CHAPTER ONE

INTRODUCTION

**1.2. History of Operations Research**

It is generally agreed that operations Research came is to existence as a discipline during World War II when there was a critical need to manage **scarce resources**. The term “Operations research” was coined as a result of research on military operations during this war. Since the war involved strategic and tactical problems which were greatly complicated, to expect adequate solutions from individual or specialists in a single discipline was unrealistic. Therefore, group of individuals who collectively were considered specialists in mathematics, Economics, statistics and probability theory, engineering, behavioral, and physical science were formed as special unit within the armed forces to deal with strategic and tactical problems of various military operations. The objective was the most effective utilization of most limited military resources by the use of quantitative techniques.

After the war ended, scientists who had been active in the military OR groups made efforts to apply the operations research approach to civilian problems, related to business, industry, research and development, and even won Nobel prizes when they returned to their peacetime disciplines.

There are **three important** **factors behind the rapid development in the use of OR**

1. The **economic and industrial boom after World War II** resulted in continuous mechanization, automation, decentralization of operations and division of management factors. This industrialization also resulted in **complex managerial** problems, and therefore application of operations research to managerial decision-making become popular.
2. Many **operation researchers continued** their research after war. Consequently, some important advancement were made in various operations research techniques: linear programming and its solution by a method known as simplex method, statistical quality control, dynamic programming, queuing theory and inventory theory were well developed during this time.
3. **Analytic power was** made available by **high-speed computers.** The use of computers made it possible to apply many OR techniques for practical decision analysis.

**1.3. Nature and Significance Operations Research**

The Operations research approach is particularly **useful** in **balancing conflicting objectives (goals or interests),** where there are many alternative courses of action available to the decision-makers. In a theoretical sense, the **optimum decision** must be one that is best for the organization as a whole.

In other words, operations research attempts to resolve the conflicts of interest among various sections of the organization and seeks the **optimal solution which may not be acceptable to one department but is in the interest of the organization as a whole.**

**1.4. Operation Research: Some definitions**

The British/Europeans refer to "operational research", the Americans to "Operations research" - but both are often shortened to just "OR" (which is the term we will use). Another term which is used for this field is "management science" ("MS"). The Americans sometimes combine the terms OR and MS together and say "OR/MS" or "ORMS". Yet other terms sometimes used are "industrial engineering" ("IE"), "Decision Science" ("DS”) and “problem solving”. In recent years there has been a move towards a standardization upon a single term for the field, namely the term "OR".

Because of the wide scope of application of operations research, giving a precise definition is difficult. However, a few definitions of OR are given below.

* Operations research is the **application of the methods of science** **to complex problems** in the direction and management of large systems of men, machines, materials and money in industry, business, government and defense. The distinctive approach is to develop a scientific model of the system incorporating measurements of factors such as chance and risk with which to predict and compare the out comes of alternative decisions, strategies, or controls.

 *Operations Research So*ciety, UK

* Operations research is concerned with scientifically defining **how to best design and operate man-machine systems** usually requiring the allocation of scarce resources.

 *Operations Research Society, America*

A few other definitions, which are commonly used and widely acceptable, are:

* Operations research is a **systematic application of quantitative methods**, techniques and tools to the analysis of problems involving the operation of systems.
* Operations research is essentially a **collection of mathematical techniques and tools** which in conjunction with systems approach, is applied to solve practical decision problems **of an economic or engineering nature.**
* Operation research seeks the **determination of the optimum course of action of a decision** problem under the restriction of limited resources.
* Operations research is the **application of a scientific approach** to solving management problems in order to help managers make better decisions.

**1.5. Features of Operations Research Approach**

From the previous discussions and various definitions of OR, important features or characteristics can be drawn. These features of OR approach to any decision and control problems can be summarized as:

 **1.5.1. Inter-disciplinary approach**

**Interdisciplinary teamwork** is essential because while attempting to solve a complex management problem, one person may not **have complete knowledge of all its aspects** such as economic, social, political, psychological, engineering, etc. This means we should not expect a desirable solution to managerial problems from a single individual or discipline. Therefore, a team of individuals specializing in mathematics, statistics, computer science, psychology, etc,

**1.5.2. Methodological Approach**

Operation research is the **application of scientific methods, techniques and tools** to problems involving the **operations of systems** so as to provide those in control of operations with optimum solutions to the problems.

☞***Note*:** A system is defined as an **arrangement of components** designed to **achieve a particular objective(s)** according to plan. The components may be either physical or conceptual or both but they share a unique relation ship with each other and with the overall objective of the system.

**1.5.3. Holistic Approach or Systems Orientate**

While arriving at a decision, an operation research team examines the relative importance of **all conflicting and multiple objectives** and the validity of claims of various departments of the organization from the perspective of the whole organization.

**1.5.4. Objectivistic Approach**

The OR approach seeks **to obtain an optimal solution** to the problem under analysis. For this, a measure of **desirability (or effectiveness)** is defined, based on objective(s) of the organization.

**1.5.5.** **Decision Making** – OR increases the **effectiveness** **of management decisions.** It is the decision science which helps management to make better decisions.

**1.5.6.** **Use of Computers:** OR often requires a **computer to solve the complex** **mathematical model** or to perform a large number of computations that are involved.

**1.5.7.** **Human factors**: In deriving quantitative solution, we do not consider human factors which doubtlessly play a great role in the problems. So study of the OR is incomplete without a study of human factors.

**1.6. Models and Modeling in Operations Research**

Both **simple and complex** systems can easily be studied by concentrating on some portion or **key features instead of concentrating on every detail of it.** This approximation or **abstraction,** maintaining only the **essential elements of the system**, which may be constructed in various forms by establishing relationships among **specified variables and parameters of the system**, is called a **model.** In general, models attempt to describe the essence of a **situation or activity by abstracting from reality** so that the decision- maker can study the relationship among relevant variables in isolation.

**A model is constructed to analyze and understand** the given system for the purpose of improving its performance. The **reliability of the solution obtained from a model** depends on the **validity of the model** in representing the system under study.

**1.6.1. Classification of OR Model**

 It can be in the form of a graph or chart, but most frequently an OR model consist of a set of mathematical relationships. These mathematical relationships are made up of numbers and symbols. There are many ways to classify models:

**i)** Classification based on **structure**

 a) **Physical Models**: These models provide a **physical appearance of the real object** under study **either reduced in size or scaled up**. Physical models are useful only in **design problems** because they are easy to observe, build, and describe. Since these models can not manipulated and are not very useful for prediction, problems such as portfolio analysis selection, media selection, production scheduling, etc cannot be analyzed by physical models.

b) **Symbolic models:** These models use **symbols (letters, numbers)** and functions to represent variables and their relationships to describe the properties of the system.

**ii)** Classification based on **function or purpose**

Models based on the **purpose of their utility** include:

a) **Descriptive models:** Descriptive models simply **describe some aspects of a situation,** based on **observation, survey, questionnaire** results or other available data of a situation and do not recommend anything. Example: Organizational chart, plant layout diagram, etc.

b) **Predictive Models:** These models indicate “If this occurs, then that follow”. They relate **dependent and independent variables and permit trying out**, “what if” questions. In other words, these models are used to **predict outcomes** due to a given set of alternatives for the problem.

For example, S = a + bA +cI is a model that describes how the sales (S) of a product changes in advertising expenditures (A) and disposal personal income (I). Here, a, b, and c are parameters whose values must be estimated.

**c) *Normative (Optimization) models:*** These models provide the **“best” or “Optimal”** solution to problems subject to **certain limitations on the use of resources**. These models provide recommended courses of action. These models are also called *prescriptive models*, because they prescribe what the decision maker ought to do.

**iii)** Classification Based **on Time Reference**

a) **Static Models:** Static models represent a **system at some specified time and do not account for changes over time.** For example, an **inventory model** can be developed and solved to determine an **economic order quantity** for the next period assuming that the demand in planning period would remain the same as that for today.

b) **Dynamic models:** In dynamic models, **time** is considered as one of the variables and allows **the impact of changes due to change in time.** Thus, sequences of interrelated decisions over a period of time are made to select the optimal course of action to optimize the given objective. Dynamic programming is an example of a dynamic model.

**iv)** Classification based **on Degree of certainty**

 a) **Deterministic Models:** If all the **parameters, constants and functional** relationships are **assumed to be known with certainty** when the decision is made, the model is said to be **deterministic.** Thus, in such a case, the **outcome associated with a particular** course of **action is known**.

b) **Probabilistic (Stochastic) models**: Models in which **at least one parameter or decision variable is a random variable** are called probabilistic (or stochastic). Since at least one decision variable is random, a dependant variable which is the function of independent variable(s) will also be random. This means consequences or pay off due to certain changes in the independent variable **can not be produced with certainty**.

**v)** Classification Based on **Method of solution or Quantification**

a***)* Heuristic Model:** These models employ some **sets of rules** which, though perhaps **not optimal,** do facilitate solutions of problems when applied in a consistent manner.

b) **Analytical Models**: These models have a **specific mathematical structure** and thus can be solved by known analytical or mathematical techniques. Any optimization model (which requires **maximization or minimization** of an objective function) is an analytical model.

c) **Simulation Models:** These models have a mathematical structure **but are not solved by applying mathematical techniques** to get a solution. Instead, a simulation model is essentially **a computer-assisted experimentation on a mathematical** structure of a real-life problem in order to describe and evaluate its behavior under certain assumptions over a period of time.

Simulation models are more **flexible than mathematical** ones and therefore, can be used to represent a complex system which otherwise **can not be represented mathematically**.

**1.6.2. Advantage of Models**

Models in general are used as an aid for analyzing complex problems. However, a model can also serve other purposes as:

1. A model **provides economy in representation of the realities** of the system. That is, models **help decision makers to visualize a system** so that he/she can understand the system’s structure or operation in a better any. For example, it easier to represent a factory **lay out on paper than to construct it**.
2. The **problem** can be **viewed in its entirety**, with all the components being considered simultaneously.
3. Models **serve as aids to transmit ideas and visualization** among people in the organization.
4. A model **allows us to analyze and experiment in a complex situation** to a degree that would be impossible in the actual system and its environment. For example, the experimental firing of satellite may be costly and require years of preparation.
5. Models simplify the investigation considerably and provide a **powerful and flexible for predicting the future state** of the process or system.

**1.6.3. Methodology/steps of Operations Research**

For effective use of OR techniques, it is essential to **follow some steps** that are helpful for decision-makers to make better solution. These are

**Step 1. Observation and defining a problem:** The first step in OR process is the **identification of a problem** that exists is a system (organization). The system must be continuously and closely observed so that problems can be identified as soon as they occur or anticipated.

**Step 2. Formulating a model**

Model formulation involves an **analysis of the system** under study, determining **objective of the** **decision-maker**, and **alternative course of action**, etc, so as to understand and describe, in precise terms, the problem that an organization faces.

The major steps which have to be taken in to consideration for **formulating the model** are:

* **Problem Components:** The **first component** of the problem to be defined is the **decision maker who is not satisfied** with the existing **state of affairs**.
* **Decision environment**

 It is desirable to know about the **resources** such as managers, employees equipments, etc which are required to **carry out the policies of the organization** considering the social and ecological environment in which the organization functions. Knowledge of such factors will help in modifying the initial set of decision-maker’s objectives.

* **Alternative courses of Action**

The problem arises only when there are **several courses of action available** for a solution. An **exhaustive list of course** of action can be prepared in process of going through the above steps of formulating the problem. Courses of action which are **not feasible** **with respect to objectives** and resources may be **ruled out.**

* **Measure of effectiveness**

A certain measure of effectiveness or **performance** is required in order **to evaluate the merit of the several courses of action.** The performance or effectiveness can be measured in different units such as birr (net profits), percentage (share of market desired), time dimension (service or waiting time).

* **Collecting Data and Constructing a Mathematical Model**

After the problem is clearly defined and understood, the next step is to collect required data and then formulate a mathematical model. Model construction consists of **hypothesizing relationships** between variables subject to and not subject to control by decision-maker. Certain basic components required in every decision problem model are:

* **Controllable (decision) Variables** - These are the issues or factors in the problem whose values are to be **determined (in the form of numerical values)** by solving the model. The possible values assigned to these variables are called decision alternatives (strategies or courses of actions).
* **Uncontrollable variable.** These are the factors **whose numerical value** depends up on the **external environment** prevailing around the organization. The values of these variables are **not under the control** of the decision-maker (state of nature).
* **Objective function**

 It is a representation of (i) the **criterion** that expresses the decision-maker’s manner of evaluating the desirability of alternative values of decision variables, and (ii) how that criterion is to be optimized (minimized or maximized. A stated objective helps to focus attention on what the problem actually is.

* **Constraints or Limitations**

These are the **restrictions** on the values of the decision variables. These restrictions can arise due **to limited resources** such as space, money, manpower, material, etc. The constraints may be in the form of equations or inequalities.

* **Functional relationships**

In a decision problem, the decision variables in the objective function and in the constraints are connected by a specific functional relationship. A general decision problem model might take the form:

Optimize (Max or Min) Z = f(x)

Subject to the constraints:

 gi(x) (≤, = ≥ ) bi; i = 1,2,…..m and x ≥ 0

Where, x = a vector of decision variables (x1, x2, x3, xn)

 f(x) = Criterion or objective function to be optimized

 gi(x) = the ith constraint

 bi = fixed amount of the ith resource

A model is referred to as a linear model if **all functional relationships among decision variables** X1, X2, Xn in f(x) and g(x) are of a linear form. But if one or more of the relationships are non – linear, the model is said to be a non-linear model.

* Parameters- These are **constants** in the functional relationships. Parameters can be deterministic or probabilistic in nature.

**Step 3. Solving the Mathematical Model**

 This involves **obtaining the numerical values** of decision variables. Obtaining these values depends on the specific form or type, of mathematical models. Solving the model requires the use of various mathematical tools and numerical procedures. In general, there are **two** categories of methods used for solving an OR model.

* **Optimization model:** These models yield the **best value** for the **decision variables** both for unconstrained and constrained problems. In constrained problems, these values simultaneously satisfy all of the constraints and provide an optimal or acceptable value for the objective function or measure of effectiveness. The solution so obtained is called the optimal solution to the Problem.
* **Heuristic Model:** These methods yield values of the variables that satisfy all the **constraints, but not necessarily provide optimal solution.** However, these values provide an acceptable value of the objective function.

Heuristic Methods are sometimes described as “**rules of thumb”** which work. These methods are used when obtaining optimal solution is either very time consuming or the model is complex.

**Difficulties in problem solving**

 Difficulties in problem solving arise due to lack of an appropriate methodology for it and psychological perceptions on the part of the problem solver.

|  |  |
| --- | --- |
| **(i) Failure to recognize the existence of a problem**  | **(iii) Failure to use all available information**  |
| - Some people tend to personalize problems- Problems arise in context which people have  had no experience.  | - The problem-solver fails to seek out information  |
| ii)Failure to define the correct problem * One situation may contain many intertwined problems.
* Obvious problems are often symptoms of much deeper problems.
* The inability to identify accurately what is going on can lead to inaccurate problem identification.
* Attitudes and beliefs can blind the problem solver to the real causes on undesirable situations.
 | iv) Failure to recognize or question assumptions. * it is assumed that there is a solution to every problem
* Rigid thinking blocks ones view point.
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**Step 4. Validating (Testing) the solution**

After solving the mathematical model, it is important to review the solution carefully to see that **values make sense and that the resulting decisions can be implemented.** Some of the **reasons** for validating the solution are:

* + 1. The mathematical model **may not have enumerated** all the limitations of the problem under consideration.
		2. Certain aspect of the problem may have been **overlooked, omitted or simplified**,
		3. The **data** may have been **incorrect estimated or recorded**, perhaps when entered in to the **computer.**

**Step 5. Implementing the solution**

The decision-maker has not only to identify good decision alternatives but also to select alternatives that are capable of being implemented. It is important to ensure that any solution implemented is continually reviewed and updated in the light of a changing environment.

**Step 6. Modifying the Model**

For a mathematical model to be useful, the degree to which it actually represents the system or problem being modeled must be established. If during validation, the solution cannot be implemented, one needs to

 (a) identify constraint that were omitted during the original problem formulation or

 (b) find if some of the original constraints were incorrect and need to be modified.

In all such cases, one must return to the **model formulation step and carefully make the appropriate modifications** to represent more accurately the given problem. A model must be applicable for a reasonable time period and should be updated from time to time, taking in to consideration the past, present, and future aspects of the problem.

**Step 7. Establishing control over the solution**

The **dynamic environment and changes** within the environment can have significant implications regarding the continuing validity of models and their solutions. Thus, a control procedure has to be established for detecting significant changes in decision variables of the problem so that suitable adjustments can be made in the solution without having to build a model every time a significant change occurs.

**1.7. Features of OR solution**

A solution that works but is quite expensive compared to the potential savings from its application should not be considered successful. Also a solution that is well within the budget but which does not accomplish the objective is not successful either. The following are features of good solution:

* **Technically appropriate:** The solution should work technically; meet the constraints and operate in the problem environment.
* **Reliable:** The solution must be useful for a reasonable period of time under the conditions for which it was designed.
* **Economically viable:** The economic value should be more that what it costs to develop and should be seen as wise investment in hiring OR talents.
* **Behaviorally appropriate:** The solution should be behaviorally appropriate and must remain valid for reasonable period of time within the organization.

**1.8. Basic Operations Research Models**

There is no unique set of problems which can be solved by using OR models or techniques. Some OR models or techniques include:

**Allocation Models**

Allocation models are used to **allocate resources** to activate in such a way that some measure of effectiveness (objective function) is optimized.

If the measure of effectiveness such as **profit, cost,** etc., is represented as a linear function of several variables and if **limitations of resources (constraints)** can be expressed as a system of linear inequalities or equalities, the allocation problem is classified as **linear programming problems.**

But, if the objective unction of **any or all constraints can not be expressed** as a system of linear equalities or inequalities, the allocation problem is classified as a **non-linear programming problem.**

**Inventory Model**

Inventory Models deal with the problem of **determination of how much to order at a point** in time and when to place an order. The main objective is to **minimize the sum of three conflicting inventory costs:** the Cost of holding or carrying extra inventory, the cost of shortage or delay in delivery of items when it is needed, a cost of ordering or set-up.

**Competitive (Game Theory) Model**

These models are used to characterize the behavior of **two or more opponents** (called players) who compete for the **achievement of conflicting goals.**

**Network Models**

These models are applied to the **management (planning, controlling and scheduling)** of large-scale projects. PERT/CPM Techniques help in identifying potential trouble spots in a project through the identification of the critical path. These techniques improve project **coordination and enable the efficient use of resources.** Network methods are also used to **determine time-cost trade off, resource allocation and updating of activity time.**

**Decision analysis Model**

These models deal with the **selection of an optimal course faction** given the possible pay offs and their associated probabilities of occurrence. These models are broadly applied to problems involving decision making under risk and uncertainty.

**1.9. Operations Research Techniques**

OR techniques can be loosely classified in to five categories.

 Management Science Techniques

Linear Mathematical Programming

Linear programming models

Graphic Analysis

Simplex Method

Post optimality

Transportation and assignment

Integer linear programming

Goal Linear Programming

Probabilistic Techniques

Probability

Decision Analysis

Game Theory

Markov Analysis

Queuing

Simulation

Forecasting

Inventory Techniques

Certain Demand

Uncertain Demand

Net work Techniques

Net work flow

CPM/PERT

Other linear and non-linear Techniques

Dynamic Programming

Break-even Analysis

Non-linear programming

*Source: Taylor, 1990, Introduction to Management Science, 3rd edition, Brown Publishe*