INTRODUCTION TO COMMUNICATION SYSTEMS

UNIT II ANGLE MODULATION

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**Foundation to understand Angle Modulation**

**Phasor Diagram**



* The generalized sinusoidal expression given as: **A(t) = Am sin(ωt ± Φ)** representing the sinusoid in the time-domain form. But when presented mathematically in this way it is sometimes difficult to visualize the angular velocity so sinusoids can also be represented graphically in the spacial or phasor-domain form by a **Phasor Diagram**, and this is achieved by using the rotating vector method.
* Basically a rotating vector, simply called a "**Phasor**" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase") which is "frozen" at some point in time. A phasor is a vector that has an arrow head at one end.
* Generally, vectors are assumed to pivot at one end around a fixed zero point known as the "point of origin" while the arrowed end representing the quantity, freely rotates in an **anti-clockwise** direction at an angular velocity, ( ω ) of one full revolution for every cycle. This anti-clockwise rotation of the vector is considered to be a positive rotation. Likewise, a clockwise rotation is considered to be a negative rotation.
* As the single vector rotates in an anti-clockwise direction, its tip at point A will rotate one complete revolution of 360o or 2π representing one complete cycle. If the length of its moving tip is transferred at different angular intervals in time to a graph as shown above, a sinusoidal waveform would be drawn starting at the left with zero time. Each position along the horizontal axis indicates the time that has elapsed since zero time, t = 0. When the vector is horizontal the tip of the vector represents the angles at 0o, 180o and at 360o.

**Phasors**

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Phasors are a complex numbers used to represent sinusoids.

**Complex Numbers**

|  |
| --- |
| http://www.ele.uri.edu/~daly/342/phasors/cnumber.GIFFigure 1   Two dimensional representation of a complex number.  |

Recall Euler's formula relating exponential and sinusoidal functions,

***w = x + jy = Re j = R cos( ) + j R sin( )***

where ***R*** is the magnitude and ******is the phase.

***R = sqrt(x2 + y2)***

**Phasors**

Consider the sinusoidal function

***v(t) = A cos(  t +  )***

A phasor representation of this function is the complex number ***V*** with a magnitude ***A*** and a phase ***.***

***V = A e j ***

If we multiply ***V*** by ***e jt*** and apply Euler's formula,

***V e j t = A e j e j t = V e i(t + ) = A cos(t +  ) + jA sin(t +  )***

The real part of ***V e j t*** is the desired sinusoidal function,

***v(t) = A cos(  t +  )***

|  |
| --- |
| Multiply the phasor by ***e j t*** and take the real part to get the sinusoidal function.  |

**I. CONCEPT OF ANGLE MODULATION**

Angle Modulation is defined as a technique/mechanism in which the total phase angle of the carrier signal is varied in accordance with the message signal ;(while keeping the amplitude of the carrier signal constant)

Let us consider a un modulated carrier signal

 c(t)= A.cos(**ωc t** + 𝜭0) ------------ eqn 1

 c(t)= A.cos(Φ) -------------eqn 2

where Φ = **ωc t** + 𝜭0 -------------eqn 3

 A = Amplitude of the carrier

 **ωc** = carrier frequency (a.k.a angular frequency or angular velocity)

 𝜭0  = some phase angle

 Φ = total phase angle of the carrier

Since the angle ‘Φ’ is varied according to the instantaneous value of the message signal;such modulation is called as ANGLE MODULATION.

**II.TYPES OF ANGLE MODULATION**

We can vary the phase angle Φ in two ways and so there are two types of angle modulation

*Angle Modulation*

 1. *Frequency Modulation*

 (Instantaneous frequency of the carrier is varied according to the message signal)

 2. *Phase Modulation*

 (Instantaneous phase angle of the carrier is varied according to the message signal)

**III.UNDERSTANDING INSTANTANEOUS FREQUENCY AND INSTANTANEOUS PHASE**

 From Eqn 2 ---- > c(t)= A.cos(Φ)

Representing eqn 2 in phasor diagram

 c (t) = Re( A.); (or)

 C = A.

Where C is the phasor of length A; rotates at constant angular velocity **ωc** ; 𝜭0 is the phase angle of the carrier at t =0.

From eqn 3 ----- > Φ = **ωc t** + 𝜭0

Differentiating eqn 3 on both sides with respect t ;we have

 = **ωc ; (or)**

**ωc**  =

Hence the angular velocity of the phasor C vary with time (rate of change of time).Therefore this time dependent angular velocity or angular frequency is called as **INSTANTANEOUS FREQUENCY**  and denoted by **ωi**  = --- > eqn 4.

On integrating eqn 4 gives us **INSTANTANEOUS PHASE** denoted by ‘Φi ‘

 Φi = --- > eqn 4a.

**IV (a) PHASE MODULATION**

*Definition*

Phase modulation is a type of angle modulation in which phase angle ‘Φ ‘is varied in accordance with a message signal x (t).This means that in phase modulation the instantaneous value of the phase angle ‘Φi ‘is equal to the phase angle of the un modulated carrier signal i.e. (**ωc t** + 𝜭0) plus a time varying component which is proportional to message signal x(t).

 Φi = **ωc t** + 𝜭0 + Kp . x(t)

i.e. Φi = **ωc t** + Kp . x(t) ----- > eqn 5 {neglecting 𝜭0}

where Kp is proportionality constant and known as phase sensitivity of the modulator

we know that expression for un modulated carrier is

 c(t)= A.cos(Φ) ---- > eqn 2

Therefore expression for phase modulated signal

 s(t) = A.cos(Φi)

 s(t)= A.cos(**ωc t** + Kp . x(t)) ---- > eqn 6 [ from eqn 5]

eqn 6 is the required mathematical expression for a phase modulated wave.

**IV (b) Frequency Modulation**

Frequency modulation is a type of angle modulation in which instantaneous frequency ‘**ωi**’ is varied in accordance with a message signal x(t).This means that in frequency modulation the instantaneous value of the angular frequency ‘**ωi** ‘ is equal to the carrier frequency ‘**ωc**’ plus a time varying component which is proportional to message signal x(t).

i.e. **ωi** = **ωc**  + Kf . x(t) ----- > eqn 7

where Kf is proportionality constant and known as frequency sensitivity of the modulator

we know that expression for un modulated carrier is

 c(t)= A.cos(Φ) ---- > eqn 2

For frequency modulation amplitude A must remain constant and only angle Φ will change Therefore expression for frequency modulated signal

 s(t) = A.cos(Φi) --- > eqn 8.

But eqn 4a gives

 Φi = --- > from eqn 4a.

 Φi = --- > from eqn 7.

 Φi = --- > eqn 9.

Putting this value of Φi in eqn 8,the expression for frequency modulated signal will be

t + ]

Now if the phase angle of the un modulated carrier is taken at t=0 ,then the limit of integration in the above equation will be 0 to t. So the expression for FM is

t + -- > eqn 10.

**V RELATIONSHIP BETWEEN PM AND FM**

From eqn 5 and eqn 9 it may be observed that PM and FM are closely related to each other because in both the cases there is variation in the total phase angle. In PM the phase angle varies linearly with x(t) and the in the case of FM the phase angle varies linearly with integral of x(t).This means that FM signal may be obtained by using PM and vice versa.

To get FM using PM ,we first integrate the message signal and then apply to the phase modulator(Figure 1). To get PM using FM ,we first differentiate the message signal and then apply to the frequency modulator (Figure 2).

Carrier Generator

A cos Wc t

X(t)

FM signal

Phase modulator

Integrator

Figure1. Generation of FM using Phase modulator

Differentiator

X(t)

PM signal

Frequency modulator

Carrier Generator

A cos Wc t

Figure2. Generation of PM using Frequency modulator

**VI (a) FREQUENCY DEVIATION**

The maximum change in instantaneous frequency from the average frequency **ωc** is called frequency deviation; denoted by ∆ω and it depends upon the magnitude of kf.x (t).

 Frequency deviation ∆ω= | kf.x (t) |max

∆ω= kf.|x (t) |max

∆ω= kf.Am -- > eqn 11

Where Am is maximum amplitude of x(t). i.e. .|x (t) |max = Am

∆ω is a useful parameter for determining the bandwidth of FM signals.

**VI (b)MODULATION INDEX**

For FM modulation index is defined as the ratio of frequency deviation to the modulating frequency.

Modulation index β=

*Points to be noted*

* No danger of over modulation in FM
* As β increased, uses more bandwidth
* As β increased, more resistant to noise
* On the basis of β FM is divided into two part

If β > 1 WIDEBAND FM

If β < 1 NARROWBAND FM

**VI (c)SINGLE TONE FM DERIVATION**

From eqn 10

t +

put x(t)= Am cos **ωm t** in eqn 10

t +

t )]

t )] (FROM EQN 11; ∆ω= kf.Am )

t ) ] -- > eqn 12

Eqn 12 is the required mathematical expression for single tone FM wave.

**VI (d)TRANSMISSION BANDWIDTH OF FM SIGNAL**

Carson’s rule provides a thumb rule formula to calculate the bandwidth of a single tone FM.According to this rule the FM BW is given by the twice the sum of the frequency deviation and the highest modulating frequency.So mathematically

 BW=2 ( + ) -- > eqn 13 ; but β=

Therefore BW= 2 (β . + )

BW= 2 (1 + β).-- > eqn 14

**VII FM DEMODULATION**

Electronic circuits that perform frequency demodulation are called FM detectors. The detector performs the extraction in 2 steps.

1. Converts FM to corresponding AM by *FREQUENCY DISCRIMINATORS*.

2. The message signal x(t) is recovered from this AM signal by using *ENVELOPE DETECTOR.*

*FREQUENCY DISCRIMINATORS*

A simple LC circuit can be used as a FM discriminator. They can be divided into 2 types.

1. Slope Detectors

 (i) simple slope detector

 (ii)balanced slope detector

2. Phase difference discriminators

 (i) Foster –Seelay discriminator

 (ii) Ratio Detector

*Slope Detectors*

1. FM signal is applied to the input of first LC circuit.
2. When carrier frequency is matched to LC circuit, then resonance phenomena occur which produce a energy in form of voltage.
3. Voltage produce by discriminator is AM wave .Finally this AM is detected by envelope detector.

**VII APPLICATION**

1. FM digital synthesizers in PC sound cards
2. FM broadcasting



