**UNIT FIVE**

**5. DECISION THEORY/ANALYSIS**

**Unit objective:**

After completing this unit, the learner should be able to:

* **D**escribe the basic characteristics of decision theory problems
* Differentiate between decision analysis under certainty and uncertainty.
* Describe the different approaches (criteria) to decision making under complete uncertainty.
1. **Maximamax**
2. **Maximin**
3. **Minimax regret**
4. **Hurwicz**

**What is decision making?**

 Decision making is the thinking process of **picking a choice between the available options.** It serves as a **purpose to remove any uncertainty** before we decide on something. The best decisions usually are the decision that can bring **success and effective results**. When doing decision making, we must look at both **positives and negatives of each option**. We should always think and consider all the **alternatives before making up a decision.** Sometimes there might be a better alternative choice that we did not put under consideration, therefore it is important for us to always keep track and record down the reason why decision are made. These tracks and record can be used in the future for reference and improvements purpose, and saves us a lot of time by preventing any past mistakes. Wikipedia (2012) explains that:

Decision making can be regarded as the **mental processes (cognitive process**) resulting in the **selection of a course of action among several alternative scenarios**.

## Decision Making Environment

The quality of the decisions made in an [organization](https://www.toppr.com/guides/business-management-entrepreneurship/organizing/structure-of-organization/) will dictate the success or failure of the said business.

Among other factors that affect these decisions is **the environment** in which they are taken. There are different types of [environments](https://www.toppr.com/guides/geography/environment/environment/) in which these decisions are made. And the type of decision making environment **has an impact on the way the decision is taken.** Broadly there are **three basic** types of decision making environment. Let us take a brief look at each of them.

### ****1] Certainty****

Such type of environment is **very sure and certain by its nature.** This means that all the **information is available and at hand**.

So the manager has **all the information** he/she may need to make an informed and well thought out decision. All the alternatives and their outcomes can also be analyzed and then the manager chooses the best alternative.

### ****2] Uncertainty****

In the decision making environment of uncertainty, the information available to the manager is **incomplete, insufficient and often unreliable**.

In an uncertain environment, everything is in a **state of flux**. Several external and random forces mean that the environment is most **unpredictable.**

In these times of chaos, all the **variables change fast**. But the manager has to make sense of this mayhem to the best of his/her ability. He/she must create some order, obtain some reliable data and make the best decision as per his [**judgment**](https://www.toppr.com/guides/reasoning-ability/statements/judgements/)**.**

### ****3] Risk****

Under the condition of risk, there is the **possibility of more than one event taking place**. Which means the manager has to first **ascertain the possibility and**[**probability**](https://www.toppr.com/guides/maths/probability/introduction-to-probability/)**of the occurrence or non-occurrence of the event.** The manager will generally rely on past experiences to make this deduction.

In this scenario too, the manager has some information available to him/her. But the **availability and the reliability of the information is not guaranteed.** He/she has to chart a few alternative courses of actions from the data he/she has.

5.1. Characteristics of Decision Theory

Decision theory problems are characterized by the following:

1. **List of alternatives**: are a set of mutually exclusive and collectively exhaustive decisions that are available to the decision maker (some times, not always, one of these alternatives will be to “do nothing”.)
2. **States of nature**: the set of possible future conditions, or events, **beyond the control of the decision maker,** that will be the **primary determinants** of the eventual consequence of the decision.
3. **Payoffs**: the payoffs might be **profits, revenues, costs,** or other measures of value. Usually the measures are financial. Usually payoffs are estimated values. The more accurate these estimates, the more useful they will be for decision making purposes and the more likely, it is that the decision maker will choose an appropriate alternative.
4. **Degree of certainty**: the approach often used by a decision maker depends on the degree of certainty that exists. There can be different degrees of certainty. One extreme is complete certainty and the other is complete uncertainty. The later exists when the likelihood of the various states of nature are unknown. Between these two extremes is risk (probabilities are unknown for the states of nature).
5. **Decision criteria**: the decision maker’s attitudes toward the decision as well as the degree of certainty that surrounds a decision. Example; **maximize/minimize** the expected payoffs.

 **5.2**. **THE PAYOFF TABLE**

A payoff table is **a device a decision maker** can use to summarize and organize information relevant to a particular decision. It includes a **list of alternatives**, the possible **future states** **of nature**, and the **payoffs** associated with each of the alternative/state of nature combinations. If **probabilities** for the states of nature are available, these can also be listed. The general format of the table is illustrated below:

 **States of nature**

 S1 S2 S3

|  |  |  |
| --- | --- | --- |
| V11 | V12 | V13 |
| V21 | V22 | V23 |
| V31 | V32 | V33 |

 A1

**Alternatives**  A2

 A3
 where:

 Ai = the ith alternative

 Sj = the jth states of nature

 Vij = the value or payoff that will be realized if alternative i is chosen and event j

 occurs.

 **5.3. DECISION MAKING UNDER CERTAINTY**

The simplest of all circumstances occurs when decision making takes place in an **environment of complete certainty.** When a decision is made under conditions of complete certainty, the attention of the decision maker is focused on the column in the payoff table that corresponds to the state of nature that will occur. The decision maker then selects the alternative that **would yield the best payoff**, given that state of nature.

**EXAMPLE**

 The following payoff table provides data about profits of the various states of nature/alternative combination.

 S1 S2 S3

|  |  |  |
| --- | --- | --- |
| 4 | 16 |  12 |
| 5 | 6 |  10 |
| -1 | 4 |  15 |

 A1

 A2

 A3
If we know that S2 will occur, the decision maker then can focus on the first raw of the payoff table. Because alternative A1 has the largest profit (16), it would be selected.

**5.4**. **DECISION MAKING UNDER COMPLETE UNCERTAINTY (With out probabilities)**

Under complete uncertainty, the decision maker **either is unable to estimate the probabilities for the occurrence of the different state of nature,** or else he or she lacks confidence in available estimates of probabilities, and for that reason, probabilities are not included in the analysis.

A decision making situation includes several components- the decision themselves and the actual event that may occur future, known as state of nature. At the time the decision is made, the decision maker is **uncertain which state of nature** will occur in the future, and has no control over them.

 There are several approaches (criteria) to decision making under complete uncertainty. Some of these discussed in this section include: **maximax, maximin, minimax regret, Hurwicz,** and **equal likelihood.**

**5.4.1. MAXIMAX**

With the maiximax criterion, the decision maker selects the decision that will result in the **maximum of the maximum payoffs** The maximax is very **optimistic.** The decision maker assumes that the most favorable state of nature for each decision alternative will occur. For example, the investor would **optimistically** assume that **good economic conditions** will prevail in the future.

 For the previous problem:

 S1 S2 S3 Row Maximum

|  |  |  |  |
| --- | --- | --- | --- |
| 4 | 16 | 12 | 16\*maximum |
| 5 | 6 | 10 | 10 |
| -1 | 4 | 15 | 15 |

 A1

 A2

 A3
 **Decision**: A1 will be chosen.

☞**Note**: If the pay off table **consists of costs** instead of profits, **the opposite selection would be indicated:** The minimum of minimum costs. For the subsequent decision criteria we encounter, the same logic in the case of costs can be used.

**5.4.2. Maximin Criteria**

This approach is the opposite of the previous one, i.e. it is **pessimistic**. This strategy is a **conservative one**; it consists of identifying the **worst (minimum) payoff** for each alternative, and, then, selecting the alternative that has the **best (maximum) of the worst payoffs.** In effect, the decision maker is setting a floor on the potential payoff by selecting **maximum of the minimum;** the **actual payoff can not be less** than this amount. It involves **selecting best of the worst.**

 For the previous problem:

 S1 S2 S3 Row minimum

|  |  |  |  |
| --- | --- | --- | --- |
| 4 | 16 | 12 | 4 |
| 5 | 6 | 10 | 5\*maximum |
| -1 | 4 | 15 | -1 |

 A1

 A2

 A3
**Decision**: A2 will be chosen.

☞**Note**: If it were cost, the conservative approach would be to select the maximum cost for each decision and select the minimum of these costs.

**5.4.3. MINIMAX REGRET**

Both the maximax and maximin strategies can be **criticized because they focus only on a single, extreme payoff and exclude the other payoffs.** Thus, the maximax strategy ignores the possibility that an alternative with a slightly smaller payoff might offer a better overall choice.

An approach that **does take all payoffs in to consideration** is Minimax regret. In order to use this approach, it is necessary to develop an **opportunity loss table**. Hence, opportunity loss amounts are **found by identifying the best payoff in a column and, then, subtracting each of the other values in the column from that payoff.** Therefore, this decision avoids the greatest regret by selecting the decision alternative that minimizes the maximum regret. This technique **depends on the state of nature** rather than alternatives.

 **EXAMPLE:**

 S1 S2 S3

|  |  |  |
| --- | --- | --- |
| 4 | 16 | 12 |
| 5 | 6 | 10 |
| -1 | 4 | 15 |

 A1

 A2

 A3

**Opportunity loss table**:

 S1 S2 S3

|  |  |  |
| --- | --- | --- |
| 5-4=1 | 16-16=0 | 15-12=3 |
| 5-5=0 | 16-6=10 | 15-10=5 |
| 5-(-1)=6 | 16-4=12 | 15-15=0 |

 A1

 A2

 A3

The **values in an opportunity loss** table can be **viewed as potential “regrets”** that might be **suffered as the result of choosing various alternatives.** A decision maker could **select an alternative** in such a way as to **minimize the maximum possible regret**. This requires **identifying the maximum opportunity loss** in each row and, then, **choosing the alternative that would yield the best (minimum) of those regrets.**

 S1 S2 S3 Max. Loss

|  |  |  |  |
| --- | --- | --- | --- |
| 5-4=1 | 16-16=0 | 15-12=3 | 3\*minimum |
| 5-5=0 | 16-6=10 | 15-10=5 | 10 |
| 5-(-1)=6 | 16-4=12 | 15-15=0 | 12 |

 A1

 A2

 A3

**Decision**: A1 will be chosen.

Although this approach makes use of more information than either Maximin or Maximax, it still ignores some information, and, therefore, can lead to a poor decision.

 **EXAMPLE:**

 Opportunity loss table

 S1 S2 S3 S4 Max. Loss

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0 | 0 | 0 | 24 | 24 |
| 15 | 15 | 15 | 0 | 15\*minimum |
| 15 | 15 | 15 | 0 | 15\*minimum |

 A1

 A2

 A3

**5.4.4. PRINCIPLE OF INSUFFICIENT REASON/ Equal likelihood/ Laplace**

The Minimax regret criterion’s weakness is the **inability to factor row differences**.

The principle of insufficient reason offers a method that incorporates more of the information. It treats the **states of nature as if each were equally likely**, and it focuses on the **average payoff** for each row, selecting the alternative that has the **highest row average.**

 **EXAMPLE**

 S1 S2 S3 S4 S5 **Row Average**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 28 | 28 | 28 | 28 | 4 | 23.2\*maximum |
| 5 | 5 | 5 | 5 | 28 | 9.6 |
| 5 | 5 | 5 | 5 | 28 | 9.6 |

 A1

 A2

 A3

**5.4.5. The Hurwitz Criterion**

The Hurwitz criterion **strikes a compromise between the maximax and maximin** criterion. The principle underlying this decision criterion is that the decision maker is **neither totally optimistic, nor totally pessimistic.** With Hurwitz criterion, the **decision payoffs are weighted by a coefficient of optimism,** a measure of a decision maker’s **optimism.** The coefficient of optimism, which is defined as α, is between zero and one (0< α<1). **If α = 1,** then the decision maker is said to be completely **optimistic,** if **α= 0,** then the decision maker is completely **pessimistic.** Given this definition, if **α is coefficient of optimism, 1-α is coefficient of pessimism.**

The Hurwitz criterion requires that for each alternative, the **maximum** payoff is multiplied by **α** and the **minimum payoff be multiplied by 1-α.**

Example: If α = 0.4 for the above example,

A1 = (0.4x16) + (0.6x4)

 = 8.8

A2 = (0.4x10) + (0.6x5)

 = 7

A3 = (0.4x15) – (0.6x1)

 = 5.4

 **Decision:** A1 is selected

A limitation of Hurwicz criterion is the fact that α must be determined by the decision maker. Regardless of how the decision maker determines α, it is still a completely a subjective measure of the decision maker’s degree of optimism. Therefore, Hurwicz criterion is a completely subjective decision making criterion.

**5.5. DECISION MAKING UNDER RISK (WITH PROBABILITIES)**

Dear learner, the decision making criteria just presented were based on the assumption **that no information regarding the likelihood of the states of the nature was available.** Thus, **no probabilities of occurrence were assigned to the states of nature,** except in the case of the equal likely hood criterion.

It is often possible for the decision maker to know enough about the future state of nature to **assign probabilities** to their occurrences. The term risk is often used in conjunction **with partial uncertainty, presence of probabilities for the occurrence of various states of nature.** The probabilities may be subjective estimates from managers or from experts in a particular field, or they may reflect historical frequencies. If they **are reasonably correct**, they provide the decision maker with **additional information that can dramatically improve** the decision making process.

Given that probabilities can be assigned, several decision criteria are available to aid the decision maker. Some of these are discussed below.

**5.5.1. EXPECTED MONETARY VALUE (EMV**)

The EMV approach provides the decision maker with a **value which represents an average payoff** for each alternative. The **best alternative** is, then, the one that has the **highest EMV**. The average or expected payoff of each alternative is a weighted average:

Note: the **sum of the probabilities** for all states of nature **must be 1.**

 **EXAMPLE:**

 Probability 0.20 0.50 0.30

 S1 S2 S3 **Expected payoff**

|  |  |  |  |
| --- | --- | --- | --- |
| 4 | 16 | 12 | 12.40\*maximum |
| 5 | 6 | 10 | 7 |
| -1 | 4 | 15 | 6.30 |

 A1

 A2

 A3
 **Decision**: A1 will be chosen.

In this case to find the best **multiply the probabilities with respective points and sum up**. Then find the **maximum value which becomes the best pay off.**

☞Note that it does not necessarily follow that the decision maker will receive a payoff equal to the expected monetary value of a chosen alternative. Simply a long-run average amount; the approximate average amount one could reasonably anticipate for a large number of identical situations.

**5.5.2. Expected Opportunity Loss (EOL)**

 Hence, the **opportunity losses** for each alternative are weighted by the probabilities of their respective state of nature to compute a long run average opportunity loss, and the alternative with the **smallest expected loss is selected as the best choice.**

It can be calculated by **taking the results of minmax regeret loss table results and multiplying with respective given probabilities. The lowest alternative will be selected since is loss matrix.**

EOL (A1) = 0.20(1) + 0.50(0) + 0.30(3) = 1.10 \*minimum

EOL (A2) = 0.20(0) + 0.50(10) + 0.30(5) = 6.50

EOL (A3) = 0.20(6) + 0.50(12) + 0.30(0) = 7.20

☞**Note:** The EOL approach resulted in **the same alternative as the EMV** approach

(**Maximizing** the payoffs is **equivalent to minimizing** the opportunity losses).

**5.5.3. Expected Value of Perfect Information (EVPI)**

 It can some times be useful for a decision maker to determine the potential benefit of knowing for certain which state of nature is going to prevail. The EVPI is the measure of the difference between the certain payoffs that could be realized under a condition involving risk.

**EXAMPLE:**

 Probability 0.20 0.50 0.30

 S1 S2 S3 **EPwoPI**

|  |  |  |  |
| --- | --- | --- | --- |
| 4 | 16 | 12 | 12.40\*maximum |
| 5 | 6 | 10 | 7 |
| -1 | 4 | 15 | 6.30 |

In case of EPwPI, the decision maker take the above table, knows that S1 will occur, A2 would be chosen with a payoff of $5. Similarly for S2 $16 (for A1) and for S3, $15 (with A3) would be chosen.

 Hence, the expected payoff under certainty (EPC) would be:

 EPC = 0.20(5) + 0.50(16) + 0.30(15) = 13.50

The difference between this figure and the expected payoff under risk (i.e., the EMV) is the expected value of perfect information. Thus:

 EVPI = EPVwPI – EMVwoPI

 = 13.50 – 12.40 = 1.10

☞**Note:** The EVPI is exactly equal to the EOL. The EOL indicates the expected opportunity loss due to **imperfect information**, which is another way of saying the expected payoff that could be achieved by having perfect information.

**5.6. DECISION TREES**

Decision trees some times are used by decision makers to obtain a **visual portrayal** of decision alternatives and their possible consequences. The term gets its name from the tree-like appearance of the diagram.

 Decision tree format:

Decision tree, like probably tree is composed of squares, circles, and lines:

* The **squares** indicate de**cision points**
* **Circles** represent **chance events**( circles and squares are called **nodes**)
* The lines (branches) emanating from **squares** **represent alternatives**.
* The lines from **circles** represent **states of nature**
* The tree is read from right to left.

It should be noted that although decision trees represent an alternative approach to payoff tables, they are not commonly used for problems that involve a single decision. Rather, their greatest benefit lies in portraying sequential decisions (i.e., a series of chronological decisions). In the case of a single decision, constructing a decision tree can be cumbersome and time consuming.

**Example**

Pay off table for Real Estate investment

State of Nature

 Good economic Poor economic

Decision conditions conditions

(Purchase) 0.6 0.4

Apartment building 50,000 30,000

Office building 100,000 -40,000

Warehouse 30,000 10,000

1

Apartment building

Good economic conditions (0.60

Poor economic conditions (0.4)

 Poor economic

 conditions (0.4)

Good economic conditions (0.60

Poor economic conditions (0.0.4)

Warehouse

Office building

Good economic conditions (0.6

 $50,000

 $100,000

 -$40,000

 $30,000

 $10,000

$30,000

 For example, in the above figure, node 1 signifies a decision to purchase an apartment building. Office building, or ware house. The circles are probability nodes, and the branches emanating from them indicate the state of nature that can occur: good economic conditions or poor economic conditions. The decision tree represents the sequence of events in a decision situation. First, one of the three decision choices is selected at node 1. Depending on the branch selected, the decision maker ar­rives at probability node 2, 3, or 4, where one of the states of nature will prevail, resulting in one of six possible payoffs.

Determining the best decision using a decision tree involves computing the expected value at each probability node. This is ac­complished by starting with the final outcomes (payoffs) and work­ing backward through the decision tree toward node 1. First, the expected value of the payoffs is computed at each probability node.

EV(node 2) = .60($ 50,000) + .40($ 30,000) = $42,000

EV(node 3) = .60($100,000) + .40($-40,000) **= $44,000**

EV(node 4) = .60($ 30,000) + .40($ 10,000) = $22,000

These values are now shown as the **expected**payoffs from each of the **three branches** emanating from node 1 in figure below. Each of these three expected values at nodes 2, 3, and 4 is the outcome of a possible decision that can occur at node 1. Moving toward node 1, we select the branch that comes from the probability node with the **highest expected** payoff. In figure below, the branch corresponding to the **highest payoff, $44,000** is from node 1 to node 3. This branch represents the decision to purchase the **office building**. The decision to purchase the office building, with an expected payoff of $44,000, is the same result we achieved earlier using the expected value crite­rion.