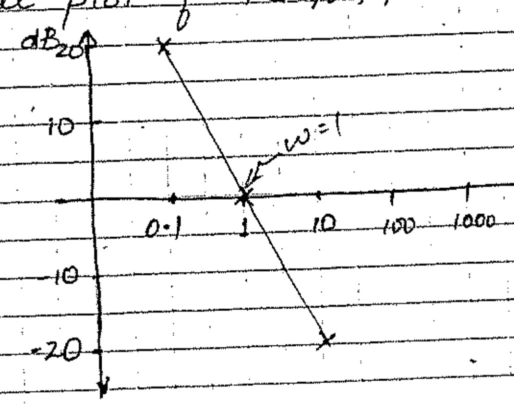
$= -20\log\omega$ slope = -20dB/decade

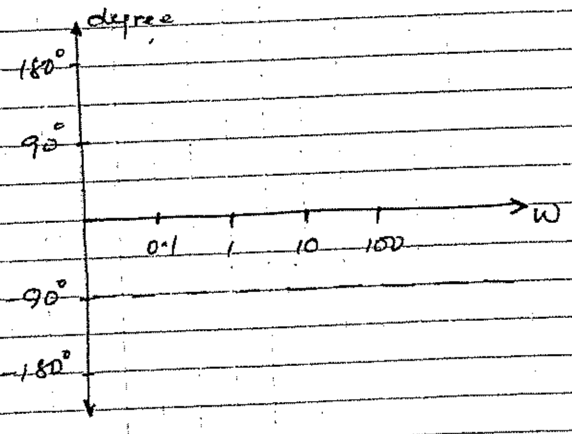
ω	$\lvert G(j\omega)\rvert$
$0.1 = 10^{-1}$	$-20\log 10^{-1} = 20$
$1 = 10^{0}$	$-20\log 10^{0} = 0$
$10 = 10^{1}$	$-20\log 10^{1} = -20$

b) phase angle of $G(j\omega)$

$\phi(\omega) = -90^\circ$

Bode plot of $|G(j\omega)|$ and $\phi(\omega)$





In Bode diagrams, frequency ratios are expressed in terms of octave or decades.

An octave is a frequency band from w_1 to $2w_1$, where w_1 is any frequency value

A decade frequency is a frequency band from w_1 to $10w_1$, where again w_1 is any frequency.

(on the logarithmic scale of semilog paper, any given frequency ratio can be represented by the same horizontal distance
eg. the horizontal distance from $\omega = 1$ to $\omega = 10$ is equal to that of from $\omega = 3$ to $\omega = 30$)

If the log-magnitude $-20\log w$ dB is plotted against w on a logarithmic scale is a straight line with a slope of

$$-20\ dB/decade \quad \text{or} \quad -6\ dB/octave.$$

(To draw this straight line we need to locate a point (0dB, $w=1$) on it)

Similarly for a multiple pole at the origin, we have

$$G(j\omega) = \frac{1}{(j\omega)^n}$$

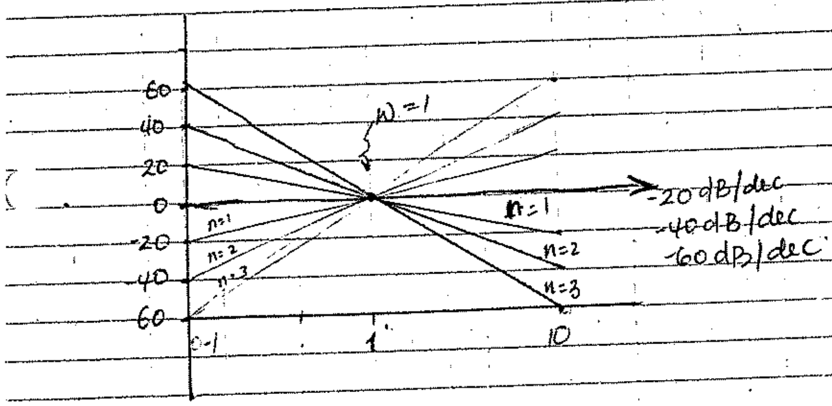
1- The magnitude

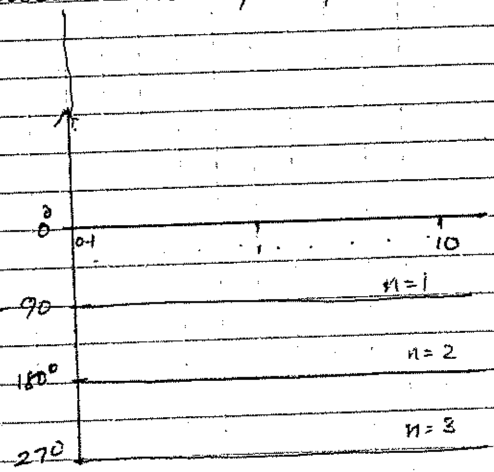
$$20\log\left|\frac{1}{(j\omega)^n}\right| = -n * 20\log w \ dB$$

2- phase angle

$$\phi(\omega) = -90^\circ \times n$$

=) The slope of the log-magnitude curve due to multiple pole is $-20n$ dB/dec





For a zero at the origin =) $G(j\omega) = j\omega$

1) The magnitude
$$20\log|j\omega| = +20\log w \ dB$$

2) The phase angle $\phi(\omega) = +90^\circ$

Pole or Zero on the real axis (pole on real axis)

Consider the sinusoidal transfer function $G(j\omega) = \frac{1}{1+j\omega T}$

$$\Rightarrow |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2 T^2}}$$

1- the log-magnitude of $G(j\omega)$

$$20 \log \left| \frac{1}{1+j\omega T} \right| = -20 \log \sqrt{1+\omega^2 T^2}$$

$$= -10 \log (1+\omega^2 T^2)$$

2- the phase angle

$$\phi(\omega) = -\tan^{-1}(\omega T)$$

For low frequencies such that $\omega \ll \frac{1}{T}$, the log magnitude may be approximated by

$$-20 \log \sqrt{1+\omega^2 T^2} = -20 \log 1 = 0\,dB.$$

$\Rightarrow$ Thus, the log-magnitude curve at low frequencies is constant 0-dB line.

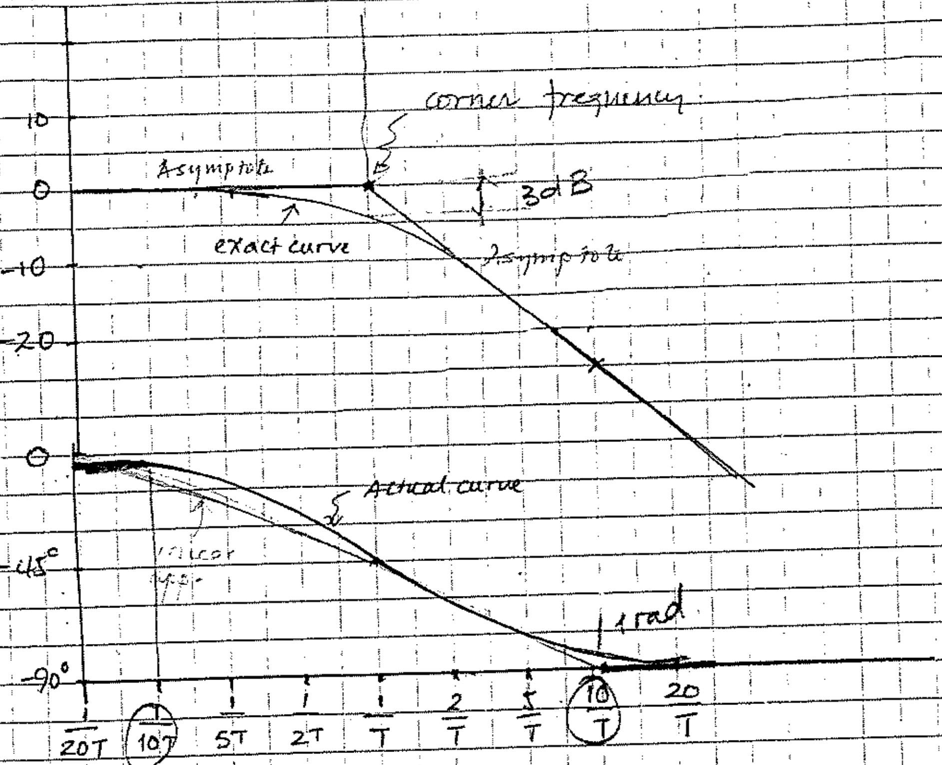
For higher frequencies, such that $\omega \gg \frac{1}{T}$

$$-20 \log \sqrt{1+\omega^2 T^2} = -20 \log \omega T \ dB$$

(where at $\omega = \frac{1}{T}$, the log magnitude equals 0dB); at $\omega = 10/T$, the log-mag is $-20dB$.

$\Rightarrow$ therefore log-mag of $\frac{1}{1+j\omega T}$ can be approximated by two straight ~~line~~ line asymptotes.

ω	$-20 \log \sqrt{(1+\omega^2 T^2)}$	$\phi(\omega) = -\tan^{-1}(\omega T)$
At low frequency $\ll \frac{1}{T} \Rightarrow \omega = 0$	$-20 \log (1) = 0 dB$	$\phi(0) = 0^\circ$
At high frequency $\omega \gg \frac{1}{T}$	$-20 \log \sqrt{\omega^2 T^2}$ dB $-20 \log \omega T$ dB	$\phi(\omega) = -\tan^{-1}(\omega T)$
At $\omega = \frac{1}{T}$	$-20 \log (1) = 0 dB$	$\phi(\omega) = \tan^{-1} 1 = \cancel{90^\circ}\ -45^\circ$
At $\omega = \frac{10}{T}$	$-20 \log (10) = -20 dB$	$\phi(\omega) = -\tan^{-1}(10)$
At $\omega = \frac{100}{T}$	$-20 \log (10^2) = -40 dB$	$\phi(\omega) = -\tan^{-1}(100) \simeq$
at $\omega \to \infty$	$20 \log \|G(j\omega)\| \to \infty$ dB	$\phi(\omega) = -90$



The frequency $\omega = \frac{1}{T}$ at which the two asymptotes meet is called the corner frequency or the break frequency, which divides the plot into two regions, a low freq. region and a high freq. range.

The actual plot can be obtained from the asymptotic approximation by applying correction for the errors introduced in the approximation.

The maximum error occurs at the corner

The error in log magnitude for $0 < \omega \le \frac{1}{T}$ is given by,

$$-10 \log (1+\omega^2 T^2) + 10 \log 1$$

Therefore the max error occurs at the corner frequency $\omega = \frac{1}{T}$

$$-10 \log (1+1) + 10 \log 1 = -10 \log 2 = -3.03 \text{ db}$$

The error at one octave below the corner frequency i.e $\omega = \frac{1}{2T}$ is

$$-20 \log \sqrt{\frac{1}{4}+1} + 20 \log 1 = -20 \log \frac{\sqrt{5}}{2} = -0.97$$

The error at one octave above the corner frequency i.e $\omega = \frac{2}{T}$

$$-20 \log \sqrt{2^2+1} + 20 \log 2 = -20 \log \frac{\sqrt{5}}{2} = -0.97 \text{ dB}$$

Thus the error at one octave below or above the corner frequency is approximately equals to -1dB

Similarly, the error at one decade below or above the corner frequency is approximately -0.04dB.

An advantage of the Bode diagram is that, for reciprocal factors, eg. the zero factor $(1+j\omega T) = \left[\frac{1}{1+j\omega T}\right]^{-1}$, the log-magnitude and phase-angle curve need only be changed in sign since

$$\text{dB mag } |1+j\omega T| = 20\log|1+j\omega T|$$
$$= -20\log\left|\frac{1}{1+j\omega T}\right|$$
$$= -20\log|(1+j\omega T)^{-1}|$$
$$= 20\log(1+j\omega T)$$

for small ωT, $\omega T \ll 1$
$dB = 20\log(\frac{1}{1})^{\frac{1}{2}} = 0$
phase angle $= 0$
For large value ωT
$dB = 20\log\omega T$
phase angle $\tan^{-1}(\omega T)$
when $\omega = \frac{1}{T}$
$dB = 20\log(\sqrt{1+1}) = 3dB$
phase $= \tan^{-1}(1) = 45$

$$\phi = \tan^{-1}(\omega T) = -\tan^{-1}\left(\frac{1}{1+j\omega T}\right)$$
$$= -\tan^{-1}(\omega T)^{-1}$$
$$= \tan^{-1}(\omega T)$$

The corner frequency is the same for both cases on the LACK *

Eg. ~~Plot its frequency~~ plot the Bode of the

$$G(s) = \frac{50(s+2)}{s(s+10)}$$

soln $G(s) = 10\left(\frac{1}{s}\right)\left(1+\frac{s}{2}\right)\left(\frac{1}{1+\frac{s}{10}}\right)$

$$G(j\omega) = 10\left(\frac{1}{j\omega}\right)\left(1+\frac{j\omega}{2}\right)\left(\frac{1}{1+\frac{j\omega}{10}}\right)$$

(The factor shows

1- A constant gain $K=10$

2- A pole at the origion $\left(\frac{1}{j\omega}\right)$

3- A pole at $\omega = 10$

4- A zero at $\omega = 2$

soln We plot the magnitude of char. for individual pole and zero factors.

1- The constant gain $= 20\log 10 = 20dB$

2. The magnitude of the pole at the origion extends from zero to inf frequency and has a slope of $-20dB$ per decade and the cross over frequency $\omega = 1$, when $Lm(j\omega) = 0dB$

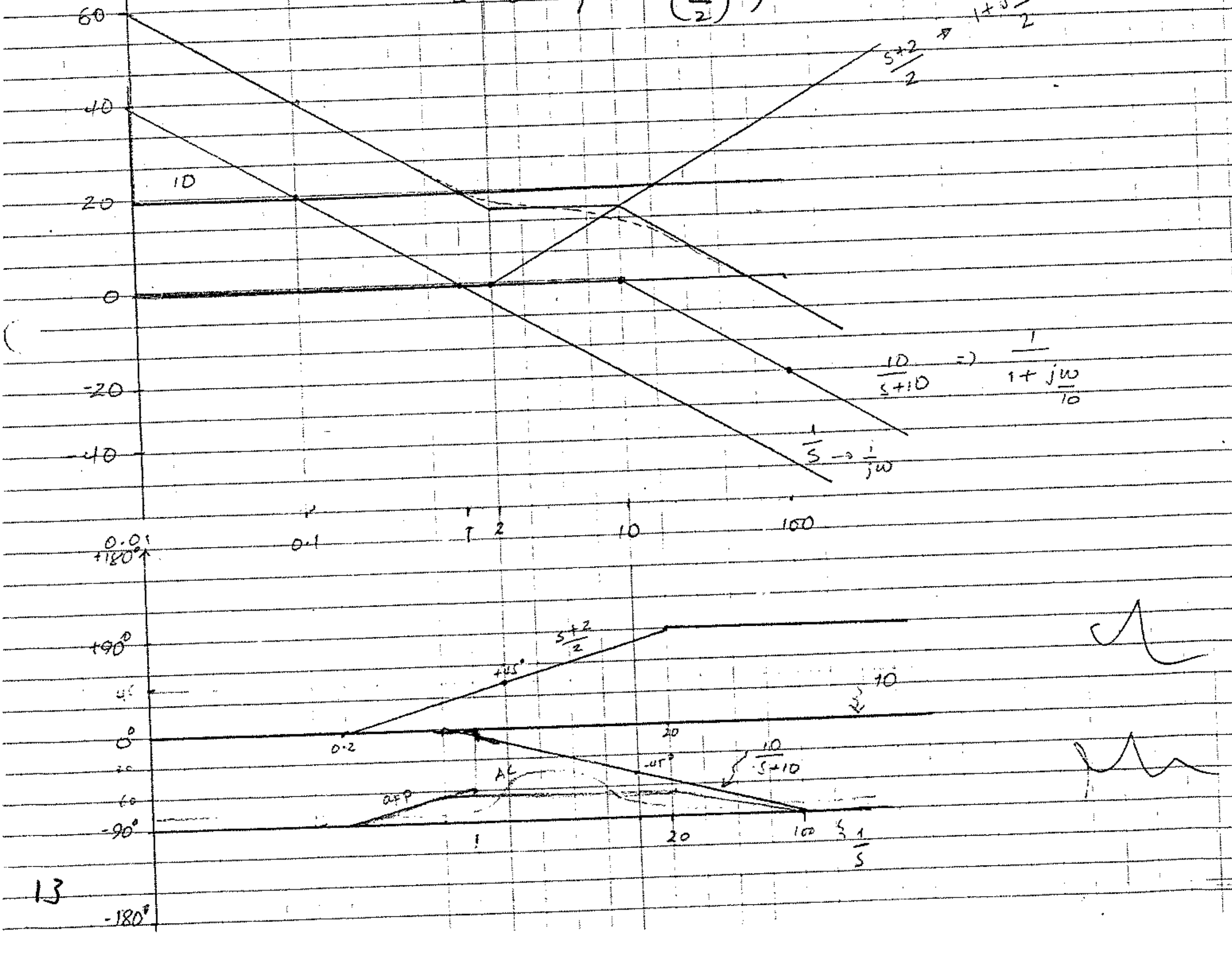
$$Lm(j\omega) = 20\log\left|\frac{1}{j\omega}\right| = 20\log \omega^{-1}$$
$$= -20\log \omega \ dB/dec.$$

3. The asymptotic approximation of the magnitude of the pole at $\omega = 10$ has a slope of $-20dB/dec$ beyond the break frequency at $\omega = 2$, the asymptotic magnitude below break frequency is 0dB

$$Lm\left(1+j\frac{\omega}{10}\right)^{-1} = 20\log\left|1+j\frac{\omega}{10}\right|^{-1}$$
$$= -20\log\left\{\left(1^2+\left(\frac{\omega}{10}\right)^2\right)\right\}^{\frac{1}{2}}$$
$$= -10\log 1+\left(\frac{\omega}{10}\right)^2$$

4. The asymptotic magnitude for the zero at $\omega = \cancel{10}\ 2$ has a slope of 20 dB per decade

$$Lm\left(1+j\frac{\omega}{2}\right) = 20\log\left|\left(1+j\frac{\omega}{2}\right)\right|$$
$$= 10\log\left(1+\left(\frac{\omega}{2}\right)^2\right)$$



. Quadratic factors $1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2$

control systems often posses quadratic factors of the form

$$G(j\omega) = \frac{1}{1+2\xi\left(j\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2}$$

~~If $\xi > 1$~~ If $0 < \xi < 1$, this quadratic factor can be expressed as a product of two ~~first-order factors~~ complex-conjugate factors.

The magnitude and phase of the quadratic factor depend on both the ~~and the corner frequency.~~ corner frequency and the damping ratio ξ ~~and the corner frequency~~.

The asymptotic frequency-response curve may be obtained as follows

$$Lm = 20\log\left|\frac{1}{1+2\xi\left(j\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2}\right| = -20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

For low frequencies such that $\omega \ll \omega_n$, the log mag. become

$$Lm \simeq -20\log 1 = 0\,dB$$

→ the low-frequency asymptote is a horizontal line at 0dB.

For high frequencies such that $\omega \gg \omega_n$, the log mag become

$$-20\log\frac{\omega^2}{\omega_n^2} = -40\log\frac{\omega}{\omega_n}\ dB$$

→ an eqn of straight line having the slope $-40\,dB/dec$

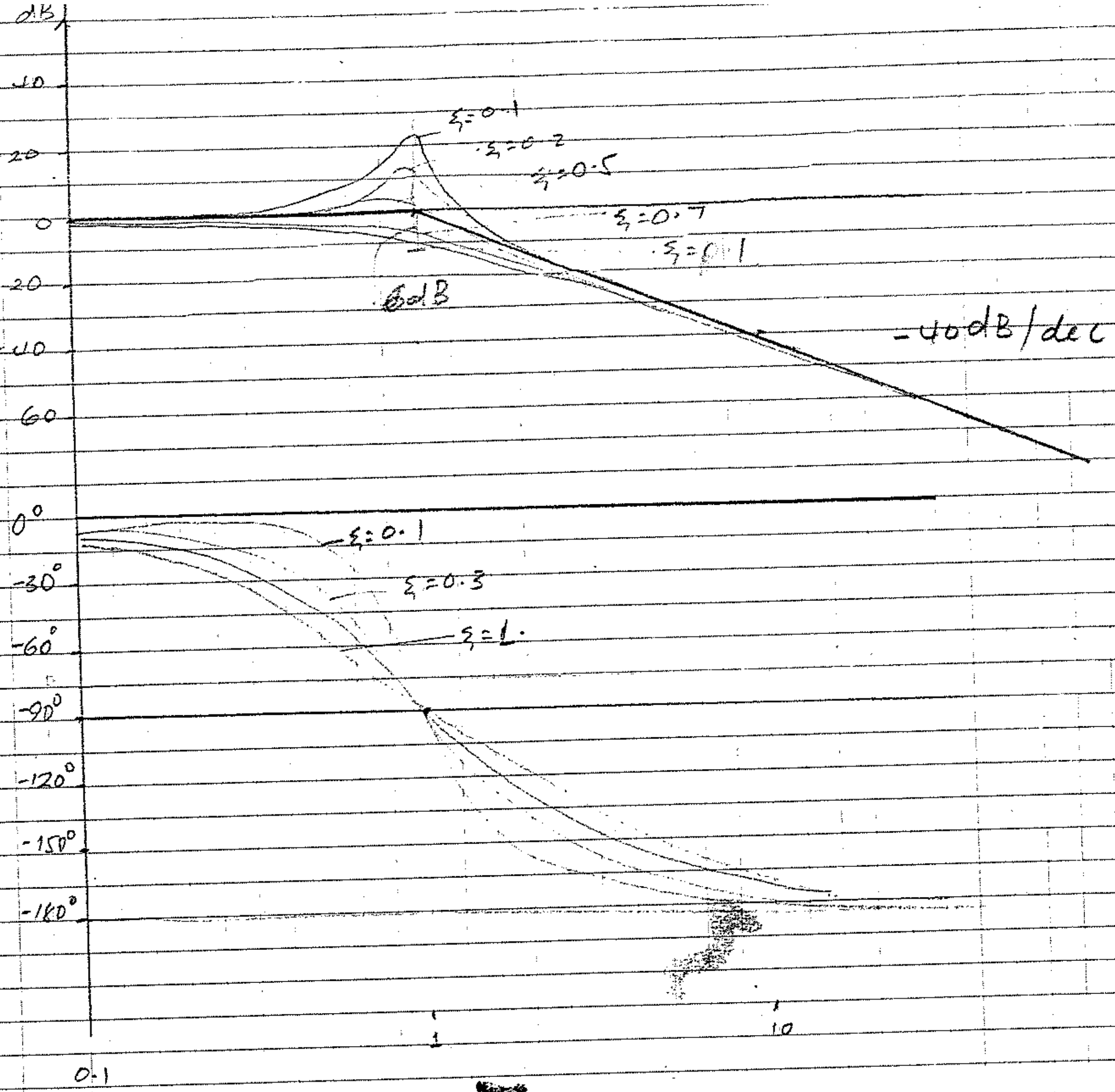
$$-40\log\frac{10\omega}{\omega_n} = -40 - 40\log\frac{\omega}{\omega_n}$$

The high-frequency asymptote intersect the low frequency one at $\omega = \omega_n$ since at this frequency

$$-40\log\frac{\omega_n}{\omega_n} = -40\log 1 = 0\,dB.$$

This frequency ω_n, is the corner frequency for the quadratic factor consider.

Near the frequency $\omega = \omega_n$, a resonant peak occurs. , the damping factor determines the ~~factor~~ magnitude of this resonant peak.



The phase angle

$$\phi(\omega) = \angle \frac{1}{1+2\xi\left(j\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2} = -\tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

- is a function of both ω and ξ

At $\omega = 0$

$$\phi(\omega=0) = 0$$

At corner frequency $\omega = \omega_n$

$$\phi(\omega=\omega_n) = -90^\circ$$

At $\omega = \infty$

$$\phi(\omega=\infty) = -180^\circ$$

The resonant frequency ω_r and the Resonant peak value M_r

$$\omega_r = \omega_n \sqrt{1-2\xi^2} \quad \text{for } 0 \le \xi \le 0.707$$

The magnitude of the resonant peak M_r

$$M_r = \left|G(j\omega)\right|_{max} = \left|G(j\omega_r)\right| = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

General Procedure For constructing Bode Plots:

The following steps are generally involved in constructing the Bode plot for a given $G(j\omega)$

1 Rewrite the sinusoidal transfer function in the time constant form (normalized form)

2. Identify the corner frequencies, draw the asymptotic associated with each factor of the transfer function.

3. Knowing the corner frequencies, draw the asymptotic magnitude plot.

This plot consists of straight line segment with line slope changing at each corner frequency by +20 db/dec for a zero and -20dB/dec for a pole ($\pm 20m$ dB/dec for zero or pole multiplicity of m).

For a complex conjugate zero or pole the slope change by ± 40 dB/dec

4. Form the error graph, and determine the correction to be applied to the asymptotic plot.

5. Draw a smooth curve through the corrected points such that it is asymptotic to the line segment. This gives the actual log-mag. plot

6. Draw phase angle curve for each factor and add them algerbically to get the ~~plot~~ phase plot.

Eg. Draw the Bode plot for the trans. function

$$G(s) = \frac{64(s+2)}{s(s+0.5)(s^2+3.2s+64)}$$

sol^n rearange the transfer function in the time-constant form gives

$$G(s) = \frac{4\left(1+\frac{s}{2}\right)}{s(1+2s)\left(1+0.05s+\frac{s^2}{64}\right)}$$

Sinusoidal transfer function:

$$G(j\omega) = \frac{4\left(1+j\frac{\omega}{2}\right)}{j\omega(1+2j\omega)\left[1+j0.4\frac{\omega}{8}-\left(\frac{\omega}{8}\right)^2\right]}$$

The factor of the transfer function are in order of their frequency occurance.

1- Constant gain $K=4$

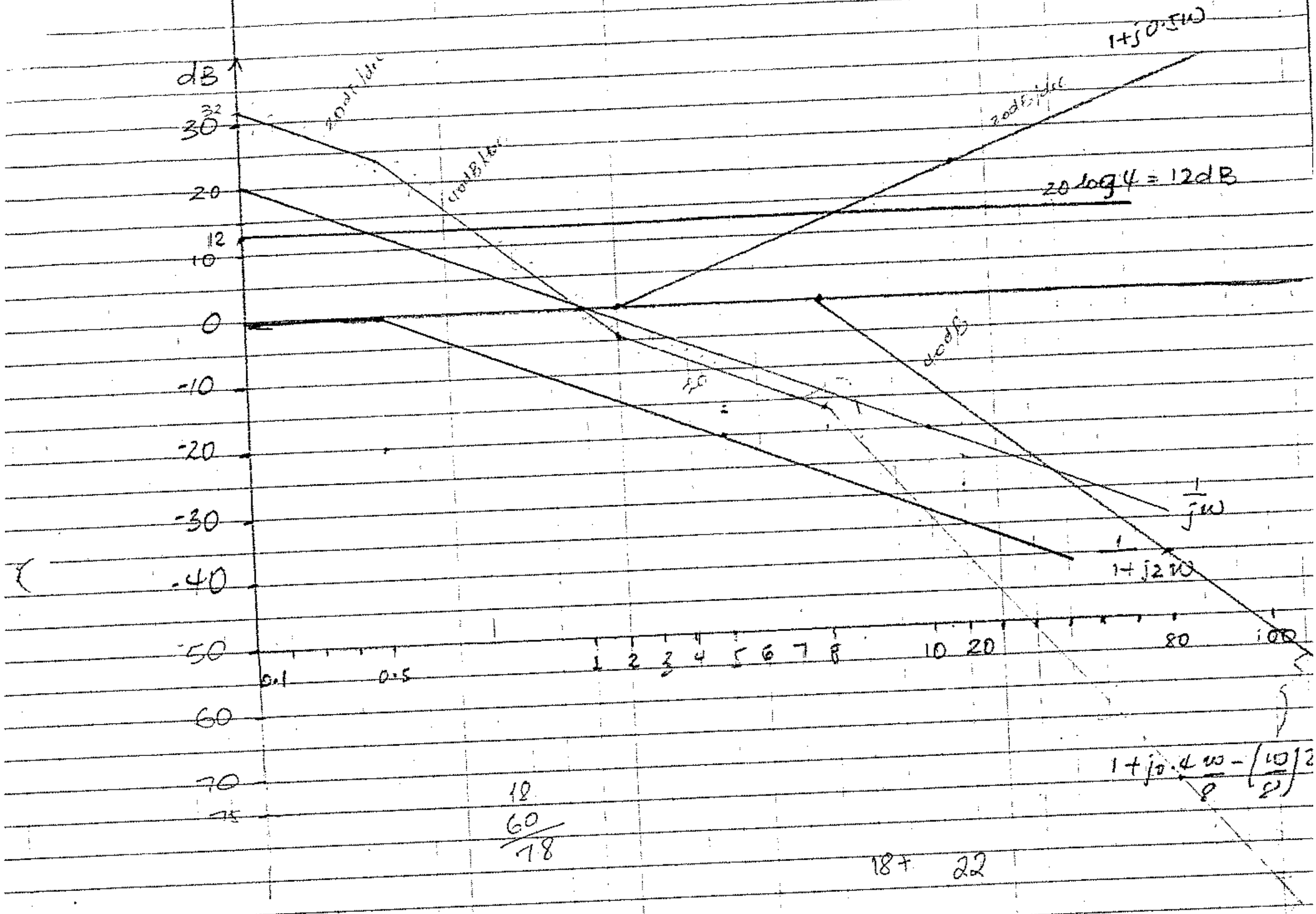
2- Pole at origion $\frac{1}{j\omega}$

3- Pole at $s=-0.5$; corner frequency $\omega_1 = 0.5$

4- Zero at $s=-2$; Corner frequency $\omega_2 = 2$

5- Pair of complex conjugate poles with $\xi = 0.2$, $\omega_n = 8$; corner frequency $\omega_3 = 8$

Factor	Corner freq.	Asym. log-mag char.	phase angle
4	None	straight line $20\log 4 = 12\,dB$ all ω	constant 0
$\frac{1}{j\omega}$	None	Straight line of constant slope -20 db/dec passing through $\omega=1$ with 0 dB	constant -90°
$\frac{1}{1+j2\omega}$ $1/1+j\omega/0.5$	$\omega_1 = 0.5$	Straight line of 0 dB for $\omega < \omega_1$, straight line of slope -20 dB/dec for $\omega > \omega_1$	Phase angle varies from 0 to -90° angle at $\omega_1 = -45$
$1+j0.5\omega$	$\omega_2 = 2$	straight line of 0 dB for $\omega < \omega_2$, straight line of slope $+20$ dB/dec for $\omega > \omega_2$	Phase angle varies from 0 to 90° angle at $\omega_2 = 45$
$\left[1+j0.4\left(\frac{\omega}{8}\right)-\left(\frac{\omega}{8}\right)^2\right]$	$\omega_3 = 8$ $\xi = 0.2$	Straight line of 0 dB for $\omega < \omega_3$, straight line of slope -40 dB/dec for $\omega > \omega_2$	



Expermental Detemination of Transfer function

I.

$$\frac{250}{s(1+0.4s)(1+0.025s)}$$