UNIT TWO: THE TIME VALUE OF MONEY

Point to be discussed

- ✓Time value of money: What and Why?
- ✓ Present value
- ✓Compound or future value
- ✓ Present and future value of annuity
- ✓ Present and future value of perpetuity
- ✓Accumulation of future sum (sink fund)
- ✓Determining yield (rate of return)

Time Value: What and Why

- One of the most important tools in personal finance and investing is, the time value of money.
- Understanding financial transactions,
- whether involving,
  - investing,
  - borrowing, or
  - lending, requires understanding the time value of money.
- Evaluating financial transactions requires valuing uncertain future cash flows; that is, determining what uncertain cash flows are worth at different points in time.

Time Value: What and Why…

- We are often concerned about what a future cash flow or a set of future cash flows are worth today
- Moving money through time—that is,
  - finding the equivalent value to money at different points in time—involves translating values from one period to another.
- Translating money from one period involves interest, which is how the time value of money and risk enter into the process.
- The basic idea of time value of money is that a birr today is worth more than a birr tomorrow.

Intuition Behind Present Value

- There are three reasons why A Birr today is more valuable than a Birr a year after. Such are
  1. Individuals prefer present consumption to future consumption. To induce people to give up present consumption you have to offer them more in the future.
  2. When there is monetary inflation, the value of currency decreases over time. The greater the inflation, the greater the difference in value between a Birr today and a Birr tomorrow.
     - Except for rare periods of significant deflation where the opposite may be true, a birr in cash is worth less today than it was yesterday, and worth more today than it will be worth tomorrow.
  3. If there is any uncertainty (risk) associated with the cash flow in the future, the less that cash flow will be valued.

Intuition Behind Present Value …

- Other things remaining equal, the value of cash flows in future time periods will decrease as
  - the preference for current consumption increases.
  - expected inflation increases.
  - the uncertainty in the cash flow increases.
- This time-value is a function of property investment characteristics, namely-
  - loss of liquidity and
  - costs associated with the management of the investment,
  - inflation and risk

Intuition Behind Present Value ...

- Most real estate decisions involves comparing the immediate cost of an action against the value of the future resulting benefits (usually quantified as cash flows).

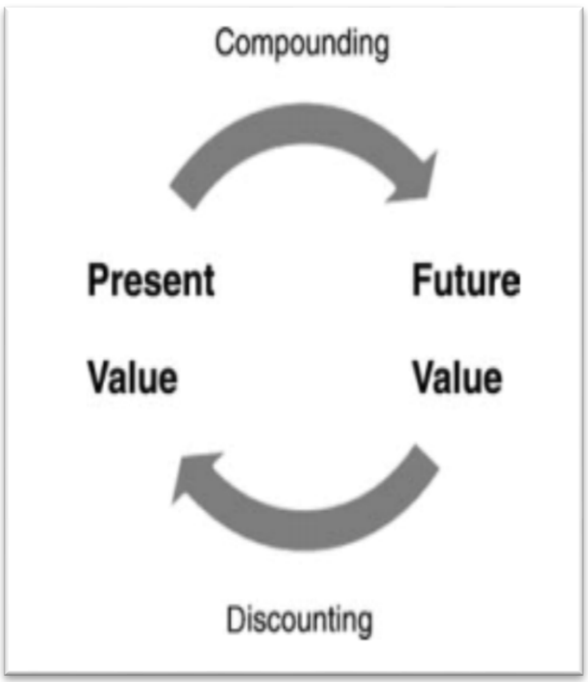
- However, the valuation of future benefits is complicated by two factors.

Intuition Behind Present Value …

1. the future benefits of a proposed investment cannot simply be added up to determine their value to investors because the value of future benefits declines with time into the future
2. value assessments are based on expected cash flows, but what actually happens is seldom, if ever, exactly what the investor expected

Intuition Behind Present Value …

Present and future value



- The operation of evaluating a present value into the future value is called a capitalization (how much will birr100 today be worth in 5 years time?
- The reverse operation which consists in evaluating the present value of a future amount of money is called a discounting.

Intuition Behind Present Value…

- The main factors that are used by investors to determine the rate of return at which they are willing to invest money include:
  - estimates of future **inflation rates**
  - estimates regarding the **risk of the investment** (e.g. how likely it is that investors will receive regular interest/dividend payments and the return of their full capital)
  - Whether or not the investors want the money available **("liquid")** for other uses.

Intuition Behind Present Value …

- present value calculations allow us to determine the value today of a stream of cash flows to be received or paid in the future, by taking into account the time value of money

Intuition Behind Present Value …

Discussion question

1. Do you receive 100 birr today or 100 birr after 1 year?
2. Do you receive 100 birr today or 105 birr after 1 year?

Time value of Money

- The time value of money is reflected in the interest rates that Banks offer for Deposits, and also in the interest rates that banks charge for loans such as home mortgages. The "Risk-free" rate is used in our discussion

- this is an important concept which is widely used both in real asset and in personal financial decision-making

Time value of money…

- The **time value of money** (TVM) is a way of calculating the value of a sum of money, at any time in the present or future. It allows us to calculate:
  1. Present Value (PV) is the present value of an amount that will be received in the future. It answers such questions as, what is the value now of a coupon bond that will pay birr1, 000 in 10 years.

Time value of money …

2. Future Value (FV) is the future worth of a present amount. It answers such questions as, how much will be in my savings account at year end, which has birr1,000 in it now, and pays 5% compounded yearly?

Time Value of Money …

3, Present Value of an Annuity (PVA)

An annuity is a constant cash flow that occurs at a regular intervals for a fixed period of time (payments or receipts)

PVA is the present value of a stream of future payments. If we define "a" to be the annuity,

$$Pva = a[(1+r)^n-1/r(1+r)^n]$$

Example: If the cash flow is CF_{1-10} = 8000 birr a year and r= 10 %. What is the PVA at the end of the 10th year?

Time Value of Money …

4. Future Value of an Annuity (FVA) is the future value of a stream of payments or receipts (annuity). When the cash flow occurs at the end of each period, the annuity is called **a regular annuity**. When the cash flow occurs at the beginning of each period, the annuity is called an **annuity due**.

In general terms the future value of an annuity is given by the following formula:

Time Value of Money …

- FVA (sinking fund) = $a[(1+r)^n-1/r]$

Example: For equal annual payments of Birr 2000 are made in to a deposited that pays 8% interest per year. What is the future value of this annuity at the end of the 4th year?

- 5. Perpetuity is an annuity that lasts "forever", or at least indefinitely.
- $C0 = a/r$

Time value of money …

- Notation in this lecture
- *PV* = present value = value today of a stream of cash flows
- *FV* = future value = value at some future time of a stream of cash flows
- *C* = cash flow
- *r* = interest rate (discount rate)
- *T* = number of years
- *m* = number of periods in a year
- *n* = total number of periods (n = mT)
- we will often apply subscripts to indicate time, e.g. *Ct is* a cash flow occurring at time *t*

Future value- One period case- Single cash flow

- In examining this problem, one should be aware that any compounding problem has four basic components:
  1. An initial deposit, or present value of an investment of money.
  2. An interest rate.
  3. Time.
  4. Value at some specified future period.

Future value- One period case- Single cash flow

- For example, if birr1 is invested at the beginning of year 1 at r rate of return, the capital accrued at the end of the year will be 1 + r
- The value after one year can be determined by examining the following relationship:
- The future value, *FV*, at the end of one year equals the deposit made at the beginning of the year, *PV*, plus the Birr amount of interest earned in the first period.
- $FV = PV + I_I$

Future Value- Multiple period case-single cash flow

Here we need to identify types of interest rate

- two types of interest: Simple Interest Vs. Compound Interest
- Simple interest - earn no interest on interest rather interest earned only on original principal

- So in this case the future value is calculated as
- FV after 1 yr =C0+C0r , where C0 = cash flow (principal)
- r = interest rate
- FV after 2 yrs =C0+C0r+C0r

...

$$\text{FV after } n \text{ yrs} = C0 + \underbrace{C0 \text{ X } r + C0 \text{ X } r + \text{------} + C0 \text{ X } r}_{n \text{ terms}}$$
$$= C0 \text{ X } (1 + n \text{ X } r)$$

Future Value – Multiple period case-single cash flow

- Simple interest is where the interest earned on invested capital is not reinvested but is spent, so the capital remains the same and the interest each year remains the same.
- Simple Interest is defined as interest that is earned on the original amount deposited in the account (ONLY the original amount)
  - I = C0rt
    - **General Formula for Simple Interest:**
    - **FV = C0(1+rt)**
- where C0 = principle amount, A = accumulated amount, r = rate of interest, and t = time in years

Future value - Compounding

- Compounding is a process of finding the future value of a present amount.
- Understanding the process of compounding in investment analysis requires the knowledge of only a few basic formulas.

- The theory of compounding assumes that when money is invested today it will accumulate interest at the end of a stated period, normally one year.
- The interest is added back to the capital and reinvested over year 2 and so on, thus the interest earns interest and the capital builds up more quickly than with simple interest.

Compound interest: interest earned on original principal and on previously earned interest

FV after 1 yr $= C0(1+r)$

FV after 2 yrs $= C0(1+r)(1+r)$

$$\text{FV after } n \text{ yrs} = C0 \times \underbrace{(1+r)\ (1+r) \text{ ------ } (1+r)}_{n \text{ terms}}$$

General formula for compound interest

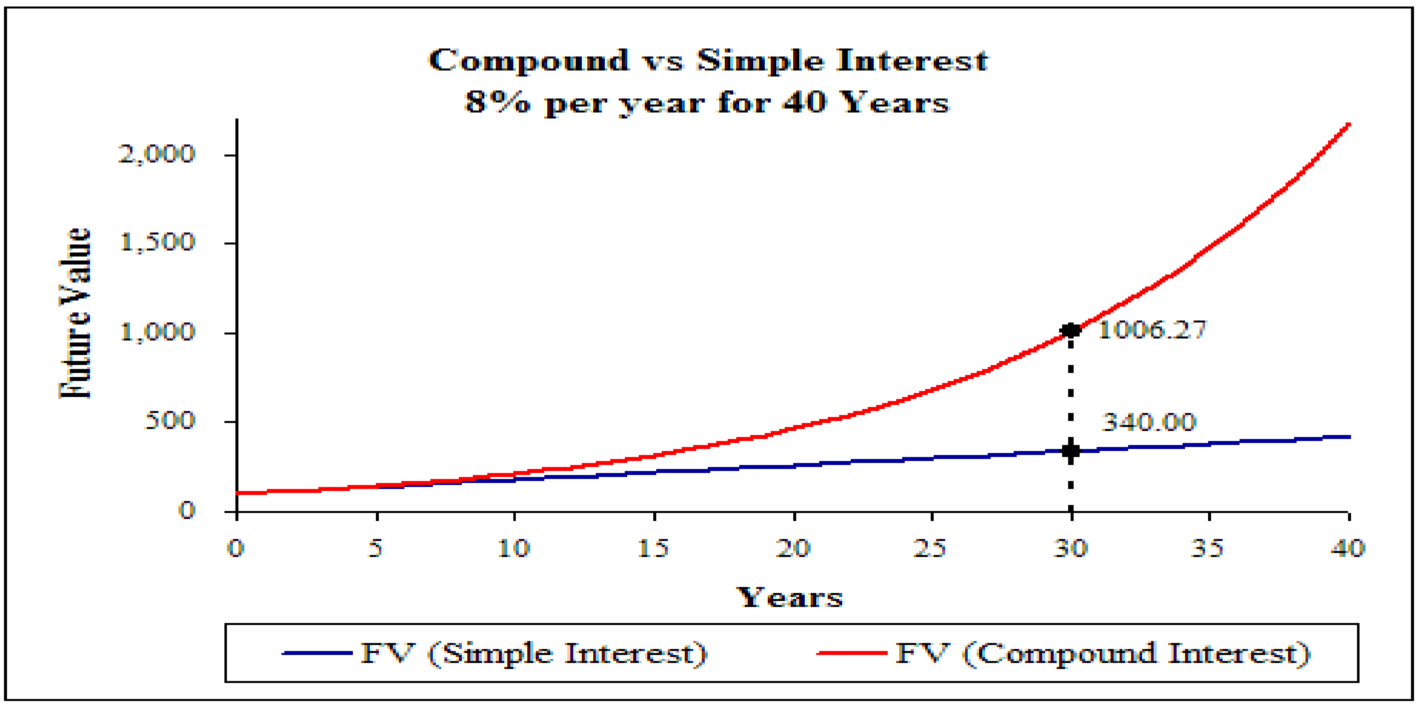
$$= C0 \times (1+r)^n$$

Example: Compound and simple interest with r = 10%.

	Simple			Compound		
Year	Initial	Interest	Ending balance	Initial	Interest	Ending balance
1	100	10	110	100.00	10.00	110.00
2	110	10	120	110.00	11.00	121.00
3	120	10	130	121.00	12.10	133.10
…	…	…	…	…	…	…
10	190	10	200	235.79	23.58	259.37
…	…	…	…	…	…	…
20	290	10	300	611.59	61.16	672.75
…	…	…	…	…	…	…
30	390	10	400	1586.31	158.63	1744.94

Future value: Compounding …

- To see this even better take a look at the following chart, this shows the difference between compound and simple interest over long periods. Notice how the future value grows exponentially with compound interest. The dotted line shows the difference in future values over 30 years



Future value: Compounding for Periods of Less than One Year

- In the preceding subsections, the discussion of compounding applies to cases where funds were compounded only once per year.
- Many savings accounts, bonds, mortgages, and other investments provide for monthly, quarterly, or semiannual compounding.
- When compounding periods other than annual are considered, a simple modification can be made to the general formula for compound interest. formula:
- $FV=PV\ (1+r)^{n}$
- Where *n* = years ,r = annual interest rate and *PV* = deposit (present value)

Future value: Compounding …

- To change the general to any compounding period, we divide the annual interest rate(r) by the desired number of compounding intervals *within* one year.
- We then increase the number of time periods *(n)* by multiplying by the desired number of compounding intervals *within* one year.
- For example, let *m* be the number of intervals *within* one year in which compounding is to occur, and let *n* be the number of years in the general formula. Then we have

$$FV = PV\left[1+\frac{r}{m}\right]^{n.m}$$

Future value: **Compounding …**

- If you compare the results of monthly compounding with those from compounding annually, we can immediately see the benefits of monthly compounding.

- If our initial deposit is compounded monthly, we would have Birr 10,616.78 at the end of the year, compared with Birr 10,600.00 when annual compounding is used.

Compounding Interval	Modified Formula
Semiannually, m = 2	FV=PV $\left[1+\frac{r}{2}\right]^{n.2}$
Quarterly, m = 4	FV=PV $\left[1+\frac{r}{4}\right]^{n.4}$
Daily, m = 365	FV=PV $\left[1+\frac{r}{365}\right]^{n.365}$

If there is inflation… Real Vs. Nominal

Real vs. Nominal Cash Flows

- In our previous calculation of PV and FV we assume that there is no risk. But the question is what if there is for example **inflation**

- Example If inflation is 4% per year and you have been promised 1.04 birr in one year, what is this really worth next year? What can 1.04 birr by next year given that prices will rise by 4%? The *real* or *inflation adjusted* value should be considered

➢ In general, at annual inflation rate of *i* we have

$$\text{(Real Cash Flow)} = \frac{\text{(Nominal Cash Flow)}}{(1+i)}$$

Real vs. Nominal Interest (Discount) Rates

- Nominal interest rates - typical market rates
- Real interest rates - interest rates adjusted for inflation.

$$(1+r_n) = (1+r_r)(1 + i)$$

Nominal interest rate Vs. Effective interest rate

- **Nominal Interest rates:**
- In cases where interest is calculated more than once a year, the annual rate quoted is the nominal annual rate or nominal rate.
- If interest were quoted at 8% per annum but paid each quarter, it would be assumed that 2% would be receivable every quarter.
- However, if 2% is compounded every quarter it will accumulate to more than 8% per annum, as shown below:
- $(1 + r_{quarter})^4 = (1 + r_{annual})$
- $(1 + r_{quarter})^4 - 1 = r_{annual}$
- $(1 + 0.02)^4 - 1 = 0.0824 = 8.24\%$ p.a.
- The 8.24% is often quoted as the annual percentage rate (APR) on loan agreements.

Nominal interest rate Vs. Effective interest rate ...

- **Effective Interest rates:**
- If the actual interest earned per year is calculated and expressed as a percentage of the relevant principal, then the so-called effective rate is obtained.
- **The effective rate** of interest is the rate per period that, when compounded, equates to the annual percentage rate.

Nominal interest rate Vs. Effective interest rate ...

- This is the equivalent annual rate of interest – that is, the rate of interest earned in one year if compounding is done on a yearly basis.
- Another way of **Converting Nominal Rate to Effective Rate:** $r_{eff} = \left(1 + \frac{r_{nom}}{k}\right)^k - 1$

where: r_{eff} : effective rate (percentage), r_{nom} : nominal rate (percentage) , k : no. of periods in one year

Nominal interest rate Vs. Effective interest rate ...

- Whenever the nominal annual interest rates offered on two investments are equal, the investment with the more frequent compounding interval within the year will always result in a higher effective annual yield.

Present value

- We were concerned with determining value at some time in the *future;* that is, we considered the case where a deposit had been made and compounded into the future to yield some unknown future value.
- Here , we are interested in the problem of knowing the future cash receipts for an investment and of determining how much should be paid for the Investment at *present.*
- The concept of **present value** is based on the idea that money has time value.
- Suppose you are offered the choice between 1 Birr today, or receiving 1 Birr in the future, which choice do you prefer?
- The proper choice will always be to receive the 1 Birr today because this 1 Birr can be invested in some opportunity that will earn interest, which is always preferable to receiving only 1 Birr in the future. In this sense, money is said to have *time value.*

Present value…

- When determining how much should be paid *today* for an investment that is expected to produce income in the *future*, we must apply an adjustment called **discounting** to income received in the future to reflect the time value of money.
- The concept of present value lays the cornerstone for calculating mortgage payments, determining the true cost of mortgage loans, and finding the value of an income property, all of which are very important concepts in real estate investment.
- Note that with compounding, we are concerned with determining the *future value* of an investment.
- With discounting, we are concerned with just the opposite concept; that is, what *present value* or *price* should be paid *today* for a particular investment, assuming a desired rate of interest is to be earned?

Graphic Illustration of present Value

Month														
		1	2	3	4	5	6	7	8	9	10	11	12	
Compounding at 6%													Future	
	Birr												Value	
	10,000												(?)	
Discounting	Present												Birr	
at 6%	Value(?)												10,600	

Present value vs. future value

- To determine future value, recall the general equation for compound interest: $FV=PV\ (1+i)^n$

- *PV*, the present value or amount we should pay for the investment today, can be easily determined by rearranging terms in the above compounding formula as follows:

  - $FV = PV\,(1 + i)^n$

  - $PV = FV \dfrac{1}{(1+i)^n}$

Present Value: Discounting for Periods of Less than one Year

- Because we can use the **discounting process to find the present value of a future value when *annual* compounding is assumed,** we can also apply the same methodology assuming *monthly* discounting.
- For example, in our illustration involving monthly compounding, the future value of 10,000 Birr at an annual rate of interest of 6 percent compounded monthly was 10,616.80 Birr.
- An important question an investor should consider is how much should be paid today for the future value of 10,616.80 Birr received at the end of one year, assuming that a 6 percent return compounded *monthly* is required?
- We could answer this question by finding the reciprocal of the formula used to compound monthly, $1 \div (1 + i/12)^{1.12}$, and multiply that result by the future value of 10,616.78 Birr to find the present value *(PV)*. We may calculate this factor

Present value of a future sum

- The present value formula is the core formula for the time value of money; each of the other formulae is derived from this formula.

- The present value (PV) formula has four variables, each of which can be solved for:

$$PV = \frac{FV}{(1+i)^n}$$

1. PV is the value at time=0
2. FV is the value at time=n
3. i is the interest rate at which the amount will be compounded each period
4. n is the number of periods (not necessarily an integer)

Annuities

An annuity is a constant cash flow that occurs at regular intervals for a fixed period of time. Or an annuity is a stream of cash flows that are equally spaced in time and of equal Amount.

There are two types of annuities : ordinary annuity Vs. annuity due . The difference between the two types of annuities is when the payment is made.

An **ordinary annuity** (also referred as **annuity-immediate**) is an annuity whose payments are made at the end of each period (e.g. a month, a year).

Annuity due and a ***deferred annuity:*** In an annuity due the cash flows are assumed to occur at the *start* of each period

Annuities…

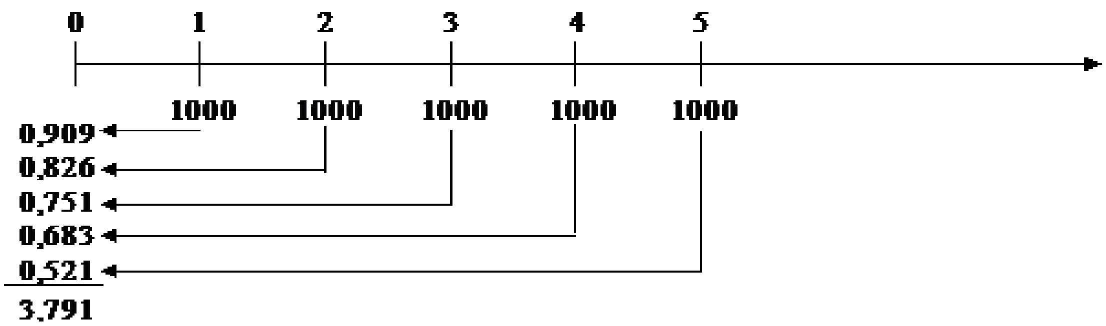
- To understand their difference looks the following application: Ordinary annuity is the case with-
  - mortgages, **where the loan payment is made in the month after the payment is due.**

- If the arrangement requires the payment in advance, such as with-
  - property rental, it is called an "annuity due". The timing of the payments slightly affect the amount of the payments

Annuities

- To realize the difference between the two type of annuity look the following Example:_Part of your portfolio is bonds. One particular bond will pay you birr 1000 for five years at the end of each year. What is the present value of the cash flows if the interest rate is 10%?

- The solution can be shown on a timeline:

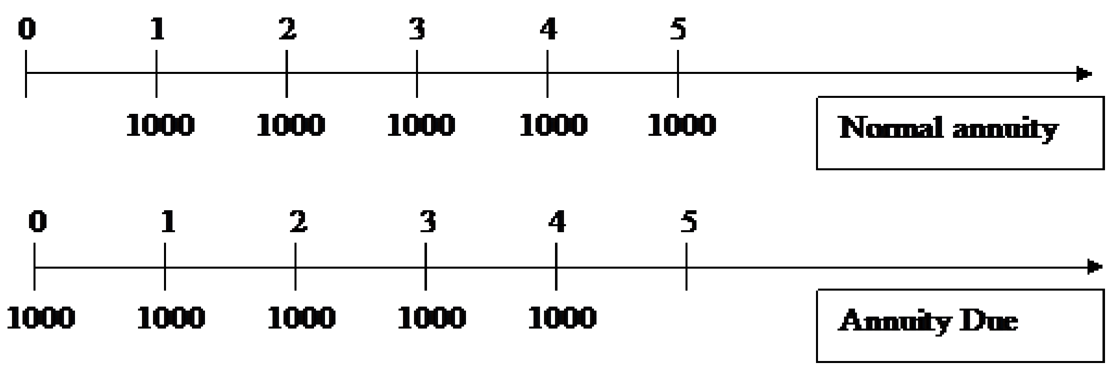


If we would calculate Present value each time, we would get 909, 826 and so on
If we add these values up, we would get 3791

Annuity due is different from normal annuity because you get each cash flow amount in the beginning of the period.

The calculation matches the one before, but a methodical difference is very important to remember.

Present value for annuity due is larger, while future value is a smaller amount than for normal annuity. To understand this we (again) look at some timelines and seek the help of present value



The present value (PV) of an annuity

- the present value of an annuity can be calculated by taking each cash flow and discounting it back to the present, and adding up the present values. Alternatively, there is a short cut that can be used in the calculation [A = Annuity; r = Discount Rate; n = Number of years]

$$PV \text{ of an Annuity } = \text{PMT}\left[\frac{1 - \frac{1}{(1+r)^n}}{r}\right]$$

- The **PV** of an annuity formula is used to calculate how much a stream of payments is worth *currently* where "currently" does not necessarily mean right now but at some time prior to a specified future date.

The present value (PV) of an annuity…

- In practice the **PV** calculation is used as a valuation mechanism.
- It evaluates a series of payments over a period of time and reduces or consolidates them into a single representative value as of a certain date.
- The **PV** calculation is useful in a variety of situations including:
  - valuing a series of retirement payments;
  - establishing the purchase price of property sold for installment payments;
  - determining the value of periodic payments under an insurance contract or a structured settlement;
  - pricing a coupon bearing bond (this is actually a composite calculation involving the present value of the periodic interest payments (an annuity) and the return of principal at maturity (a single sum).

Solving for Other Variables

- The **PV** of an annuity equation above can be rearranged algebraically to solve for the payment amount (**PMT**) that will amortize (pay off) a loan or equate to a current sales price.
- While the basic **PV** of an annuity formula presented above allows us to calculate **PV**, we often need to calculate one of the other variables in the equation such as the number of compounding periods (**n**), the payment amount (**PMT**), or the interest rate (**i**). These calculations are illustrated below.
- **a. Number of compounding periods (n)**
- Solving for **n** is a simple matter of algebraic rearrangement of the basic **PV** of an annuity formula for which the following algebraic identity is helpful...

If $x = b^y$ then $y = \dfrac{\ln x}{\ln b}$

Solving for Other Variables …

- …rearranging for **n**, the **PV** of an annuity equation looks like this…

$$n = -\frac{\ln\left(1 - \frac{i\,PV}{PMT}\right)}{\ln(1+i)}$$

- …and using the values for the other variables from our earlier example, we calculate the number of compounding periods (**n**) as…

$$n = -\frac{\ln\left(1 - \frac{PVi}{PMT}\right)}{\ln(1+i)} = -\frac{\ln\left(1 - \frac{(131.22)(0.07)}{50}\right)}{\ln(1.07)} = 3.0$$

b. Payment amount (PMT)

- Rearranging the basic **PV** of an annuity formula to solve for **PMT** is a little easier than it was for **n**. What we end up with looks like this..

$$PMT = \frac{iPV}{1-(1+i)^{-n}}$$

C. Interest rate (i): Unfortunately there is no easy way to isolate the interest rate (**i**) variable in the basic **PV** of an annuity equation. For practical purposes, **i** is normally computed using a calculator or computer program rather than through manual iteration

The present value (PV) of an annuity…

- The equation for the **PV** of an annuity presented at the top of this page assumes annual compounding.
- The same procedure used for discounting annuities for amounts deposited or paid annually can also be applied to monthly annuities A very simple modification can be made to the formulation used for annual annuities by substituting r / 12 in place of r and multiplying number of years by 12 and the formulation is as follows:
-

The future value of annuity

- The future value of an end-of-the-period annuity can also be calculated as follows-

$$\text{FV of an Annuity} = \text{FV}(A,r,n) = A\left[\frac{(1+r)^n - 1}{r}\right]$$

- The same procedure used for discounting annuities for amounts deposited or paid annually can also be applied to monthly annuities.
- A very simple modification can be made to the formulation used for annual annuities by substituting r / 12 in place of r and multiplying number of years by 12 and the formulation is as follows:
- Future value of Annuity (FV_A) $= \dfrac{(1+r/12)12^n - 1}{r/12}$

Present value of growing annuity

- In this case each cash flow grows by a factor of (1+g).
- Similar to the formula for an annuity, the present value of a growing annuity (PVGA) uses the same variables with the addition of *g* as the rate of growth of the annuity.
- This is a calculation that is rarely provided for on financial calculators
- **Where i ≠ g :**

$$PV = \frac{A}{(i-g)}\left[1-\left(\frac{1+g}{1+i}\right)^{n}\right]$$

To get the PV of a growing annuity due, multiply the above equation by (1 + *i*).

Where i = g :

$$PV = \frac{A*n}{1+i}$$

Perpetuity-definition

Definition:

A perpetuity is an annuity that provides payments indefinitely. Since this type of annuity is unending, its sum or future value cannot be calculated.

Examples of perpetuity:

Local governments set aside monies so that funds will be available on a regular basis for cultural activities.

A children's charity club set up a fund designed to provide a flow of regular payments indefinitely to needy children.

Therefore, what happens in perpetuity is that once the initial fund has been established the payments will flow from the fund indefinitely which implies that these payments are nothing more than annual interest payments.

Present value of perpetuity

The present value of a perpetuity can be calculated by taking the limit of the above formula as *n* approaches infinity. The bracketed term reduces to one leaving:

Recall the formula for PVA

$$PV \text{ of an Annuity} = \text{PV(A, r, n)} = \text{A}\left[\frac{1 - \dfrac{1}{(1+\text{r})^{\text{n}}}}{\text{r}}\right]$$

Where A = Annuity; r = Discount Rate; n = Number of years

When $n \rightarrow \infty$, the PV of a perpetuity (a perpetual annuity) formula becomes simple division.

$$PVP = \frac{A}{r}$$

Present value of growing perpetuity

When the perpetual annuity payment grows at a fixed rate (*g*) the value is theoretically determined according to the following formula

In practice, there are few securities with precise characteristics, and the application of this valuation approach is subject to various qualifications and modifications.

Most importantly, it is rare to find a growing perpetual annuity with fixed rates of growth and true perpetual cash flow generation.

Despite these qualifications, the general approach may be used in valuations of real estate, equities, and other assets.

Present value of growing perpetuity is given as: $PVP = \frac{A}{r - g}$

How many years is needed to double money

Solving for the period needed to double money

Using the algebraic identity that if:

$$x = b^y$$

then

$$y = \frac{\log(x)}{\log(b)}$$

The present value formula can be rearranged such that:

$$Y = \frac{\log\left(\frac{FV}{PV}\right)}{\log(1+r)}$$

Example : Consider a deposit of birr 100 placed at 10% (annual). How many years are needed for the value of the deposit to double to 200 birr?

$$y = \frac{\log\left(\frac{FV}{PV}\right)}{\log(1+i)} = \frac{\log\left(\frac{200}{100}\right)}{\log(1.10)} = 7.27$$

What return is needed to double money

What return is needed to double money?
Similarly, the present value formula can be rearranged to determine what rate of return is needed to accumulate a given amount from an investment.

$$i = \frac{\left(\frac{FV}{PV}\right)1/n}{n} - 1$$

For example, birr 100 is invested today and birr 200 return is expected in five years; what rate of return (interest rate) does this represent? The present value formula restated in terms of the interest rate is:

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1 = \left(\frac{200}{100}\right)^{\frac{1}{5}} - 1 = 2^{0.20} - 1 = 0.15 = 15\%$$

The rule 72

For the period of time needed to double an investment, the Rule of 72 is a useful shortcut that gives a reasonable approximation of the period needed

The Rule of 72 is a quick and simple technique for estimating one of two things:

1. the time it takes for a single amount of money to double with a known interest rate, or
2. the rate of interest you need to earn for an amount to double within a known time period.

The rule states that an investment or a cost will double when:
[Investment Rate per year as a percent] x **[Number of Years] = 72.**

The rule 72…

- When interest is compounded annually, a single amount will double in each of the following situations:
- When interest is compounded annually, a single amount will double in each of the following situations
- The Rule of 72 is an often useful tool that can be used to approximate how long it will take to double your money at a particular interest rate:
- Years to double money = 72 ÷ interest rate

The rule 72…

- So, using the rule we can see that at 10% it will take about 7.2 years to double your money:
  - Years to double money = 72 ÷ 10% = 7.2 years
- Alternatively, the rule can tell us what interest rate is needed to double your money in a particular number of years:
- Interest Rate to Double Money in N Years = 72 ÷ N So, to double your money every 5 years you would need to earn about 15% per year: Interest Rate to Double Money in 5Years = 72 ÷ 5 = 15% per year
- This rule can often be used to mentally check answers or to quickly see if a statement makes sense.

Accumulation of a future sum (Sinking fund)

- In addition to compounding and discounting single payments on annuities, it is necessary to determine a series of payments necessary to *accumulate a future sum*, taking into account the fact that such payments will be accumulating interest as they are deposited.
- In popular usage the term "sinking fund" is applied almost indiscriminately to any method of providing for repayment of a long-term loan during its life, by setting aside a predetermined amount at regular periods for that purpose.
- This process is known in more technical language as "amortization." In its proper sense, amortization includes four principal methods as follows:
- The amount of birr1 per annum calculates the future accumulations from the investment of birr1 at the end of every year accumulating at compound interest.

Accumulation of a future sum (Sinking fund) …

- Amount of birr1 per annum = $\frac{(1+r)^{n} - 1}{r}$
- Where the amount of birr1 per annum formula calculates the future total from an annual investment of birr1, the annual sinking fund formula calculates the amount of annual investment necessary in order to accumulate to birr1 in the future.
- The annual sinking fund is the reciprocal of the amount of birr1 per annum formula, as seen below. SF formula can be derived from the FV birr 1 pa formula.

Accumulation of a future sum (Sinking fund) …

- If *A* was invested at the end of each year at *r* rate of interest for *n* periods in order to accumulate to birr 1 at the end of the total number of periods, we can rearrange Equation
- FV of birr1 per annum = $A\frac{(1+r)^n - 1}{r}$
- to solve for *A*. Substituting birr1 as the amount to which the FV birr1 pa must accrue
- $1 = A\frac{(1+r)^n - 1}{r}$

$$A = \frac{r}{(1+r)n - 1}$$

- *A* is the periodic amount that must be invested (the *SF*) to accumulate to birr1. The formula is the reciprocal of the FV birr1 pa formula

Accumulation of a future sum (Sinking fund) …

Sff $= \frac{r}{(1+r)^n - 1}$

- Annual Sinking Fund
  - ✦ the annual sinking fund may be used to calculate the annual amount to be set aside to meet a known future liability or expense.
  - ✦ This is the annual sum, SF, required to be invested at the end of each year to accumulate to birr1 in n years at i compounded interest.
  - ✦ Since the present value of birr1 is the reciprocal of the future amount of birr1, so that the Annual Sinking Fund is the reciprocal of the Future Amount of birr1 pa. That is $SF = \frac{r}{(1+r)^n - 1}$
- The purpose of this formula is to calculate how much should be saved (the sinking fund) at the end of each year in order to accumulate to a specific required sum in the future.

Application of Annuity and Sinking fund : Methods of loan payment

Loan is an arrangement in which a lender gives money or property to a borrower and the borrower agrees to return the property or repay the money, usually along with interest, at some future point(s) in time.

Usually, there is a predetermined time for repaying a loan, and generally the lender has to bear the risk that the borrower may not repay a loan (though modern capital markets have developed many ways of managing this risk)

In the amortization method, the borrower repays the lender by means of installment payments at periodic intervals. Examples include car loan, mortgage repayment.

In the sinking fund method, the borrower repays the lender by means of a lump-sum payment at the end of the term of the loan. The borrower pays interest on the loan in installments over this period.

- It is also assumed that the borrower makes periodic payments into a fund, called a “sinking fund”, which will accumulate to the amount of the loan to be repaid at the end of the term of the loan.

Amortization - Outstanding balance

- At each moment of time, you need to know the present value of your outstanding liabilities. This is of great practical importance.
- Suppose that you have just paid the installment number t. If you want to completely repay your loan, in addition to the current installment, you must pay the present value of the remaining future installments,
- **Sinking fund method**
- Institutional borrowers have higher credibility than individual borrowers, so they can be trusted to repay the loan in one lump sum at the end of the loan term.
- They may also make smaller regular payments to pay the interest accumulated over the year.

Determining Rates of Return or Yields on Investments

- Up to now, this unit has demonstrated how to determine future values in the case of compounding and present values in the case of discounting.
- In other words, the concepts illustrated in the compounding and discounting processes can also be used to determine rates of return, or yields, on investments, mortgage loans, and so on.
- These concepts must be mastered because procedures used here will form the basis for much of what follows in succeeding units.
- We have concentrated previously on determining the future value of an investment made today when compounded at some given rate of interest, or the present investment value of a stream of cash returns received in the future when discounted at a given rate of interest.
- In this lecture, we are concerned with problems where we know what an investment will cost today and what the future stream of cash returns will be, but we do not know what yield or rate of return will be earned if the investment is made.

Determining Rates of Return or Yields on Investments…

- Determining your **return on investment** is a very important part of any investment review.
- Whether you're investing in savings accounts, stocks, real estate, capital upgrades, or new business ventures, estimating a return on investment will aid you in choosing among investment options.
- Steps on how you can calculate your return on investment in terms of APY:
- **Step 1. Calculate all the costs associated with an investment.**
- Ask yourself the following questions about the investment:
  - What are the initial upfront costs associated with the investment?
  - What are the maintenance costs?
  - Are there any fees or taxes associated with the investment?
  - What kind of research costs will you incur to properly evaluate (*i.e.*, due diligence) the investment?
  - How much of your time will this investment consume? Your time is valuable, so a complicated project could have real opportunity costs.

Determining Rates of Return or Yields on Investments…

- **Step 2. Estimate or calculate your returns.**
  - How much do you expect to gain from the investment?
  - When do you expect returns to happen?
- Determine how much you expect to gain from the investment. Detail specifically all the individual returns you expect to receive from the investment.
- Since returns from investments are often uncertain, you might also want to dot down what you think the probability of each return occurring when you thought they would.
- Be sure to also specify when these returns will occur and for how long. This will be important for the next step.

Determining Rates of Return or Yields on Investments…

- **Step 3. Establish a timeline for costs and returns.**
- Draw a simple timeline or just list in chronological order all the costs and returns you discovered in steps 1 and 2. Costs should be listed as negative dollars and returns as positive dollars.
- For example:
  - 1/1/2006 Initial investment cost: - birr100,000
  - 9/21/2006 Sell investment: birr120,000
- **Step 4. Calculate annualized return of investment or APY.**
  - This is the meat of the process and the most challenging step of calculating the return on an investment.

Determining Rates of Return or Yields on Investments…

- Example Let's assume you made an investment on January 1, 2006. That investment cost you birr100,000 including fees. Today, September 21, 2006, you decide to sell that investment and you receive birr120,000 after all expenses.
- Now we want to find the APY of this investment so we can compare it to other investments or even your savings account rate.
- In finance, this is often called calculating your **internal rate of return**.
- Technically this process involves determining a **discount rate** at which the **present value** of a series of investments is equal to the present value of the returns on those investments. For a simple situation like the example above:

Let me show you a two examples:

Example #1 – Monthly returns

Date		Payment	Note
1-Jan-2006	$	(100,000)	Initial investment
1-Feb-2006	$	10,000	Dividend/profit
1-Mar-2006	$	8,000	Dividend/profit
1-Apr-2006	$	6,000	Dividend/profit
1-May-2006	$	4,000	Dividend/profit
1-Jun-2006	$	1,000	Dividend/profit
1-Sep-2006	$	(1,000)	Taxes
31-Dec-2006	$	100,000	Sell investment
	$	28,000	Profit
		36.3%	**APY**

Example #2 – Yearly returns

Date		Payment	Note
1-Jan-2006	$	(100,000)	Initial investment
1-Jan-2007	$	10,000	Dividend/profit
1-Jan-2008	$	8,000	Dividend/profit
1-Jan-2009	$	6,000	Dividend/profit
1-Jan-2010	$	4,000	Dividend/profit
1-Jan-2011	$	1,000	Dividend/profit
1-Jan-2012	$	(1,000)	Taxes
1-Jan-2013	$	100,000	Sell investment
	$	28,000	Profit
		4.3%	**APY**

Notice how the return in Example #1 is much higher than that of Example #2 even though the payment amounts and total profit are the same. The key difference is the timing of the payments.

Because the returns in Example #1 come monthly, the annualized rate of return or APY is much higher. In Example #2, profit dividends only come in annually. That dramatically reduces the APY.

Concepts that you should be clear with : Rate of Return, interest rate, yield

Rate of Return

- The rate of return is an internal measure of the return on money invested in a project.
  - The rate of return is the rate at which the project's discounted profits equal the upfront investment.
  - Consider a project that requires an upfront investment of birr100 and returns profits of birr65 at the end of the first year and birr75 at the end of the second year.
  - When birr65 and birr75 are discounted at 25 percent compounded annually, the sum is birr100. In this case, the internal rate of return equals 25 percent.

Concepts that you should be clear with : Rate of Return, interest rate, yield…

- **Interest Rate:** The interest rate is the external rate at which money can be borrowed from lenders.
  - The interest rate is the rate charged by a lender on a loan for the project. The interest rate is based on the borrower's credit rating and the bank's assessment of project feasibility and profits.
- **Yield:** The yield represents the total annual amount earned in interest expressed as a percent.
- **Considerations:** Loan financing makes sense if the internal rate of return is higher than the interest rate. If the rate of return is 25 percent and the bank charges 15 percent, the project will be profitable even after paying off interest expenses..

Concepts that you should be clear with : Rate of Return, interest rate, yield…

- Yield is a term also used for other investments, including stocks. Here, it also represents the total earnings as a percent of the base investment

Function

- Use rate of return to select projects competing for investment dollars.
- Banks require information on a project's investment, profits and rate of return before approving loans. Banks are more likely to lend money if the rate of return is higher than the bank's interest rate.
- Interest rates that are compounded during the year earn more overall. The annualized yield represents the actual earnings.

Concepts that you should be clear with : Rate of Return, interest rate, yield…

- When selecting the best investment from competing alternatives having similar risk, an investor should prefer the one having the largest expected rate of return.
- For direct comparison, each expected rate of return should be calculated in the same way. What is a satisfactory measure of the expected rate of return? Consider,
- For example, the consequences of placing birr1,000 on deposit with a savings institution that promises annual interest of 6 percent compounded quarterly
- If, at the end of the year, the depositor receives birr 61.36 interest, has the depositor received a 6 Percent return, compounded quarterly? Yes;

Concepts that you should be clear with : Rate of Return, interest rate, yield…

- however, if the investor received the birr61.36 interest but could not recover the initial birr1, 000 deposit because the savings institution failed, what is the return?
- Although highly unlikely, under these circumstances the depositor would not receive a 6 percent return—in fact, the return is negative.
- An investment's rate of return is a combination of the **return on** and the **return of** the investment.

Thank You!!!